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Solving Semi-Fully Fuzzy Linear Programming Problems

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Abstract

The present paper aims to propose an alternative solution approach in obtaining the fuzzy optimal solution to a semi-fully fuzzy linear programming problem. In this paper, the semi-fully fuzzy linear programming problem is transformed into equivalent semi-fully interval linear programming problems. The solutions to these interval linear programming problems are then obtained with the help of linear programming technique. A set of seven random numerical examples has been solved using the proposed approach.

Keywords: Interval numbers, Fuzzy numbers, Linear programming.

1 | Introduction

Linear programming is a most widely and successfully used decision tool in the quantitative analysis of practical problems where rational decisions have to be made. In order to solve a Linear Programming Problem, the decision parameters of the model must be fixed at crisp values. But to model real-life problems and perform computations we must deal with uncertainty and inexactness. These uncertainty and inexactness are due to measurement inaccuracy, simplification of physical models, variations of the parameters of the system, computational errors etc. Interval and fuzzy analysis are an efficient and reliable tool that allows us to handle such problems effectively.

Several researchers have carried out investigations on the semi-fully fuzzy linear programming problems with interval numbers, triangular fuzzy numbers, trapezoidal fuzzy numbers, pentagonal fuzzy numbers, hexagonal fuzzy numbers, heptagonal fuzzy numbers, octagonal fuzzy numbers, nonagonal fuzzy numbers, decagonal fuzzy numbers, hendecagonal fuzzy numbers and dodecagonal fuzzy numbers.

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Ramesh and Ganesan [1] proposed the solution concepts of primal and dual linear programming problems involving interval numbers without converting them to classical linear programming problems. Nasseri et al. [4] proposed a method to find the fuzzy optimal solution of fully fuzzy linear programming problems with equality constraints having lr flat fuzzy numbers. Siddi [12] proposed a method for solving fuzzy linear programming problem with pentagonal fuzzy number by using a ranking function and compared the solutions with fully fuzzy linear programming problem. Ingle and Ghadle [13] presented fully fuzzy linear programming problem with hexagonal fuzzy number is solved by new ranking function. They converted the fully fuzzy linear programming problem to a crisp valued problem then can be solved using Simplex/Big-M method. Slevakumari and Tamilarasi [14] presented a paper aims at solving linear programming problems in which the parameters are octagonal fuzzy numbers with the help of robust ranking method. Das [16] proposed an approach to optimize the cost of transportation problem based on triangular fuzzy programming problem. Das et al. [17] presented a mathematical model for solving fully fuzzy linear programming problem with trapezoidal fuzzy numbers. Das et al. [17] presented a modified ranking function of linear programming problem directly approach to fuzzy environment.

In this paper, a new improved method for solving the semi-fully fuzzy linear programming problems is proposed. This new method finds the fuzzy optimal solution of semi-fully fuzzy linear programming problems. Moreover, the new method improves the existing methods for solving the interval transportation problems and fully fuzzy transportation problems [6] and [15].

In general, most of the existing techniques provide only crisp solutions for the semi-fully fuzzy linear programming problems. In contrast to most existing approaches, our method of transforming a fuzzy number into interval numbers is the first. So, our method proposed is the first. Also, the fuzzy optimal solution, obtained by using the new method mentioned, will always exactly satisfy the centers of all the constraints and some constraints.

The contributions of the present study are summarized as follows: (a) we introduce new technique for improve the methods for solving the semi-fully interval linear programming problems *Eq. (6)*. (b) We introduce a formulation of semi-fully fuzzy linear programming problems *Eq. (12)*. (c) According to the proposed approach, the *Eq. (12)* is converted into classical linear programming problems and/or interval linear programming problems. The integration of the interval optimal solutions of the sub-problems provides the fuzzy optimal solution of the problem *Eq. (12)*. (d) An algorithm for the new proposed method and is developed to find the fuzzy optimal solution of the problem *Eq. (12)*. (e) The complexity of computation is greatly reduced compared with commonly used existing methods in the literature.

The rest of this paper is organized as follows. In Section 2, some basic definition, arithmetic operations and semi-fully interval linear programming problems are reviewed. Furthermore, we attempt to introduce a formulation of semi-fully fuzzy linear problem. In Section 3, we propose a simple method for solving semi-fully fuzzy linear programming problems and a new fuzzy arithmetic on fuzzy or interval numbers. In Section 4, seven numerical examples are presented to illustrate the proposed method. Advantages of the proposed method over the existing methods are discussed in Section 5. Finally, concluding remarks and future research directions are presented in Section 6.

2 | Materials and Methods

In this section, some basic definitions, arithmetic operations for closed Intervals numbers and of linear programming problems involving interval and fuzzy numbers are presented.

2.1 | A New Interval Arithmetic

In this section, some arithmetic operations for two intervals are presented [1].

Let $\Re = \left\{ \bar{a} = \left[a^1, a^2 \right] : a^1 \le a^2, \; a^1, a^2 \in \mathbb{R} \right\}$ be the set of all proper intervals. We shall use the terms "interval" and "interval number" interchangeably. The mid-point and width (or half-width) of an interval number $\bar{a} = \left[a^1, a^2 \right]$ are defined as $m\left(\bar{a}\right) = \dfrac{a^2 + a^1}{2}$ and $w\left(\bar{a}\right) = \dfrac{a^2 - a^1}{2}$.

The interval number $\bar{a}$ can also be expressed in terms of its midpoint and width as

$$\bar{a} = \left[a^1, a^2 \right] = \langle m\left(\bar{a}\right), w\left(\bar{a}\right) \rangle = \langle \frac{a^2 + a^1}{2}, \frac{a^2 - a^1}{2} \rangle. \tag{1}$$

For any two intervals $\bar{a} = \left[a^1, a^2 \right] = \langle m\left(\bar{a}\right), w\left(\bar{a}\right) \rangle$ and $\bar{b} = \left[b^1, b^2 \right] = \langle m\left(\bar{b}\right), w\left(\bar{b}\right) \rangle$, the arithmetic operations on $\bar{a}$ and $\bar{b}$ are defined as:

$$\text{Addition: } \bar{a} + \bar{b} = \langle m\left(\bar{a}\right) + m\left(\bar{b}\right), w\left(\bar{a}\right) + w\left(\bar{b}\right) \rangle. \tag{2}$$

$$\text{Subtraction: } \bar{a} - \bar{b} = \langle m\left(\bar{a}\right) - m\left(\bar{b}\right), w\left(\bar{a}\right) + w\left(\bar{b}\right) \rangle. \tag{3}$$

$$\text{Multiplication: } \alpha\bar{a} = \begin{cases} \langle \alpha m\left(\bar{a}\right), \alpha w\left(\bar{a}\right) \rangle \text{ if } \alpha \ge 0 \\ \langle \alpha m\left(\bar{a}\right), -\alpha w\left(\bar{a}\right) \rangle \text{ if } \alpha < 0 \end{cases}. \tag{4}$$

$$\bar{a} \times \bar{b} = \begin{cases} \langle m\left(\bar{a}\right) m\left(\bar{b}\right) + w\left(\bar{a}\right) w\left(\bar{b}\right), m\left(\bar{a}\right) w\left(\bar{b}\right) + m\left(\bar{b}\right) w\left(\bar{a}\right) \rangle \text{ if } a^1 \ge 0, b^1 \ge 0 \\ \langle m\left(\bar{a}\right) m\left(\bar{b}\right) + m\left(\bar{a}\right) w\left(\bar{b}\right), m\left(\bar{b}\right) w\left(\bar{a}\right) + w\left(\bar{b}\right) w\left(\bar{a}\right) \rangle \text{ if } a^1 < 0, b^1 \ge 0 \\ \langle m\left(\bar{a}\right) m\left(\bar{b}\right) - w\left(\bar{a}\right) w\left(\bar{b}\right), m\left(\bar{b}\right) w\left(\bar{a}\right) - m\left(\bar{a}\right) w\left(\bar{b}\right) \rangle \text{ if } a^2 \le 0, b^1 \ge 0 \end{cases} \tag{5}$$

2.2 | Formulation of Semi-Fully Interval Linear Programming Problem

We consider the linear programming problem involving interval numbers as follows [1]:

$$\begin{cases} \text{Max / Min } \bar{Z}^{pq}\left(\bar{x}^{pq}\right) \approx \sum_{j=1}^{n} \bar{c}_j^{pq} \bar{x}_j^{pq} \\ \quad \text{Subject to the constraints} \\ \sum_{j=1}^{n} a_{ij} \bar{x}_j^{pq} \begin{pmatrix} \approx \end{pmatrix} \bar{b}_i^{pq}, \text{ for } i = 1, 2, \ldots, m. \end{cases} \tag{6}$$

Where:

I. p and q are integers $\left(\mathbb{N}\right)$ with $q \ge p$.

II. $\bar{c}_j^{pq} = \left[c_j^p, c_j^q \right]$ are non-negatives interval numbers.

III. $\bar{x}_j^{pq} = \left[x_j^p, x_j^q \right]$ and $\bar{b}_i^{pq} = \left[b_i^p, b_i^q \right]$ are unrestricted interval numbers.

IV. a_{ij} are real numbers $\left(\mathbb{R}\right)$.

Object function transformation.

$$\bar{Z}^{pq}\left(\bar{x}^{pq}\right)\approx\langle m\left(\bar{Z}^{pq}\left(\bar{x}^{pq}\right)\right),w\left(\bar{Z}^{pq}\left(\bar{x}^{pq}\right)\right)\rangle$$

$$Max/Min\bar{Z}^{pq}\left(\bar{x}^{pq}\right)\approx\sum_{j=1}^{n}\bar{c}_j^{pq}\bar{x}_j^{pq}=\sum_{j=1}^{n}\langle m\left(\bar{c}_j^{pq}\bar{x}_j^{pq}\right),w\left(\bar{c}_j^{pq}\bar{x}_j^{pq}\right)\rangle \text{ where}$$

$$m\left(\bar{c}_j^{pq}\bar{x}_j^{pq}\right)=\begin{cases} m\left(\bar{c}_j^{pq}\right)m\left(\bar{x}_j^{pq}\right)+w\left(\bar{c}_j^{pq}\right)w\left(\bar{x}_j^{pq}\right) \text{ if } x_j^{p}\geq 0 \\ m\left(\bar{c}_j^{pq}\right)m\left(\bar{x}_j^{pq}\right)+w\left(\bar{c}_j^{pq}\right)m\left(\bar{x}_j^{pq}\right) \text{ if } x_j^{p}\left\langle 0 \text{ and } x_j^{q}\right\rangle 0. \\ m\left(\bar{c}_j^{pq}\right)m\left(\bar{x}_j^{pq}\right)-w\left(\bar{c}_j^{pq}\right)w\left(\bar{x}_j^{pq}\right) \text{ if } x_j^{q}<0 \end{cases} \quad (7)$$

And

$$w\left(\bar{c}_j^{pq}\bar{x}_j^{pq}\right)=\begin{cases} m\left(\bar{c}_j^{pq}\right)w\left(\bar{x}_j^{pq}\right)+m\left(\bar{c}_j^{pq}\right)w\left(\bar{x}_j^{pq}\right) \text{ if } x_j^{p}\geq 0 \\ m\left(\bar{c}_j^{pq}\right)w\left(\bar{x}_j^{pq}\right)+w\left(\bar{c}_j^{pq}\right)w\left(\bar{x}_j^{pq}\right) \text{ if } x_j^{p}\left\langle 0 \text{ and } x_j^{q}\right\rangle 0. \\ m\left(\bar{c}_j^{pq}\right)w\left(\bar{x}_j^{pq}\right)-m\left(\bar{c}_j^{pq}\right)w\left(\bar{x}_j^{pq}\right) \text{ if } x_j^{q}<0 \end{cases} \quad (8)$$

Transformation of constraints.

$$\sum_{j=1}^{n}a_{ij}\bar{x}_j^{pq}\left(\approx\right)\bar{b}_i^{pq}\Leftrightarrow\sum_{j=1}^{n}\langle a_{ij}m\left(\bar{x}_j^{pq}\right),w\left(a_{ij}\bar{x}_j^{pq}\right)\rangle\left(\approx\right)\langle m\left(\bar{b}_i^{pq}\right),w\left(\bar{b}_i^{pq}\right)\rangle \text{ then}$$

$$\langle\sum_{j=1}^{n}a_{ij}m\left(\bar{x}_j^{pq}\right),\sum_{j=1}^{n}w\left(a_{ij}\bar{x}_j^{pq}\right)\rangle\left(\approx\right)\langle m\left(\bar{b}_i^{pq}\right),w\left(\bar{b}_i^{pq}\right)\rangle.$$

We can write the following remark

Remark 1. For $k\in\left[1,m\right]$, we have

- $\sum_{j=1}^{n}a_{kj}\bar{x}_j^{pq}=\bar{b}_k^{pq}$ *if and only if* $\sum_{j=1}^{n}a_{kj}m\left(\bar{x}_j^{pq}\right)=m\left(\bar{b}_k^{pq}\right)$ *and* $\sum_{j=1}^{n}w\left(a_{kj}\bar{x}_j^{pq}\right)=w\left(\bar{b}_k^{pq}\right)$.
- $\sum_{j=1}^{n}a_{kj}\bar{x}_j^{pq}\neq\bar{b}_k^{pq}$ *if and only if* $\sum_{j=1}^{n}a_{kj}m\left(\bar{x}_j^{pq}\right)=m\left(\bar{b}_k^{pq}\right)$ *and* $\sum_{j=1}^{n}w\left(a_{kj}\bar{x}_j^{pq}\right)\neq w\left(\bar{b}_k^{pq}\right)$.

Remark 2. For $k\in\left[1,m\right]$, we have

- $\sum_{j=1}^{n}a_{kj}\bar{x}_j^{pq}=\bar{b}_k^{pq}$ *if and only if the slack variable* $x_{n+k}^{pq}=0$.
- $\sum_{j=1}^{n}a_{kj}\bar{x}_j^{pq}\neq\bar{b}_k^{pq}$ *if and only if the slack variable* $x_{n+k}^{pq}\neq 0$.

From *Remark 1* and *2*, we can say that

$$\begin{cases} \text{Max / Min } \bar{Z}^{pq}\left(\bar{x}^{pq}\right) \approx \sum_{j=1}^{n} \bar{c}_j^{pq}\bar{x}_j^{pq} = \sum_{j=1}^{n} \langle m\left(\bar{c}_j^{pq}\bar{x}_j^{pq}\right), w\left(\bar{c}_j^{pq}\bar{x}_j^{pq}\right)\rangle \\ \qquad\qquad \text{Subject to the constraints} \\ \langle \sum_{j=1}^{n} m\left(a_{ij}\bar{x}_j^{pq}\right), \sum_{j=1}^{n} w\left(a_{ij}\bar{x}_j^{pq}\right)\rangle \left(\approx\right) \langle m\left(\bar{b}_i^{pq}\right), w\left(\bar{b}_i^{pq}\right)\rangle, \text{ for } i = 1,2,\ldots,m. \end{cases} \tag{9}$$

From *Eqs. (7), (8)* and *(9)*, we can get:

$$\begin{cases} \text{Max / Min } m\left(\bar{Z}^{pq}\left(\bar{x}^{pq}\right)\right) = \sum_{j=1}^{n} m\left(\bar{c}_j^{pq}\bar{x}_j^{pq}\right) = \sum_{j=1}^{n} m\left(\bar{c}_j^{pq}\right) m\left(\bar{x}_j^{pq}\right) \\ \qquad\qquad \text{Subject to the constraints} \\ \sum_{j=1}^{n} a_{ij} m\left(\bar{x}_j^{pq}\right) \begin{pmatrix} = \\ \geq \\ \leq \end{pmatrix} m\left(\bar{b}_i^{pq}\right), \text{ for } i = 1,2,\ldots,m. \end{cases} \tag{10}$$

Where $\sum_{j=1}^{n} w\left(a_{kj}\bar{x}_j^{pq}\right) = w\left(\bar{b}_k^{pq}\right)$ or $\sum_{j=1}^{n} w\left(a_{kj}\bar{x}_j^{pq}\right) \neq w\left(\bar{b}_k^{pq}\right)$ for $k \in \left[1, m\right]$.

And *Eq. (10)* is equivalent to

$$\begin{cases} \text{Max / Min } m\left(\bar{Z}^{pq}\left(\bar{x}^{pq}\right)\right) = \sum_{j=1}^{n} \left(\dfrac{c_j^p + c_j^q}{2}\right) x_j^{pq} \\ \qquad\qquad \text{Subject to the constraints} \\ \sum_{j=1}^{n} a_{ij} x_j^{pq} \begin{pmatrix} = \\ \geq \\ \leq \end{pmatrix} \dfrac{b_i^p + b_i^q}{2}, \text{ for } i = 1,2,\ldots,m. \end{cases} \tag{11}$$

Optimal solution according to the choice of the decision maker:

$\textit{Max / Min } \bar{Z}^{pq}\left(\bar{x}^{pq}\right) \approx \sum_{j=1}^{n} \bar{c}_j^{pq}\bar{x}_j^{pq}$ with $\bar{x}_j^{pq} = \left[x_j^{pq} - w\left(\bar{x}_j^{pq}\right), x_j^{pq} + w\left(\bar{x}_j^{pq}\right)\right]$ and

$w\left(\bar{x}_j^{pq}\right) = \dfrac{x_j^q - x_j^p}{2}$.

For $x_{n+k}^{pq} = 0$, we have $\sum_{j=1}^{n} a_{kj}\bar{x}_j^{pq} = \bar{b}_k^{pq}$ and $\sum_{j=1}^{n} w\left(a_{kj}\bar{x}_j^{pq}\right) = w\left(\bar{b}_k^{pq}\right)$.

2.3 | Formulation of Semi-Fully Fuzzy Linear Programming Problems

Since the fuzziness may appear in many ways for the parameters of linear programming models, hence the definition of fuzzy linear programming is not unique. One of these models is Semi-Fully Fuzzy Linear Programming problem where the coefficients in the objective function, the right hand side vector and the decision variables are a kind of fuzzy numbers, simultaneity. This paper is assigned to these type of problems. Consider the Semi-fully Fuzzy Linear Programming Problems as follows [3] and [4]:

$$\begin{cases} \text{Max / Min } \tilde{Z}(\tilde{x}) \approx \sum_{j=1}^{n} \tilde{c}_j \tilde{x}_j \\ \text{Subject to the constraints} \\ \sum_{j=1}^{n} a_{ij} \tilde{x}_j \begin{pmatrix} \approx \\ \\ \end{pmatrix} \tilde{b}_i, \text{ for } i = 1, 2, .., m. \end{cases} \tag{12}$$

Where non-negatives fuzzy numbers, and are unrestricted fuzzy numbers and are real numbers.

3 | Results

In this section, a solution procedure for solving the problem *Eq. (6)* via *Eq. (11)* is developed in the following steps:

Step 1. Construct the fuzzy linear programming problem *Eq. (12)*, and then convert it into an interval linear programming problem *Eq. (6)* based on the new arithmetic of fuzzy or interval numbers.

Step 2. Convert the problem *Eq. (6)* into the corresponding classical linear programming problems *Eq. (11)* based on the new arithmetic of fuzzy or interval numbers, and then solving *Eq. (11)*:

$$Max / Min\, \left(\bar{Z}^{pq}\left(\bar{x}^{pq}\right)\right) = \sum_{j=1}^{n} c_j^{pq} x_j^{pq} \text{ subject to the constraints } \sum_{j=1}^{n} a_{ij} x_j^{pq} \begin{pmatrix} = \\ \leq \\ \geq \end{pmatrix} \frac{b_i^q + b_i^p}{2}, x_j^{pq} \geq 0.$$

Step 3. Determine $w\left(\bar{x}_j^{pq}\right)$ with $\bar{x}_j^{pq} = \left[x_j^{pq} - w\left(\bar{x}_j^{pq}\right), x_j^{pq} + w\left(\bar{x}_j^{pq}\right)\right] = \left[x_j^p, x_j^q\right]$ for $j = 1, \ldots, n$ by applying the following conditions: if and only if the slack variable $x_{n+k}^{pq} = 0$ for $k \in \left[1, m\right]$. Considering the $\sum_{j=1}^{n} a_{kj} w\left(\bar{x}_j^{pq}\right) = w\left(\bar{b}_k^{pq}\right)$ following cases:

Case 1. t is odd or even:

I. If t is odd, then $p = q = \frac{t+1}{2}$ and $w\left(\bar{x}_j^{pq}\right) = 0$ with $w\left(\bar{x}_j^{pq}\right) = 0$ if and only if $x_j^{pq} = 0$, and go to Case 2.

II. If t is even, then $p = \frac{t}{2}$ and $q = \frac{t+2}{2}$ do:

a) If $x_j^{pq} = 0$, then $w\left(\bar{x}_j^{pq}\right) = 0$. Else, choose between (b) or (c) or (d):

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b) **Very important decision:** if $\sum_{j=1}^{n} a_{kj} w\left(\bar{x}_j^{pq}\right) = w\left(\bar{b}_k^{pq}\right)$ for all $k \in \left[1, m\right]$, then the current solution is optimal and go to Case 2.

c) **Very important decision:** if $\sum_{j=1}^{n} a_{kj} w\left(\bar{x}_j^{pq}\right) = w\left(\bar{b}_k^{pq}\right)$ for some $k \in \left[1, m\right]$, then the current solution is optimal and go to Case 2.

d) **Important decision:** choose an index k such that $\sum_{j=1}^{n} a_{kj} w\left(\bar{x}_j^{pq}\right) = w\left(\bar{b}_k^{pq}\right)$, then go to Case 2.

Case 2. For $p = q \neq \dfrac{t+1}{2}$, $p \neq \dfrac{t}{2}$ and $q \neq \dfrac{t+2}{2}$, then choose between (a) or (b) or (c):

a) **Very important decision:** if $\sum_{j=1}^{n} a_{kj} w\left(\bar{x}_j^{pq}\right) = w\left(\bar{b}_k^{pq}\right)$ for all $k \in \left[1, m\right]$ with $\left|x_j^{pq} - x_j^{(p+1)(q-1)}\right| + w\left(\bar{x}_j^{(p+1)(q-1)}\right) \leq w\left(\bar{x}_j^{pq}\right)$, then the current solution is optimal.

b) **Very important decision:** if $\sum_{j=1}^{n} a_{kj} w\left(\bar{x}_j^{pq}\right) = w\left(\bar{b}_k^{pq}\right)$ for some $k \in \left[1, m\right]$ with $\left|x_j^{pq} - x_j^{(p+1)(q-1)}\right| + w\left(\bar{x}_j^{(p+1)(q-1)}\right) \leq w\left(\bar{x}_j^{pq}\right)$, then the current solution is optimal.

c) **Important decision:** choose an index k such that $\sum_{j=1}^{n} a_{kj} w\left(\bar{x}_j^{pq}\right) = w\left(\bar{b}_k^{pq}\right)$ with $x_{n+k}^{pq} = 0$ and $\left|x_j^{pq} - x_j^{(p+1)(q-1)}\right| + w\left(\bar{x}_j^{(p+1)(q-1)}\right) \leq w\left(\bar{x}_j^{pq}\right)$ otherwise $w\left(\bar{x}_j^{pq}\right) = \left|x_j^{pq} - x_j^{(p+1)(q-1)}\right| + w\left(\bar{x}_j^{(p+1)(q-1)}\right)$.

3.1 | Solution Procedure for Semi-Fully Interval Linear Programming Problem (t= 2)

For all the rest of this paper, we will consider the following semi-fully interval linear programming problem as follows *Eq. (6)* and $\left(t=2\right)$ [4] where $\bar{x}_j^{12} = \left[x_j^1, x_j^2\right]$ and $\bar{b}_1^{12} = \left[b_i^1, b_i^2\right]$.

The steps of our method for solving the semi-fully interval linear programming problem as follows *Eqs. (6) and (11)*:

Step 1. Solving *Eq. (6)* via *Eq. (11)*. We have $p = \dfrac{t}{2} = 1$ and $q = \dfrac{t+2}{2} = 2$. We get $\bar{x}_j^{12} = \left[x_j^{12} - w\left(\bar{x}_j^{12}\right), x_j^{12} + w\left(\bar{x}_j^{12}\right)\right] = \left[x_j^1, x_j^2\right]$ for $j = 1,\ldots,n$ and $Max / Min \left(\bar{Z}^{12}\left(\bar{x}^{12}\right)\right) = \sum_{j=1}^{n} c_j^{12} x_j^{12}$

subject to the constraints $\sum_{j=1}^{n} a_{ij} x_j^{12} \begin{pmatrix} = \\ \leq \\ \geq \end{pmatrix} \dfrac{b_i^2 + b_i^1}{2}, x_j^{12} \geq 0.$

Step 2. The optimal solution according to the choice of the decision maker is $Max / Min \bar{Z}^{12}\left(\bar{x}^{12}\right) \approx \sum_{j=1}^{n} \bar{c}_j^{12} \bar{x}_j^{12}$ with $\bar{x}_j^{12} = \left[x_j^1, x_j^2\right]$.

3.2 | Solution Procedure for Semi-Fully Fuzzy Linear Programming Problem with Triangular Fuzzy Numbers (t= 3)

For all the rest of this paper, we will consider the following semi-fully fuzzy linear programming problem with Triangular fuzzy numbers as follows *Eq. (12)* and $\left(t=3\right)$ [3] and [4] where $\tilde{x}_j = \left(x_j^1, x_j^2, x_j^3\right) = \left(x_j^2; \bar{x}_j^{13}\right)$ and $\tilde{b}_i = \left(b_i^1, b_i^2, b_i^3\right) = \left(b_i^2; \bar{b}_i^{13}\right)$. The steps of our method for solving the semi-fully fuzzy linear programming problem with Triangular fuzzy numbers as follows:

Step 1. Solving *Eq. (6)* via *Eq. (11)*. We have $p=q=\frac{t+1}{2}=2$. We get $\bar{x}_j^{22}=\left[x_j^2,x_j^2\right]=x_j^2$ for $j=1,\ldots,n$ and $Max/Min\,\Re\left(\bar{Z}^2\left(\bar{x}^2\right)\right)=\sum_{j=1}^{n}c_j^2x_j^2$ subject to the constraints $\sum_{j=1}^{n}a_{ij}x_j^2\begin{pmatrix}=\\ \leq\\ \geq\end{pmatrix}b_i^2,\ x_j^2\geq 0$.

Step 2. Solving *Eq. (6)* via *Eq. (11)*. We have $p=1,\ q=3$.
We get $\bar{x}_j^{13}=\left[x_j^{13}-w\left(\bar{x}_j^{13}\right),x_j^{13}+w\left(\bar{x}_j^{13}\right)\right]=\left[x_j^1,x_j^3\right]$ for $j=1,\ldots,n$ and

$$Max/Min\,\Re\left(\bar{Z}^{13}\left(\bar{x}^{13}\right)\right)=\sum_{j=1}^{n}c_j^{13}x_j^{13} \text{ subject to the constraints } \sum_{j=1}^{n}a_{ij}x_j^{13}\begin{pmatrix}=\\ \leq\\ \geq\end{pmatrix}\frac{b_i^3+b_i^1}{2},\ x_j^{13}\geq 0.$$

Step 3. The optimal solution according to the choice of the decision maker is Max/ $Min\,\tilde{\Re}\left(\tilde{x}\right)\approx\sum_{j=1}^{n}\tilde{c}_j\tilde{x}_j$ with $\tilde{x}_j=\left(x_j^2;\bar{x}_j^{13}\right)=\left(x_j^1,x_j^2,x_j^3\right)$.

3.3 | Solution Procedure for Semi-Fully Fuzzy Linear Programming Problem with Trapezoidal Fuzzy Numbers (t= 4)

For all the rest of this paper, we will consider the following semi-fully fuzzy linear programming problem with Trapezoidal fuzzy numbers as follows *Eq. (12)* and $(t=4)$ [3] and [4] where $\tilde{x}_j=\left(x_j^1,x_j^2,x_j^3,x_j^4\right)=\left(\bar{x}_j^{23};\bar{x}_j^{14}\right)$ and $\tilde{b}_i=\left(b_i^1,b_i^2,b_i^3,b_i^4\right)=\left(\bar{b}_i^{23};\bar{b}_i^{14}\right)$. The steps of our method for solving the semi-fully fuzzy linear programming problem with Trapezoidal fuzzy numbers as follows:

Step 1. Solving *Eq. (6)* via *Eq. (11)*. We have $p=\frac{t}{2}=2,q=\frac{t+2}{2}=3$. We get
$\bar{x}_j^{23}=\left[x_j^{23}-w\left(\bar{x}_j^{23}\right),x_j^{23}+w\left(\bar{x}_j^{23}\right)\right]=\left[x_j^2,x_j^3\right]$ for $j=1,\ldots,n$ and

$$Max/Min\,\Re\left(\bar{Z}^{23}\left(\bar{x}^{23}\right)\right)=\sum_{j=1}^{n}c_j^{23}x_j^{23} \text{ subject to the constraints } \sum_{j=1}^{n}a_{ij}x_j^{23}\begin{pmatrix}=\\ \leq\\ \geq\end{pmatrix}\frac{b_i^3+b_i^2}{2},\ x_j^{23}\geq 0.$$

Step 2. Solving *Eq. (6)* via *Eq. (11)*. We have $p=1,\ q=4$. We get
$\bar{x}_j^{14}=\left[x_j^{14}-w\left(\bar{x}_j^{14}\right),x_j^{14}+w\left(\bar{x}_j^{14}\right)\right]=\left[x_j^1,x_j^4\right]$ for $j=1,\ldots,n$ and

$$Max/Min\,\Re\left(\bar{Z}^{14}\left(\bar{x}^{14}\right)\right)=\sum_{j=1}^{n}c_j^{14}x_j^{14} \text{ subject to the constraints } \sum_{j=1}^{n}a_{ij}x_j^{14}\begin{pmatrix}=\\ \leq\\ \geq\end{pmatrix}\frac{b_i^4+b_i^1}{2},\ x_j^{14}\geq 0.$$

Step 3. The optimal solution according to the choice of the decision maker is $Max/Min\,\tilde{\Re}\left(\tilde{x}\right)\approx\sum_{j=1}^{n}\tilde{c}_j\tilde{x}_j$ with $\tilde{x}_j=\left(\bar{x}_j^{23};\bar{x}_j^{14}\right)=\left(x_j^1,x_j^2,x_j^3,x_j^4\right)$.

3.4 | Solution Procedure for Semi-Fully Fuzzy Linear Programming Problem with Pentagonal Fuzzy Numbers (t= 5)

For all the rest of this paper, we will consider the following semi-fully fuzzy linear programming problem with Pentagonal fuzzy numbers as follows *Eq. (12)* and $(t=5)$ [5] where $\tilde{x}_j=(x_j^1,x_j^2,x_j^3,x_j^4,x_j^5)=(x_j^3;\bar{x}_j^{24};\bar{x}_j^{15})$ and $\tilde{b}_i=(b_i^1,b_i^2,b_i^3,b_i^4,b_i^5)=(b_i^3;\bar{b}_i^{24};\bar{b}_i^{15})$. The steps of our method for solving the semi-fully fuzzy linear programming problem with Pentagonal fuzzy numbers as follows:

Step 1. Solving *Eq. (6)* via *Eq. (11)*. We have $p=q=\dfrac{t+1}{2}=3$. We get $\bar{x}_j^3=[x_j^3,x_j^3]=x_j^3$ for $j=1,\dots,n$ and $Max/Min\,\bar{Z}^3(\bar{x}^3)=\sum_{j=1}^{n}c_j^3x_j^3$ subject to the constraints $\sum_{j=1}^{n}a_{ij}x_j^3\begin{pmatrix}=\\ \le\\ \ge\end{pmatrix}b_i^3,\ x_j^3\ge 0$.

Step 2. Solving *Eq. (6)* via *Eq. (11)*. We have $p=2$ and $q=4$. We get $\bar{x}_j^{24}=[x_j^{24}-w(\bar{x}_j^{24}),x_j^{24}+w(\bar{x}_j^{24})]=[x_j^2,x_j^4]$ for $j=1,\dots,n$ and $Max/Min\,\bar{Z}^{24}(\bar{x}^{24})=\sum_{j=1}^{n}c_j^{24}x_j^{24}$ subject to the constraints $\sum_{j=1}^{n}a_{ij}x_j^{24}\begin{pmatrix}=\\ \le\\ \ge\end{pmatrix}\dfrac{b_i^4+b_i^2}{2},\ x_j^{24}\ge 0$.

Step 3. Solving *Eq. (6)* via *Eq. (11)*. We have $p=1$ and $q=5$. We get $\bar{x}_j^{15}=[x_j^{15}-w(\bar{x}_j^{15}),x_j^{15}+w(\bar{x}_j^{15})]=[x_j^1,x_j^5]$ for $j=1,\dots,n$ and $Max/Min\,\bar{Z}^{15}(\bar{x}^{15})=\sum_{j=1}^{n}c_j^{15}x_j^{15}$ subject to the constraints $\sum_{j=1}^{n}a_{ij}x_j^{15}\begin{pmatrix}=\\ \le\\ \ge\end{pmatrix}\dfrac{b_i^5+b_i^1}{2},\ x_j^{15}\ge 0$.

Step 4. The optimal solution according to the choice of the decision maker is $Max/Min\,\tilde{Z}(\tilde{x})\approx\sum_{j=1}^{n}\tilde{c}_j\tilde{x}_j$ with $\tilde{x}_j=(x_j^1,x_j^2,x_j^3,x_j^4,x_j^5)=(x_j^3;\bar{x}_j^{24};\bar{x}_j^{15})$.

3.5 | Solution Procedure for Semi-Fully Fuzzy Linear Programming Problem with Hexagonal Fuzzy Numbers (t= 6)

For all the rest of this paper, we will consider the following semi-fully fuzzy linear programming problem with Hexagonal fuzzy numbers as follows *Eq. (12)* and $(t=6)$ [6] where $\tilde{x}_j=(x_j^1,x_j^2,x_j^3,x_j^4,x_j^5,x_j^6)=(\bar{x}_{ij}^{34};\bar{x}_{ij}^{25};\bar{x}_{ij}^{16})$ and $\tilde{b}_i=(b_i^1,b_i^2,b_i^3,b_i^4,b_i^5,b_i^6)=(\bar{b}_i^{34};\bar{b}_i^{25};\bar{b}_i^{16})$. The steps of our method for solving the semi-fully fuzzy linear programming problem with Hexagonal fuzzy numbers as follows:

Step 1. Solving *Eq. (6)* via *Eq. (11)*. We have $p=\frac{t}{2}=3$ and $q=\frac{t+2}{2}=4$. We get $\bar{x}_j^{34}=\left[x_j^{34}-w\left(\bar{x}_j^{34}\right),x_j^{34}+w\left(\bar{x}_j^{34}\right)\right]=\left[x_j^3,x_j^4\right]$ for $j=1,\ldots,n$ and

$$Max/Min\left(\bar{Z}^{34}\left(\bar{x}^{34}\right)\right)=\sum_{j=1}^{n}c_j^{34}x_j^{34} \text{ subject to the constraints } \sum_{j=1}^{n}a_{ij}x_j^{34}\begin{pmatrix} = \\ \le \\ \ge \end{pmatrix}\frac{b_i^4+b_i^3}{2},\ x_j^{34}\ge 0.$$

Step 2. Solving *Eq. (6)* via *Eq. (11)*. We have $p=2$ and $q=5$. We get $\bar{x}_j^{25}=\left[x_j^{25}-w\left(\bar{x}_j^{25}\right),x_j^{25}+w\left(\bar{x}_j^{25}\right)\right]=\left[x_j^2,x_j^5\right]$ for $j=1,\ldots,n$ and

$$Max/Min\left(\bar{Z}^{25}\left(\bar{x}^{25}\right)\right)=\sum_{j=1}^{n}c_j^{25}x_j^{25} \text{ subject to the constraints } \sum_{j=1}^{n}a_{ij}x_j^{25}\begin{pmatrix} = \\ \le \\ \ge \end{pmatrix}\frac{b_i^5+b_i^2}{2},\ x_j^{25}\ge 0.$$

Step 3. Solving *Eq. (6)* via *Eq. (11)*. We have $p=1$ and $q=6$. We get $\bar{x}_j^{16}=\left[x_j^{16}-w\left(\bar{x}_j^{16}\right),x_j^{16}+w\left(\bar{x}_j^{16}\right)\right]=\left[x_j^1,x_j^6\right]$ for $j=1,\ldots,n$ and

$$Max/Min\left(\bar{Z}^{16}\left(\bar{x}^{16}\right)\right)=\sum_{j=1}^{n}c_j^{16}x_j^{16} \text{ subject to the constraints } \sum_{j=1}^{n}a_{ij}x_j^{16}\begin{pmatrix} = \\ \le \\ \ge \end{pmatrix}\frac{b_i^6+b_i^1}{2},\ x_j^{16}\ge 0.$$

Step 4. The optimal solution according to the choice of the decision maker is $Max/Min\,\tilde{Z}(\tilde{x})\approx\sum_{j=1}^{n}\tilde{c}_j\tilde{x}_j$ with $\tilde{x}_j=\left(x_j^1,x_j^2,x_j^3,x_j^4,x_j^5,x_j^6\right)=\left(\bar{x}_{ij}^{34};\bar{x}_{ij}^{25};\bar{x}_{ij}^{16}\right)$.

3.6 | Solution Procedure for Semi-Fully Fuzzy Linear Programming Problem with Heptagonal Fuzzy Numbers (t= 7)

For all the rest of this paper, we will consider the following semi-fully fuzzy linear programming problem with Heptagonal fuzzy numbers as follows *Eq. (12)* and $(t=7)$ [8] where $\tilde{x}_j=\left(x_j^1,x_j^2,x_j^3,x_j^4,x_j^5,x_j^6,x_j^7\right)=\left(x_j^4;\bar{x}_j^{35};\bar{x}_j^{26};\bar{x}_j^{17}\right)$ and

$\tilde{b}_i=\left(b_i^1,b_i^2,b_i^3,b_i^4,b_i^5,b_i^6,b_i^7\right)=\left(b_i^4;\bar{b}_i^{35};\bar{b}_i^{26};\bar{b}_i^{17}\right)$. The steps of our method for solving the semi-fully fuzzy linear programming problem with Heptagonal fuzzy numbers as follows:

Step 1. Solving *Eq. (6)* via *Eq. (11)*. We have $p=q=\frac{t+1}{2}=4$. We get $\bar{x}_j^{44}=\left[x_j^4,x_j^4\right]=x_j^4$ for $j=1,\ldots,n$ and $Max/Min\left(\bar{Z}^4\left(\bar{x}^4\right)\right)=\sum_{j=1}^{n}c_j^4x_j^4$ subject to the constraints $\sum_{j=1}^{n}a_{ij}x_j^4\begin{pmatrix} = \\ \le \\ \ge \end{pmatrix}b_i^4,\ x_j^4\ge 0.$

Step 2. Solving *Eq. (6)* via *Eq. (11)*. We have $p=3$ and $q=5$. We get $\bar{x}_j^{35}=\left[x_j^{35}-w\left(\bar{x}_j^{35}\right),x_j^{35}+w\left(\bar{x}_j^{35}\right)\right]=\left[x_j^{3},x_j^{5}\right]$ for $j=1,\ldots,n$ and $Max/Min\,\bar{Z}^{35}\left(\bar{x}^{35}\right)=\sum_{j=1}^{n}c_j^{35}x_j^{35}$

subject to the constraints $\sum_{j=1}^{n}a_{ij}x_j^{35}\begin{pmatrix}=\\\le\\\ge\end{pmatrix}\dfrac{b_i^5+b_i^3}{2}$, $x_j^{35}\ge 0$.

Step 3. Solving *Eq. (6)* via *Eq. (11)*. We have $p=2$ and $q=6$. We get $\bar{x}_j^{26}=\left[x_j^{26}-w\left(\bar{x}_j^{26}\right),x_j^{26}+w\left(\bar{x}_j^{26}\right)\right]=\left[x_j^{2},x_j^{6}\right]$ for $j=1,\ldots,n$ and $Max/Min\,\bar{Z}^{26}\left(\bar{x}^{26}\right)=\sum_{j=1}^{n}c_j^{26}x_j^{26}$

subject to the constraints $\sum_{j=1}^{n}a_{ij}x_j^{26}\begin{pmatrix}=\\\le\\\ge\end{pmatrix}\dfrac{b_i^6+b_i^2}{2}$, $x_j^{26}\ge 0$.

Step 4. Solving *Eq. (6)* via *Eq. (11)*. We have $p=1$ and $q=7$. We get $\bar{x}_j^{17}=\left[x_j^{17}-w\left(\bar{x}_j^{17}\right),x_j^{17}+w\left(\bar{x}_j^{17}\right)\right]=\left[x_j^{1},x_j^{7}\right]$ for $j=1,\ldots,n$ and

$Max/Min\,\bar{Z}^{17}\left(\bar{x}^{17}\right)=\sum_{j=1}^{n}c_j^{17}x_j^{17}$ subject to the constraints $\sum_{j=1}^{n}a_{ij}x_j^{17}\begin{pmatrix}=\\\le\\\ge\end{pmatrix}\dfrac{b_i^7+b_i^1}{2}$, $x_j^{17}\ge 0$.

Step 5. The optimal solution according to the choice of the decision maker is $Max/Min\,\tilde{Z}\left(\tilde{x}\right)\approx\sum_{j=1}^{n}\tilde{c}_j\tilde{x}_j$ with $\tilde{x}_j=\left(x_j^1,x_j^2,x_j^3,x_j^4,x_j^5,x_j^6,x_j^7\right)=\left(x_j^4;\bar{x}_j^{35};\bar{x}_j^{26};\bar{x}_j^{17}\right)$.

3.7 | Solution Procedure for Semi-Fully Fuzzy Linear Programming Problem with Octagonal Fuzzy Numbers (t= 8)

For all the rest of this paper, we will consider the following semi-fully fuzzy linear programming problem with Octagonal fuzzy numbers as follows *Eq. (12)* and $(t=8)$ [7] where $\tilde{x}_j=\left(x_j^1,x_j^2,x_j^3,x_j^4,x_j^5,x_j^6,x_j^7,x_j^8\right)=\left(\bar{x}_j^{45};\bar{x}_j^{36};\bar{x}_j^{27};\bar{x}_j^{18}\right)$ and

$\tilde{b}_i=\left(b_i^1,b_i^2,b_i^3,b_i^4,b_i^5,b_i^6,b_i^7,b_i^8\right)=\left(\bar{b}_i^{45};\bar{b}_i^{36};\bar{b}_i^{27};\bar{b}_i^{18}\right)$. The steps of our method for solving the semi-fully fuzzy linear programming problem with Octagonal fuzzy numbers as follows:

Step 1. Solving *Eq. (6)* via *Eq. (11)*. We have $p=\dfrac{t}{2}=4$ and $q=\dfrac{t+2}{2}=5$. We get $\bar{x}_j^{45}=\left[x_j^{45}-w\left(\bar{x}_j^{45}\right),x_j^{45}+w\left(\bar{x}_j^{45}\right)\right]=\left[x_j^{4},x_j^{5}\right]$ for $j=1,\ldots,n$ and $Max/Min\,\bar{Z}^{45}\left(\bar{x}^{45}\right)=\sum_{j=1}^{n}c_j^{45}x_j^{45}$

subject to the constraints $\sum_{j=1}^{n}a_{ij}x_j^{45}\begin{pmatrix}=\\\le\\\ge\end{pmatrix}\dfrac{b_i^5+b_i^4}{2}$, $x_j^{45}\ge 0$.

Step 2. Solving *Eq. (6)* via *Eq. (11)*. We have $p=3$ and $q=6$. We get $\bar{x}_j^{36}=\left[x_j^{36}-w\left(\bar{x}_j^{36}\right),x_j^{36}+w\left(\bar{x}_j^{36}\right)\right]=\left[x_j^{3},x_j^{6}\right]$ for $j=1,\ldots,n$ and

$$Max/Min\,\bar{Z}^{36}\left(\bar{x}^{36}\right)=\sum_{j=1}^{n}c_j^{36}x_j^{36} \text{ subject to the constraints } \sum_{j=1}^{n}a_{ij}x_j^{36}\begin{pmatrix}=\\ \le\\ \ge\end{pmatrix}\frac{b_i^{6}+b_i^{3}}{2},\ x_j^{36}\ge 0.$$

Step 3. Solving *Eq. (6)* via *Eq. (11)*. We have $p=2$ and $q=7$. We get $\bar{x}_j^{27}=\left[x_j^{27}-w\left(\bar{x}_j^{27}\right),x_j^{27}+w\left(\bar{x}_j^{27}\right)\right]=\left[x_j^{2},x_j^{7}\right]$ for $j=1,\ldots,n$ and

$$Max/Min\,\bar{Z}^{27}\left(\bar{x}^{27}\right)=\sum_{j=1}^{n}c_j^{27}x_j^{27} \text{ subject to the constraints } \sum_{j=1}^{n}a_{ij}x_j^{27}\begin{pmatrix}=\\ \le\\ \ge\end{pmatrix}\frac{b_i^{7}+b_i^{2}}{2},\ x_j^{27}\ge 0.$$

Step 4. Solving *Eq. Eq. (6)* via *Eq. (11)*. We have $p=1$ and $q=8$. We get $\bar{x}_j^{18}=\left[x_j^{18}-w\left(\bar{x}_j^{18}\right),x_j^{18}+w\left(\bar{x}_j^{18}\right)\right]=\left[x_j^{1},x_j^{8}\right]$ for $j=1,\ldots,n$ and $Max/Min\,\bar{Z}^{18}\left(\bar{x}^{18}\right)=\sum_{j=1}^{n}c_j^{18}x_j^{18}$

subject to the constraints $\sum_{j=1}^{n}a_{ij}x_j^{18}\begin{pmatrix}=\\ \le\\ \ge\end{pmatrix}\frac{b_i^{8}+b_i^{1}}{2},\ x_j^{18}\ge 0.$

Step 5. The optimal solution according to the choice of the decision maker is $Max/Min\,\tilde{Z}\left(\tilde{x}\right)\approx\sum_{j=1}^{n}\tilde{c}_j\tilde{x}_j$ with $\tilde{x}_j=\left(x_j^{1},x_j^{2},x_j^{3},x_j^{4},x_j^{5},x_j^{6},x_j^{7},x_j^{8}\right)=\left(\bar{x}_j^{45};\bar{x}_j^{36};\bar{x}_j^{27};\bar{x}_j^{18}\right)$.

3.8 | Solution Procedure for Semi-Fully Fuzzy Linear Programming Problem with Nonagonal Fuzzy Numbers (t= 9)

For all the rest of this paper, we will consider the following semi-fully fuzzy linear programming problem with Nonagonal fuzzy numbers as follows *Eq. (12)* and $(t=9)$ [9] where $\tilde{x}_j=\left(x_j^{1},x_j^{2},x_j^{3},x_j^{4},x_j^{5},x_j^{6},x_j^{7},x_j^{8},x_j^{9}\right)=\left(x_j^{5};\bar{x}_j^{46};\bar{x}_j^{37};\bar{x}_j^{28};\bar{x}_j^{19}\right)$ and

$\tilde{b}_i=\left(b_i^{1},b_i^{2},b_i^{3},b_i^{4},b_i^{5},b_i^{6},b_i^{7},b_i^{8},b_i^{9}\right)=\left(b_i^{5};\bar{b}_i^{46};\bar{b}_i^{37};\bar{b}_i^{28};\bar{b}_i^{19}\right)$. The steps of our method for solving the semi-fully fuzzy linear programming problem with Nonagonal fuzzy numbers as follows:

Step 1. Solving *Eq. (6)* via *Eq. (11)*. We have $p=q=\frac{t+1}{2}=5$. We get $\bar{x}_j^{55}=\left[x_j^{5},x_j^{5}\right]=x_j^{5}$ for $j=1,\ldots,n$

and $Max/Min\,\bar{Z}^{5}\left(\bar{x}^{5}\right)=\sum_{j=1}^{n}c_j^{5}x_j^{5}$ subject to the constraints $\sum_{j=1}^{n}a_{ij}x_j^{5}\begin{pmatrix}=\\ \le\\ \ge\end{pmatrix}b_i^{5},\ x_j^{5}\ge 0.$

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Step 2. Solving *Eq. (6)* via *Eq. (11)*. We have $p = 4$ and $q = 6$. We get $\bar{x}_j^{46} = \left[x_j^{46} - w\left(\bar{x}_j^{46}\right), x_j^{46} + w\left(\bar{x}_j^{46}\right)\right] = \left[x_j^4, x_j^6\right]$ for $j = 1,\ldots,n$ and $Max / Min\left(\bar{Z}^{46}\left(\bar{x}^{46}\right)\right) = \sum_{j=1}^{n} c_j^{46} x_j^{46}$ subject to the constraints $\sum_{j=1}^{n} a_{ij} x_j^{46} \begin{pmatrix} = \\ \le \\ \ge \end{pmatrix} \dfrac{b_i^6 + b_i^4}{2}$, $x_j^{46} \ge 0$.

Step 3. Solving *Eq. (6)* via *Eq. (11)*. We have $p = 3$ and $q = 7$. We get $\bar{x}_j^{37} = \left[x_j^{37} - w\left(\bar{x}_j^{37}\right), x_j^{37} + w\left(\bar{x}_j^{37}\right)\right] = \left[x_j^3, x_j^7\right]$ for $j = 1,\ldots,n$ and

$Max / Min\left(\bar{Z}^{37}\left(\bar{x}^{37}\right)\right) = \sum_{j=1}^{n} c_j^{37} x_j^{37}$ subject to the constraints $\sum_{j=1}^{n} a_{ij} x_j^{37} \begin{pmatrix} = \\ \le \\ \ge \end{pmatrix} \dfrac{b_i^7 + b_i^3}{2}$, $x_j^{37} \ge 0$.

Step 4. Solving *Eq. (6)* via *Eq. (11)*. We have $p = 2$ and $q = 8$. We get $\bar{x}_j^{28} = \left[x_j^{28} - w\left(\bar{x}_j^{28}\right), x_j^{28} + w\left(\bar{x}_j^{28}\right)\right] = \left[x_j^2, x_j^8\right]$ for $j = 1,\ldots,n$ and $Max / Min\left(\bar{Z}^{28}\left(\bar{x}^{28}\right)\right) = \sum_{j=1}^{n} c_j^{28} x_j^{28}$ subject to the constraints $\sum_{j=1}^{n} a_{ij} x_j^{28} \begin{pmatrix} = \\ \le \\ \ge \end{pmatrix} \dfrac{b_i^8 + b_i^2}{2}$, $x_j^{28} \ge 0$.

Step 5. Solving *Eq. (6)* via *Eq. (11)*. We have $p = 1$ and $q = 9$. We get $\bar{x}_j^{19} = \left[x_j^{19} - w\left(\bar{x}_j^{19}\right), x_j^{19} + w\left(\bar{x}_j^{19}\right)\right] = \left[x_j^1, x_j^9\right]$ for $j = 1,\ldots,n$ and $Max / Min\left(\bar{Z}^{19}\left(\bar{x}^{19}\right)\right) = \sum_{j=1}^{n} c_j^{19} x_j^{19}$ subject to the constraints $\sum_{j=1}^{n} a_{ij} x_j^{19} \begin{pmatrix} = \\ \le \\ \ge \end{pmatrix} \dfrac{b_i^9 + b_i^1}{2}$, $x_j^{19} \ge 0$.

Step 6. The optimal solution according to the choice of the decision maker is $Max / Min\tilde{Z}\left(\tilde{x}\right) \approx \sum_{j=1}^{n} \tilde{c}_j \tilde{x}_j$ with $\tilde{x}_j = \left(x_j^1, x_j^2, x_j^3, x_j^4, x_j^5, x_j^6, x_j^7, x_j^8, x_j^9\right) = \left(x_j^5; \bar{x}_j^{46}; \bar{x}_j^{37}; \bar{x}_j^{28}; \bar{x}_j^{19}\right)$.

3.9 | Solution Procedure for Semi-Fully Fuzzy Linear Programming Problem with Decagonal Fuzzy Numbers (t= 10)

For all the rest of this paper, we will consider the following semi-fully fuzzy linear programming problem with Decagonal fuzzy numbers as follows *Eq. (12)* and $(t = 10)$ [10] where $\tilde{x}_j = \left(x_j^1, x_j^2, x_j^3, x_j^4, x_j^5, x_j^6, x_j^7, x_j^8, x_j^9, x_j^{10}\right) = \left(\bar{x}_j^{56}; \bar{x}_j^{47}; \bar{x}_j^{38}; \bar{x}_j^{29}; \bar{x}_j^{110}\right)$ and

$\tilde{b}_i = \left(b_i^1, b_i^2, b_i^3, b_i^4, b_i^5, b_i^6, b_i^7, b_i^8, b_i^9, b_i^{10}\right) = \left(\bar{b}_i^{56}; \bar{b}_i^{47}; \bar{b}_i^{38}; \bar{b}_i^{29}; \bar{b}_i^{110}\right)$. The steps of our method for solving the semi-fully fuzzy linear programming problem with Decagonal fuzzy numbers as follows:

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Step 1. Solving *Eq. (6)* via *Eq. (11)*. We have $p=\frac{t}{2}=5$ and $q=\frac{t+2}{2}=6$. We get

$\bar{x}_j^{56}=\left[x_j^{56}-w\left(\bar{x}_j^{56}\right),x_j^{56}+w\left(\bar{x}_j^{56}\right)\right]=\left[x_j^5,x_j^6\right]$ for $j=1,\ldots,n$ and

$$Max/Min\left(\bar{Z}^{56}\left(\bar{x}^{56}\right)\right)=\sum_{j=1}^{n}c_j^{56}x_j^{56} \text{ subject to the constraints } \sum_{j=1}^{n}a_{ij}x_j^{56}\begin{pmatrix} = \\ \leq \\ \geq \end{pmatrix}\frac{b_i^6+b_i^5}{2},\ x_j^{56}\geq 0.$$

Step 2. Solving *Eq. (6)* via *Eq. (11)*. We have $p=4$ and $q=7$. We get

$\bar{x}_j^{47}=\left[x_j^{47}-w\left(\bar{x}_j^{47}\right),x_j^{47}+w\left(\bar{x}_j^{47}\right)\right]=\left[x_j^4,x_j^7\right]$ for $j=1,\ldots,n$ and

$$Max/Min\left(\bar{Z}^{47}\left(\bar{x}^{47}\right)\right)=\sum_{j=1}^{n}c_j^{47}x_j^{47} \text{ subject to the constraints } \sum_{j=1}^{n}a_{ij}x_j^{47}\begin{pmatrix} = \\ \leq \\ \geq \end{pmatrix}\frac{b_i^7+b_i^4}{2},\ x_j^{47}\geq 0.$$

Step 3. Solving *Eq. (6)* via *Eq. (11)*. We have $p=3$ and $q=8$. We get

$\bar{x}_j^{38}=\left[x_j^{38}-w\left(\bar{x}_j^{38}\right),x_j^{38}+w\left(\bar{x}_j^{38}\right)\right]=\left[x_j^3,x_j^8\right]$ for $j=1,\ldots,n$ and

$$Max/Min\left(\bar{Z}^{38}\left(\bar{x}^{38}\right)\right)=\sum_{j=1}^{n}c_j^{38}x_j^{38} \text{ subject to the constraints } \sum_{j=1}^{n}a_{ij}x_j^{38}\begin{pmatrix} = \\ \leq \\ \geq \end{pmatrix}\frac{b_i^8+b_i^3}{2},\ x_j^{38}\geq 0.$$

Step 4. Solving *Eq. (6)* via *Eq. (11)*. We have $p=2$ and $q=9$. We get

$\bar{x}_j^{29}=\left[x_j^{29}-w\left(\bar{x}_j^{29}\right),x_j^{29}+w\left(\bar{x}_j^{29}\right)\right]=\left[x_j^2,x_j^9\right]$ for $j=1,\ldots,n$ and

$$Max/Min\left(\bar{Z}^{29}\left(\bar{x}^{29}\right)\right)=\sum_{j=1}^{n}c_j^{29}x_j^{29} \text{ subject to the constraints } \sum_{j=1}^{n}a_{ij}x_j^{29}\begin{pmatrix} = \\ \leq \\ \geq \end{pmatrix}\frac{b_i^9+b_i^2}{2},\ x_j^{29}\geq 0.$$

Step 5. Solving *Eq. (6)* via *Eq. (11)*. We have $p=1$ and $q=10$. We get

$\bar{x}_j^{110}=\left[x_j^{110}-w\left(\bar{x}_j^{110}\right),x_j^{110}+w\left(\bar{x}_j^{110}\right)\right]=\left[x_j^1,x_j^{10}\right]$ for $j=1,\ldots,n$ and

$$Max/Min\left(\bar{Z}^{110}\left(\bar{x}^{110}\right)\right)=\sum_{j=1}^{n}c_j^{110}x_j^{110} \text{ subject toconstraints } \sum_{j=1}^{n}a_{ij}x_j^{110}\begin{pmatrix} = \\ \leq \\ \geq \end{pmatrix}\frac{b_i^{10}+b_i^1}{2},\ x_j^{110}\geq 0.$$

Step 6. The optimal solution according to the choice of the decision maker $Max/Min\tilde{Z}\left(\tilde{x}\right)\approx\sum_{j=1}^{n}\tilde{c}_j\tilde{x}_j$ with $\tilde{x}_j=\left(x_j^1,x_j^2,x_j^3,x_j^4,x_j^5,x_j^6,x_j^7,x_j^8,x_j^9,x_j^{10}\right)=\left(\bar{x}_j^{56};\bar{x}_j^{47};\bar{x}_j^{38};\bar{x}_j^{29};\bar{x}_j^{110}\right)$.

3.10 | Solution Procedure for Semi-Fully Fuzzy Linear Programming Problem with Decagonal Fuzzy Numbers (t= 11)

For all the rest of this paper, we will consider the following semi-fully fuzzy linear programming problem with Hendecagonal fuzzy numbers as follows *Eq. (12)* and $(t=11)$ [11] where $\tilde{x}_j=\left(x_j^1,x_j^2,x_j^3,x_j^4,x_j^5,x_j^6,x_j^7,x_j^8,x_j^9,x_j^{10},x_j^{11}\right)=\left(x_j^6;\bar{x}_j^{57};\bar{x}_j^{48};\bar{x}_j^{39};\bar{x}_j^{210};\bar{x}_j^{111}\right)$ and

$\tilde{b}_i=\left(b_i^1,b_i^2,b_i^3,b_i^4,b_i^5,b_i^6,b_i^7,b_i^8,b_i^9,b_i^{10},b_i^{11}\right)=\left(b_i^6;\bar{b}_i^{57};\bar{b}_i^{48};\bar{b}_i^{39};\bar{b}_i^{210};\bar{b}_i^{111}\right)$. The steps of our method for solving the semi-fully fuzzy linear programming problem with Hendecagonal fuzzy numbers as follows:

Step 1. Solving *Eq. (6)* via *Eq. (11)*. We have $p=q=\dfrac{t+1}{2}=6$. We get $\bar{x}_j^{66}=\left[x_j^6,x_j^6\right]=x_j^6$ for $j=1,\ldots,n$

and $Max / Min\,\square m\left(\bar{Z}^6\left(\bar{x}^6\right)\right)=\sum_{j=1}^{n}c_j^6x_j^6$ subject to the constraints $\sum_{j=1}^{n}a_{ij}x_j^6\begin{pmatrix}=\\ \leq\\ \geq\end{pmatrix}b_i^6$, $x_j^6\geq 0$.

Step 2. Solving *Eq. (6)* via *Eq. (11)*. We have $p=5$ and $q=7$. We get $\bar{x}_j^{57}=\left[x_j^{57}-w\left(\bar{x}_j^{57}\right),x_j^{57}+w\left(\bar{x}_j^{57}\right)\right]=\left[x_j^5,x_j^7\right]$ for $j=1,\ldots,n$ and

$Max / Min\,\square m\left(\bar{Z}^{57}\left(\bar{x}^{57}\right)\right)=\sum_{j=1}^{n}c_j^{57}x_j^{57}$ subject to the constraints $\sum_{j=1}^{n}a_{ij}x_j^{57}\begin{pmatrix}=\\ \leq\\ \geq\end{pmatrix}\dfrac{b_i^7+b_i^5}{2}$, $x_j^{57}\geq 0$.

Step 3. Solving *Eq. (6)* via *Eq. (11)*. We have $p=4$ and $q=8$. We get $\bar{x}_j^{48}=\left[x_j^{48}-w\left(\bar{x}_j^{48}\right),x_j^{48}+w\left(\bar{x}_j^{48}\right)\right]=\left[x_j^4,x_j^8\right]$ for $j=1,\ldots,n$ and $Max / Min\,\square m\left(\bar{Z}^{48}\left(\bar{x}^{48}\right)\right)=\sum_{j=1}^{n}c_j^{48}x_j^{48}$ subject to the constraints $\sum_{j=1}^{n}a_{ij}x_j^{48}\begin{pmatrix}=\\ \leq\\ \geq\end{pmatrix}\dfrac{b_i^8+b_i^4}{2}$, $x_j^{48}\geq 0$.

Step 4. Solving *Eq. (6)* via *Eq. (11)*. We have $p=3$ and $q=9$. We get $\bar{x}_j^{39}=\left[x_j^{39}-w\left(\bar{x}_j^{39}\right),x_j^{39}+w\left(\bar{x}_j^{39}\right)\right]=\left[x_j^3,x_j^9\right]$ for $j=1,\ldots,n$ and $Max / Min\,\square m\left(\bar{Z}^{39}\left(\bar{x}^{39}\right)\right)=\sum_{j=1}^{n}c_j^{39}x_j^{39}$ subject to the constraints $\sum_{j=1}^{n}a_{ij}x_j^{39}\begin{pmatrix}=\\ \leq\\ \geq\end{pmatrix}\dfrac{b_i^9+b_i^3}{2}$, $x_j^{39}\geq 0$.

Step 5. Solving *Eq. (6)* via *Eq. (11)*. We have $p=2$ and $q=10$. We get $\bar{x}_j^{210}=\left[x_j^{210}-w\left(\bar{x}_j^{210}\right),x_j^{210}+w\left(\bar{x}_j^{210}\right)\right]=\left[x_j^2,x_j^{10}\right]$ for $j=1,\ldots,n$ and

$Max / Min\,\square m\left(\bar{Z}^{210}\left(\bar{x}^{210}\right)\right)=\sum_{j=1}^{n}c_j^{210}x_j^{210}$ subject to constraints $\sum_{j=1}^{n}a_{ij}x_j^{210}\begin{pmatrix}=\\ \leq\\ \geq\end{pmatrix}\dfrac{b_i^{10}+b_i^2}{2}$, $x_j^{210}\geq 0$.

Step 6. Solving *Eq. (6)* via *Eq. (11)*. We have $p=1$ and $q=11$. We get

$\bar{x}_j^{111}=\left[x_j^{111}-w\left(\bar{x}_j^{111}\right),x_j^{111}+w\left(\bar{x}_j^{111}\right)\right]=\left[x_j^1,x_j^{11}\right]$ for $j=1,\dots,n$ and

$$Max/Min\left(\bar{Z}^{111}\left(\bar{x}^{111}\right)\right)=\sum_{j=1}^{n}c_j^{111}x_j^{111} \text{ subject to constraints } \sum_{j=1}^{n}a_{ij}x_j^{111}\begin{pmatrix}=\\ \leq\\ \geq\end{pmatrix}\frac{b_i^{111}+b_i^{111}}{2},\ x_j^{111}\geq 0.$$

Step 7. The optimal solution according to the choice of the decision maker $Max/Min\,\tilde{Z}\left(\tilde{x}\right)\approx\sum_{j=1}^{n}\tilde{c}_j\tilde{x}_j$ with $\tilde{x}_j=\left(x_j^1,x_j^2,x_j^3,x_j^4,x_j^5,x_j^6,x_j^7,x_j^8,x_j^9,x_j^{10},x_j^{11}\right)=\left(x_j^6;\bar{x}_j^{57};\bar{x}_j^{48};\bar{x}_j^{39};\bar{x}_j^{210};\bar{x}_j^{111}\right)$.

3.11 | Solution Procedure for Semi-Fully Fuzzy Linear Programming Problem with Dodecagonal Fuzzy Numbers (t= 12)

For all the rest of this paper, we will consider the following semi-fully fuzzy linear programming problem with Dodecagonal fuzzy numbers as follows *Eq. (12)* and $(t=12)$ [11] where $\tilde{x}_j=\left(x_j^1,x_j^2,x_j^3,x_j^4,x_j^5,x_j^6,x_j^7,x_j^8,x_j^9,x_j^{10},x_j^{11},x_j^{12}\right)=\left(\bar{x}_j^{67};\bar{x}_j^{58};\bar{x}_j^{49};\bar{x}_j^{310};\bar{x}_j^{211};\bar{x}_j^{112}\right)$ and

$\tilde{b}_i=\left(b_i^1,b_i^2,b_i^3,b_i^4,b_i^5,b_i^6,b_i^7,b_i^8,b_i^9,b_i^{10},b_i^{11},b_i^{12}\right)=\left(\bar{b}_i^{67};\bar{b}_i^{58};\bar{b}_i^{49};\bar{b}_i^{310};\bar{b}_i^{211};\bar{b}_i^{112}\right)$. The steps of our method for solving the semi-fully fuzzy linear programming problem with Dodecagonal fuzzy numbers as follows:

Step 1. Solving *Eq. (6)* via *Eq. (11)*. We have $p=\frac{t}{2}=6$ and $q=\frac{t+2}{2}=7$. We get

$\bar{x}_j^{67}=\left[x_j^{67}-w\left(\bar{x}_j^{67}\right),x_j^{67}+w\left(\bar{x}_j^{67}\right)\right]=\left[x_j^6,x_j^7\right]$ for $j=1,\dots,n$ and

$$Max/Min\left(\bar{Z}^{67}\left(\bar{x}^{67}\right)\right)=\sum_{j=1}^{n}c_j^{67}x_j^{67} \text{ subject to the constraints } \sum_{j=1}^{n}a_{ij}x_j^{67}\begin{pmatrix}=\\ \leq\\ \geq\end{pmatrix}\frac{b_i^7+b_i^6}{2},\ x_j^{76}\geq 0.$$

Step 2. Solving *Eq. (6)* via *Eq. (11)*. We have $p=5$ and $q=8$. We get

$\bar{x}_j^{58}=\left[x_j^{58}-w\left(\bar{x}_j^{58}\right),x_j^{58}+w\left(\bar{x}_j^{58}\right)\right]=\left[x_j^5,x_j^8\right]$ for $j=1,\dots,n$ and

$$Max/Min\left(\bar{Z}^{58}\left(\bar{x}^{58}\right)\right)=\sum_{j=1}^{n}c_j^{58}x_j^{58} \text{ subject to the constraints } \sum_{j=1}^{n}a_{ij}x_j^{58}\begin{pmatrix}=\\ \leq\\ \geq\end{pmatrix}\frac{b_i^8+b_i^5}{2},\ x_j^{58}\geq 0.$$

Step 3. Solving *Eq. (6)* via *Eq. (11)*. We have $p=4$ and $q=9$. We get

$\bar{x}_j^{49}=\left[x_j^{49}-w\left(\bar{x}_j^{49}\right),x_j^{49}+w\left(\bar{x}_j^{49}\right)\right]=\left[x_j^4,x_j^9\right]$ for $j=1,\dots,n$ and

$$Max/Min\left(\bar{Z}^{49}\left(\bar{x}^{49}\right)\right)=\sum_{j=1}^{n}c_j^{49}x_j^{49} \text{ subject to the constraints } \sum_{j=1}^{n}a_{ij}x_j^{49}\begin{pmatrix}=\\ \leq\\ \geq\end{pmatrix}\frac{b_i^9+b_i^4}{2},\ x_j^{49}\geq 0.$$

Step 4. Solving *Eq. (6)* via *Eq. (11)*. We have $p=3$ and $q=10$. We get $\bar{x}_j^{310}=\left[x_j^{310}-w\left(\bar{x}_j^{310}\right),x_j^{310}+w\left(\bar{x}_j^{310}\right)\right]=\left[x_j^{3},x_j^{10}\right]$ for $j=1,\ldots,n$ and

$$Max/Min\,\bar{Z}^{310}\left(\bar{x}^{310}\right)=\sum_{j=1}^{n}c_j^{310}x_j^{310} \text{ subject to constraints } \sum_{j=1}^{n}a_{ij}x_j^{310}\begin{pmatrix}=\\\leq\\\geq\end{pmatrix}\frac{b_i^{10}+b_i^{3}}{2},$$

Step 5. Solving *Eq. (6)* via *Eq. (11)*. We have $p=2$ and $q=11$. We get $\bar{x}_j^{211}=\left[x_j^{211}-w\left(\bar{x}_j^{211}\right),x_j^{211}+w\left(\bar{x}_j^{211}\right)\right]=\left[x_j^{2},x_j^{11}\right]$ for $j=1,\ldots,n$ and

$$Max/Min\,\bar{Z}^{211}\left(\bar{x}^{211}\right)=\sum_{j=1}^{n}c_j^{211}x_j^{211} \text{ subject to constraints } \sum_{j=1}^{n}a_{ij}x_j^{211}\begin{pmatrix}=\\\leq\\\geq\end{pmatrix}\frac{b_i^{11}+b_i^{2}}{2},\ x_j^{211}\geq 0$$

Step 6. Solving *Eq. (6)* via *Eq. (11)*. We have $p=1$ and $q=12$. We get $\bar{x}_j^{112}=\left[x_j^{112}-w\left(\bar{x}_j^{112}\right),x_j^{112}+w\left(\bar{x}_j^{112}\right)\right]=\left[x_j^{1},x_j^{12}\right]$ for $j=1,\ldots,n$ and

$$Max/Min\,\bar{Z}^{112}\left(\bar{x}^{112}\right)=\sum_{j=1}^{n}c_j^{112}x_j^{112} \text{ subject to constraints } \sum_{j=1}^{n}a_{ij}x_j^{112}\begin{pmatrix}=\\\leq\\\geq\end{pmatrix}\frac{b_i^{12}+b_i^{1}}{2},\ x_j^{112}\geq 0$$

Step 7. The optimal solution according to the choice of the decision maker $Max/Min\,\tilde{Z}\left(\tilde{x}\right)\approx\sum_{j=1}^{n}\tilde{c}_j\tilde{x}_j$ with $\tilde{x}_j=\left(x_j^1,x_j^2,x_j^3,x_j^4,x_j^5,x_j^6,x_j^7,x_j^8,x_j^9,x_j^{10},x_j^{11},x_j^{12}\right)=\left(\bar{x}_j^{67};\bar{x}_j^{58};\bar{x}_j^{49};\bar{x}_j^{310};\bar{x}_j^{211};\bar{x}_j^{112}\right)$.

4 | Examples

Example 1. Consider the following semi-fully interval linear programming problem [1]

Min $\bar{Z}^{12}\left(\bar{x}^{12}\right)\approx\left[25,27\right]\bar{x}_1^{12}+\left[6,8\right]\bar{x}_2^{12}$ subject to the constraints $6\bar{x}_1^{12}+4\bar{x}_2^{12}\ \left[29,31\right]$, $5\bar{x}_1^{12}+2\bar{x}_2^{12}\ \left[22,24\right]$ and $3\bar{x}_1^{12}+5\bar{x}_2^{12}\ \left[28,30\right]$.

Step 1. Solving $Eq.\left(6\right)$ via $Eq.\left(10\right)$. We have $p=1$, $q=2$. We get Min $Z^{12}\left(x^{12}\right)=26x_1^{12}+7x_2^{12}$ subject to the constraints $6x_1^{12}+4x_2^{12}\geq 30$, $5x_1^{12}+2x_2^{12}\geq 23$ and $3x_1^{12}+5x_2^{12}\geq 29$. Optimal solution: $x_1^{12}=0$ and $x_2^{12}=\frac{23}{2}$. Slack variables values: $x_3^{12}=16$, $x_4^{12}=0$ and $x_5^{12}=\frac{57}{2}$.

Very important decision: For $x_4^{12}=0$, we have $5w\left(\bar{x}_1^{12}\right)+2w\left(\bar{x}_2^{12}\right)=1$. We get $w\left(\bar{x}_1^{12}\right)=0$, ($x_1^{12}=0$) and $w\left(\bar{x}_2^{12}\right)=\frac{1}{2}$. Therefore, we get $\bar{x}_1^{12}=\left[0,0\right]$ and $\bar{x}_2^{12}=\left[11,12\right]$.

Step 2. The optimal solution according to the choice of the decision maker is $Min\bar{Z}^{12}\left(\bar{x}^{12}\right)\approx\left[66,96\right]$ where $\bar{x}_1^{12}=\left[0,0\right]$ and $\bar{x}_2^{12}=\left[11,12\right]$. Then the corresponding dual problem is given by: $Max\bar{W}^{12}\left(\bar{y}^{12}\right)\approx\left[66,96\right]$ where $\bar{y}_1^{12}=\left[0,0\right]$, $\bar{y}_2^{12}=\left[3,4\right]$ and $\bar{y}_3^{12}=\left[0,0\right]$.

Remark 3. We see that both primal and dual problems have interval optimal solutions and the two interval optimal values are equal. In contrast to most existing approaches [1], the centers of all constraints are saturated and some constraint is saturated.

Example 2. Consider the following linear programming problem with variables given as Triangular fuzzy numbers [2]:

Max $\tilde{Z}\left(\tilde{x}\right)\approx\left(5,6,8\right)\tilde{x}_1+\left(4,4,4\right)\tilde{x}_2$ Subject to the constraints $3\tilde{x}_1+2\tilde{x}_2 \quad \left(140,150,150\right)$ and $4\tilde{x}_1+3\tilde{x}_2 \quad \left(155,160,165\right)$.

Step 1. Solving $Eq.\left(6\right)$ via $Eq.\left(11\right)$. We have $p=2$, $q=2$. We get Max $Z^2\left(x^2\right)=6x_1^2+4x_2^2$ subject to the constraints $3x_1^2+2x_2^2\le 150$ and $4x_1^2+3x_2^2\le 160$. Optimal solution: $x_1^2=40$ and $x_2^2=0$. Slack variables values: $x_3^2=30$ and $x_4^2=0$.

Step 2. Solving $Eq.\left(6\right)$ via $Eq.\left(11\right)$. We have $p=1, q=3$. We get Max $Z^{13}\left(x^{13}\right)=\frac{13}{2}x_1^{13}+4x_2^{13}$ subject to the constraints $3x_1^{13}+2x_2^{13}\le 145$ and $4x_1^{13}+3x_2^{13}\le 160$. Optimal solution: $x_1^{13}=40$ and $x_2^{13}=0$. Slack variables values: $x_3^{13}=25$ and $x_4^{13}=0$. Very important decision: For $x_4^{13}=0$, we have $4w\left(\bar{x}_1^{13}\right)+3w\left(\bar{x}_2^{13}\right)=w\left(\bar{b}_1^{13}\right)=5$. We get $w\left(\bar{x}_1^{13}\right)=\frac{5}{4}$ and $w\left(\bar{x}_2^{13}\right)=0$ with $\left|x_1^{13}-x_1^2\right|\le\frac{5}{4}$. Therefore, we get $\bar{x}_1^{13}=\left[\frac{155}{4},\frac{165}{4}\right]$ and $\bar{x}_2^{13}=\left[0,0\right]$.

Step 3. The optimal solution according to the choice of the decision maker is $Max\tilde{Z}\left(\tilde{x}\right)\approx\sum_{j=1}^{n}c_j\tilde{x}_j$ with $\tilde{x}_j=\left(x_j^2;\bar{x}_j^{13}\right)=\left(x_j^1,x_j^2,x_j^3\right)$: $Max\tilde{Z}\left(\tilde{x}\right)\approx\left(\frac{775}{4},240,330\right)$ where $\tilde{x}_1=\left(\frac{155}{4},40,\frac{165}{4}\right)$ and $\tilde{x}_2=\left(0,0,0\right)$. Then the corresponding dual problem is given by: $Min\tilde{W}\left(y\right)\approx\left(\frac{775}{4},240,330\right)$ where $\tilde{y}_1=\tilde{0}$ and $\tilde{y}_2=\left(\frac{5}{4},\frac{3}{2},2\right)$.

Remark 4. We see that both primal and dual problems have fuzzy optimal solutions and the two fuzzy optimal values are equal. In contrast to most existing approaches [2], the centers of all constraints are saturated and some constraint is saturated.

Example 3. Consider the following linear programming problem with variables given as Trapezoidal fuzzy numbers [3] and [4]:

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Min $\tilde{Z}=(1,4,8,11)\tilde{x}_1+(2,4,6,8)\tilde{x}_2+(1,2,4,5)\tilde{x}_3$ Subject to the constraints $3\tilde{x}_1+4\tilde{x}_2+2\tilde{x}_3 \quad (3,6,10,13)$, $4\tilde{x}_1+2\tilde{x}_2+\tilde{x}_3 \quad (2,4,6,8)$ and $2\tilde{x}_1+\tilde{x}_2+3\tilde{x}_3 \quad (1,2,6,7)$.

Step 1. Solving $Eq.(6)$ via $Eq.(10)$. We have $p=2$, $q=3$. We get Min $Z^{23}(x^{23})=6x_1^{23}+5x_2^{23}+3x_3^{23}$ subject to the constraints $3x_1^{23}+4x_2^{23}+2x_3^{23}\geq 8$, $4x_1^{23}+2x_2^{23}+x_3^{23}\geq 5$ and $2x_1^{23}+x_2^{23}+3x_3^{23}\geq 4$. Optimal solution: $x_1^{23}=\frac{2}{5}$, $x_2^{23}=\frac{7}{5}$ and $x_3^{23}=\frac{3}{5}$. Slack variables values: $x_4^{23}=0$, $x_5^{23}=0$ and $x_6^{23}=0$. Very important decision: For $x_4^{23}=0$, $x_5^{23}=0$ and $x_6^{23}=0$, we have $3w(\bar{x}_1^{23})+4w(\bar{x}_2^{23})+2w(\bar{x}_3^{23})=2$, $4w(\bar{x}_1^{23})+2w(\bar{x}_2^{23})+w(\bar{x}_3^{23})=1$ and $2w(\bar{x}_1^{23})+w(\bar{x}_2^{23})+3w(\bar{x}_3^{23})=2$. We get $w(\bar{x}_1^{23})=0$, $w(\bar{x}_2^{23})=\frac{1}{5}$ and $w(\bar{x}_3^{23})=\frac{3}{5}$. Therefore, we get $\bar{x}_1^{23}=\left[\frac{2}{5},\frac{2}{5}\right]$, $\bar{x}_2^{23}=\left[\frac{6}{5},\frac{8}{5}\right]$ and $\bar{x}_3^{23}=\left[0,\frac{6}{5}\right]$.

Step 2. Solving $Eq.(6)$ via $Eq.(10)$. We have $p=1$, $q=4$. We get Min $Z^{14}(x^{14})=6x_1^{14}+5x_2^{14}+3x_3^{14}$ subject to the constraints $3x_1^{14}+4x_2^{14}+2x_3^{14}\geq 8$, $4x_1^{14}+2x_2^{14}+x_3^{14}\geq 5$ and $2x_1^{14}+x_2^{14}+3x_3^{14}\geq 4$. Optimal solution: $x_1^{14}=\frac{2}{5}$, $x_2^{14}=\frac{7}{5}$ and $x_3^{14}=\frac{3}{5}$. Slack variables values: $x_4^{14}=0$, $x_5^{14}=0$ and $x_6^{14}=0$. Very important decision: For $x_4^{14}=0$, $x_5^{14}=0$ and $x_6^{14}=0$, we have $3w(\bar{x}_1^{14})+4w(\bar{x}_2^{14})+2w(\bar{x}_3^{14})=5$, $4w(\bar{x}_1^{14})+2w(\bar{x}_2^{14})+w(\bar{x}_3^{14})=3$ and $2w(\bar{x}_1^{14})+w(\bar{x}_2^{14})+3w(\bar{x}_3^{14})=3$. We get $w(\bar{x}_1^{14})=\frac{1}{5}$, $w(\bar{x}_2^{14})=\frac{4}{5}$ and $w(\bar{x}_3^{14})=\frac{3}{5}$ with $|x_1^{14}-x_1^{23}|+0=0\leq\frac{1}{5}$, $|x_2^{14}-x_2^{23}|+\frac{1}{5}=\frac{1}{5}\leq\frac{4}{5}$ and $|x_3^{14}-x_3^{23}|+\frac{3}{5}=\frac{3}{5}\leq\frac{3}{5}$. Therefore, we get $\bar{x}_1^{14}=\left[\frac{1}{5},\frac{3}{5}\right]$, $\bar{x}_2^{14}=\left[\frac{3}{5},\frac{11}{5}\right]$ and $\bar{x}_3^{14}=\left[0,\frac{6}{5}\right]$.

Step 3. The optimal solution according to the choice of the decision maker is $Min\tilde{Z}(\tilde{x})\approx\sum_{j=1}^{n}c_j\tilde{x}_j$ with $\tilde{x}_j=(\bar{x}_j^{23};\bar{x}_j^{13})=(x_j^1,x_j^2,x_j^3,x_j^4)$. $Min\tilde{Z}(\tilde{x})\approx\left(\frac{7}{5},\frac{32}{5},\frac{88}{5},\frac{151}{5}\right)$ where $\tilde{x}_1=\left(\frac{1}{5},\frac{2}{5},\frac{2}{5},\frac{3}{5}\right)$, $\tilde{x}_2=\left(\frac{3}{5},\frac{6}{5},\frac{8}{5},\frac{11}{5}\right)$ and $\tilde{x}_3=\left(0,0,\frac{6}{5},\frac{6}{5}\right)$. Then the corresponding dual problem is given by: $Max\tilde{W}(y)\approx\left(\frac{1}{5},\frac{32}{5},\frac{88}{5},\frac{151}{5}\right)$ where $\tilde{y}_1=\left(\frac{3}{5},\frac{4}{5},\frac{4}{5},1\right)$, $\tilde{y}_2=\left(\frac{-1}{5},\frac{2}{5},\frac{6}{5},\frac{9}{5}\right)$ and $\tilde{y}_3=\left(0,0,\frac{2}{5},\frac{2}{5}\right)$

Remark 5. We see that both primal and dual problems have fuzzy optimal solutions and the two fuzzy optimal values are equal. In contrast to most existing approaches [3] and [4], the centers of all constraints are saturated and some constraint is saturated.

Example 4. Consider the following linear programming problem with variables given as Trapezoidal fuzzy numbers [3] and [4]:

Max $\tilde{Z}=(11,13,15,17)\tilde{x}_1+(9,12,14,17)\tilde{x}_2+(13,15,17,19)\tilde{x}_3$ Subject to the constraints

$12\tilde{x}_1+13\tilde{x}_2+12\tilde{x}_3 \quad (469,475,505,511)$, $14\tilde{x}_1+13\tilde{x}_3 \quad (452,460,480,488)$ and

$12\tilde{x}_1+15\tilde{x}_2 \quad (460,465,495,500)$.

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Step 1. Solving $Eq.(6)$ via $Eq.(10)$. We have $p=2$, $q=3$. We get Max $Z^{23}\left(x^{23}\right)=14x_1^{23}+13x_2^{23}+16x_3^{23}$ subject to the constraints $12x_1^{23}+13x_2^{23}+12x_3^{23}\le 490$, $14x_1^{23}+13x_3^{23}\le 470$ and $12x_1^{23}+15x_2^{23}\le 480$. Optimal solution: $x_1^{23}=0$, $x_2^{23}=\frac{730}{169}$ and $x_3^{23}=\frac{470}{13}=\frac{6110}{169}$. Slack variables values: $x_4^{23}=0$, $x_5^{23}=0$ and $x_6^{23}=\frac{70170}{169}$. Very important decision: For $x_4^{23}=0$ and $x_5^{23}=0$, we have $12w\left(\bar{x}_1^{23}\right)+13w\left(\bar{x}_2^{23}\right)+12w\left(\bar{x}_3^{23}\right)=w\left(\bar{b}_1^{23}\right)=15$ and $14w\left(\bar{x}_1^{23}\right)+13w\left(\bar{x}_3^{23}\right)=w\left(\bar{b}_2^{23}\right)=10$. We get $w\left(\bar{x}_1^{23}\right)=0, w\left(\bar{x}_2^{23}\right)=\frac{75}{169}$ and $w\left(\bar{x}_3^{23}\right)=\frac{10}{13}$. Therefore, we get $\bar{x}_1^{23}=\bar{0}$, $\bar{x}_2^{23}=\left[\frac{655}{169},\frac{805}{169}\right]$ and $\bar{x}_3^{23}=\left[\frac{460}{13},\frac{480}{13}\right]$.

Step 2. Solving $Eq.(6)$ via $Eq.(10)$. We have $p=1$, $q=4$. We get Max $Z^{14}\left(x^{14}\right)=14x_1^{14}+13x_2^{14}+16x_3^{14}$ subject to the constraints $12x_1^{14}+13x_2^{14}+12x_3^{14}\le 490$, $14x_1^{14}+13x_3^{14}\le 470$ and $12x_1^{14}+15x_2^{14}\le 480$. Optimal solution: $x_1^{14}=0$, $x_2^{14}=\frac{730}{169}$ and $x_3^{14}=\frac{470}{13}=\frac{6110}{169}$. Slack variables values: $x_4^{14}=0$, $x_5^{14}=0$ and $x_6^{14}=\frac{70170}{169}$. Important decision: For $x_4^{14}=0$, we have $12w\left(\bar{x}_1^{14}\right)+13w\left(\bar{x}_2^{14}\right)+12w\left(\bar{x}_3^{14}\right)=w\left(\bar{b}_1^{14}\right)=21$. We get $w\left(\bar{x}_1^{14}\right)=0$, $w\left(\bar{x}_2^{14}\right)=\frac{21}{26}$ and $w\left(\bar{x}_3^{14}\right)=\frac{21}{24}$ with $\left|x_2^{14}-x_2^{23}\right|+\frac{75}{169}=\frac{75}{169}\le\frac{21}{26}$ and $\left|x_3^{14}-x_3^{23}\right|+\frac{10}{13}=\frac{10}{13}\le\frac{21}{24}$. Therefore, we get $\bar{x}_1^{14}=\bar{0}$, $\bar{x}_2^{14}=\left[\frac{15431}{4394},\frac{22529}{4394}\right]$ and $\bar{x}_3^{14}=\left[\frac{143091}{4056},\frac{150189}{4056}\right]$.

Step 3. The optimal solution according to the choice of the decision maker is $Max\,\tilde{Z}\left(\tilde{x}\right)\approx\left(\frac{51697854}{105456},\frac{97560}{169},\frac{117350}{169},\frac{83385198}{105456}\right)$ where $\tilde{x}_1=\tilde{0}$, $\tilde{x}_2=\left(\frac{15431}{4394},\frac{655}{169},\frac{805}{169},\frac{22529}{4394}\right)$ and $\tilde{x}_3=\left(\frac{143091}{4056},\frac{460}{13},\frac{480}{13},\frac{150189}{4056}\right)$. Then the corresponding dual problem is given by: $Min\,\tilde{W}\left(y\right)\approx\left(\frac{52443066}{105456},\frac{97560}{169},\frac{117350}{169},\frac{82396626}{105456}\right)$ where $\tilde{y}_1=\left(\frac{21}{24},\frac{12}{23},\frac{14}{13},\frac{27}{24}\right)$, $\tilde{y}_2=\left(\frac{5}{26},\frac{51}{169},\frac{53}{169},\frac{11}{26}\right)$ and $\tilde{y}_3=\tilde{0}$.

Remark 6. We see that both primal and dual problems have fuzzy optimal solutions and the two fuzzy optimal values are equal. In contrast to most existing approaches [3] and [4], the centers of all constraints are saturated and some constraint is saturated.

Example 5. Consider the following linear programming problem with variables given as Hexagonal fuzzy numbers [6]: Max $\tilde{Z}\left(\tilde{x}\right)\approx\left(11,13,15,17,19,21\right)\tilde{x}_1+\left(31,33,35,37,39,41\right)\tilde{x}_2$ subject to the constraints $69\tilde{x}_1+99\tilde{x}_2\quad\left(151,153,155,157,159,161\right)$ and

$129\tilde{x}_1+159\tilde{x}_2\quad\left(271,273,275,277,279,281\right)$.

Step 1. Solving $Eq.(6)$ via $Eq.(10)$. We have $p=3$, $q=4$. We get Max $Z^{34}\left(x^{34}\right)=16x_1^{34}+36x_2^{34}$ subject to the constraints $69x_1^{34}+99x_2^{34}\le 156$ and $129x_1^{34}+159x_2^{34}\le 276$. Optimal solution: $x_1^{34}=0$ and $x_2^{34}=\frac{52}{33}$. Slack variables values: $x_3^{34}=0$ and $x_4^{34}=\frac{280}{11}$. Very important decision: For $x_3^{34}=0$, we have $66w\left(\bar{x}_1^{34}\right)+99w\left(\bar{x}_2^{34}\right)=w\left(\bar{b}_2^{34}\right)=1$. We get $w\left(\bar{x}_1^{34}\right)=0$ and $w\left(\bar{x}_2^{34}\right)=\frac{1}{99}$. Therefore, we get $\bar{x}_1^{34}=[0,0]$ and $\bar{x}_2^{34}=\left[\frac{155}{99},\frac{157}{99}\right]$.

Step 2. Solving $Eq.(6)$ via $Eq.(10)$. We have $p=2$, $q=5$. We get Max $Z^{25}\left(x^{25}\right)=16x_1^{25}+36x_2^{25}$ subject to the constraints $69x_1^{25}+99x_2^{25}\le 156$ and $129x_1^{25}+159x_2^{25}\le 276$. Optimal solution: $x_1^{25}=0$ and $x_2^{25}=\frac{52}{33}$. Slack variables values: $x_3^{25}=0$ and $x_4^{25}=\frac{280}{11}$. We get $w\left(\bar{x}_1^{25}\right)=0$ and $w\left(\bar{x}_2^{25}\right)=\frac{1}{33}$ with $\left|x_2^{25}-x_2^{32}\right|=0\le\frac{1}{33}$. Therefore, we get $\bar{x}_1^{25}=[0,0]$ and $\bar{x}_2^{25}=\left[\frac{17}{11},\frac{53}{33}\right]$.

Step 3. Solving $Eq.(6)$ via $Eq.(10)$. We have $p=1$, $q=6$. We get Max $Z^{16}\left(x^{16}\right)=16x_1^{16}+36x_2^{16}$ subject to the constraints $69x_1^{16}+99x_2^{16}\le 156$ and $129x_1^{16}+159x_2^{16}\le 276$. Optimal solution: $x_1^{16}=0$ and $x_2^{16}=\frac{52}{33}$. Slack variables values: $x_3^{16}=0$ and $x_4^{16}=\frac{280}{11}$. Very important decision: For $x_3^{16}=0$, we have $66w\left(\bar{x}_1^{16}\right)+99w\left(\bar{x}_2^{16}\right)=w\left(\bar{b}_1^{16}\right)=5$. We get $w\left(\bar{x}_1^{16}\right)=0$ and $w\left(\bar{x}_2^{16}\right)=\frac{5}{99}$ with $\left|x_2^{16}-x_2^{25}\right|+\frac{1}{33}=\frac{1}{33}\le\frac{5}{99}$. Therefore, we get $\bar{x}_1^{16}=[0,0]$ and $\bar{x}_2^{16}=\left[\frac{151}{99},\frac{161}{99}\right]$.

Step 4. The optimal solution according to the choice of the decision maker is $Max\tilde{Z}(\tilde{x})\approx\left(\frac{4681}{99},\frac{5049}{99},\frac{5425}{99},\frac{5809}{99},\frac{6201}{99},\frac{6601}{99}\right)$ with $\tilde{x}_1=\tilde{0}$ and $\tilde{x}_2=\left(\frac{151}{99},\frac{153}{99},\frac{155}{99},\frac{157}{99},\frac{159}{99},\frac{161}{99}\right)$. Then the corresponding dual problem is given by: $Min\tilde{W}(y)\approx\left(\frac{4681}{99},\frac{5049}{99},\frac{5425}{99},\frac{5809}{99},\frac{6201}{99},\frac{6601}{99}\right)$ where $\tilde{y}_1=\left(\frac{31}{99},\frac{33}{99},\frac{35}{99},\frac{37}{99},\frac{39}{99},\frac{41}{99}\right)$ and $y_2=\tilde{0}$.

Remark 7. We see that both primal and dual problems have fuzzy optimal solutions and the two fuzzy optimal values are equal. In contrast to most existing approaches [6], the centers of all constraints are saturated and some constraint is saturated.

Example 6. Consider the following linear programming problem with variables given as Octagonal fuzzy numbers [19]: Min $\tilde{Z}(\tilde{x})\approx(2,3,4,5,7,8,9,10)\tilde{x}_1+(3,4,5,7,9,11,12,13)\tilde{x}_2$ subject to the constraint

$$20\tilde{x}_1+30\tilde{x}_2 \quad (885,886,888,890,910,912,914,915)$$ and

$$40\tilde{x}_1+30\tilde{x}_2 \quad (1190,1191,1193,1195,1205,1207,1209,1210).$$

Step 1. Solving $Eq.(6)$ via $Eq.(10)$. We have $p=4$, $q=5$. We get Min $Z^{45}\left(x^{45}\right)=6x_1^{45}+8x_2^{45}$ subject to the constraints $20x_1^{45}+30x_2^{45}\geq 900$ and $40x_1^{45}+30x_2^{45}\geq 1200$. Optimal solution: $x_1^{45}=15$ and $x_2^{45}=20$. Slack variables values: $x_3^{45}=0 \triangleright$ and $x_4^{45}=0$. Important decision: For $x_4^{45}=0$, we have $40w\left(\bar{x}_1^{45}\right)+30w\left(\bar{x}_2^{45}\right)=w\left(\bar{b}_2^{45}\right)=5$. We get $w\left(\bar{x}_1^{45}\right)=\frac{1}{20}$ and $w\left(\bar{x}_2^{45}\right)=\frac{2}{20}$. Therefore, we get $\bar{x}_1^{45}=\left[\frac{299}{20},\frac{301}{20}\right]$ and $\bar{x}_2^{45}=\left[\frac{398}{20},\frac{402}{20}\right]$.

Step 2. Solving $Eq.(6)$ via $Eq.(10)$. We have $p=3$, $q=6$. We get Min $Z^{36}\left(x^{36}\right)=6x_1^{36}+8x_2^{36}$ subject to the constraints $20x_1^{36}+30x_2^{36}\geq 900$ and $40x_1^{36}+30x_2^{36}\geq 1200$. Optimal solution: $x_1^{36}=15$ and $x_2^{36}=20$. Slack variables values: $x_3^{36}=0$ and $x_4^{36}=0$. Important decision: For $x_4^{36}=0$, we have $40w\left(\bar{x}_1^{36}\right)+30w\left(\bar{x}_2^{36}\right)=w\left(\bar{b}_2^{36}\right)=7$. We get $w\left(\bar{x}_1^{36}\right)=\frac{2}{20}$ and $w\left(\bar{x}_2^{36}\right)=\frac{2}{20}$ with $\left|x_1^{36}-x_1^{45}\right|+\frac{1}{20}=\frac{1}{20}\leq\frac{2}{20}$ and $\left|x_2^{36}-x_2^{45}\right|+\frac{2}{20}=\frac{2}{20}\leq\frac{2}{20}$. Therefore, we get $\bar{x}_1^{36}=\left[\frac{298}{20},\frac{302}{20}\right]$ and $\bar{x}_2^{36}=\left[\frac{398}{20},\frac{402}{20}\right]$.

Step 3. Solving $Eq.(6)$ via $Eq.(10)$. We have $p=2$, $q=7$. We get Min $Z^{27}\left(x^{27}\right)=6x_1^{27}+8x_2^{27}$ subject to the constraints $20x_1^{27}+30x_2^{27}\geq 900$ and $40x_1^{27}+30x_2^{27}\geq 1200$. Optimal solution: $x_1^{27}=15$ and $x_2^{27}=20$. Slack variables values: $x_3^{27}=0$ and $x_4^{27}=0$. Important decision: For $x_4^{27}=0$, we have $40w\left(\bar{x}_1^{27}\right)+30w\left(\bar{x}_2^{27}\right)=w\left(\bar{b}_2^{27}\right)=9$. We get $w\left(\bar{x}_1^{27}\right)=\frac{2}{20}$ and $w\left(\bar{x}_2^{27}\right)=\frac{1}{6}$ with $\left|x_1^{27}-x_1^{36}\right|+\frac{2}{20}=\frac{2}{20}\leq\frac{2}{20}$ and $\left|x_2^{27}-x_2^{36}\right|+\frac{2}{20}=\frac{2}{20}\leq\frac{1}{6}$. Therefore, we get $\bar{x}_1^{27}=\left[\frac{298}{20},\frac{302}{20}\right]$ and $\bar{x}_2^{27}=\left[\frac{1190}{60},\frac{1210}{60}\right]$.

Step 4. Solving $Eq.(6)$ via $Eq.(10)$. We have $p=1$, $q=8$. We get Min $Z^{18}\left(x^{18}\right)=6x_1^{18}+8x_2^{18}$ subject to the constraints $20x_1^{18}+30x_2^{18}\geq 900$ and $40x_1^{18}+30x_2^{18}\geq 1200$. Optimal solution: $x_1^{18}=15$ and $x_2^{18}=20$. Slack variables values: $x_3^{18}=0$ and $x_4^{18}=0$. Important decision: For $x_4^{18}=0$, we have $40w\left(\bar{x}_1^{18}\right)+30w\left(\bar{x}_2^{18}\right)=w\left(\bar{b}_2^{18}\right)=10$. We get $w\left(\bar{x}_1^{18}\right)=\frac{5}{40}$ and $w\left(\bar{x}_2^{18}\right)=\frac{1}{6}$ with $\left|x_1^{18}-x_1^{27}\right|+\frac{2}{20}=\frac{2}{20}\leq\frac{5}{40}$ and $\left|x_2^{18}-x_2^{27}\right|+\frac{1}{6}=\frac{1}{6}\leq\frac{1}{6}$. Therefore, we get $\bar{x}_1^{18}=\left[\frac{595}{40},\frac{605}{40}\right]$ and $\bar{x}_2^{18}=\left[\frac{119}{6},\frac{121}{6}\right]$.

Step 5. The optimal solution according to the choice of the decision maker is $Min\tilde{Z}\left(\tilde{x}\right)\approx\left(\frac{10710}{120},\frac{14884}{120},\frac{19116}{120},\frac{25686}{120},\frac{34350}{120},\frac{41028}{120},\frac{45348}{120},\frac{49610}{120}\right)$ with $\tilde{x}_1=\left(\frac{1785}{120},\frac{1788}{120},\frac{1794}{120},\frac{1794}{120},\frac{1806}{120},\frac{1812}{120},\frac{1812}{120},\frac{1815}{120}\right)$ and

$\tilde{x}_2 = \left(\frac{2380}{120}, \frac{2380}{120}, \frac{2388}{120}, \frac{2388}{120}, \frac{2412}{120}, \frac{2412}{120}, \frac{2420}{120}, \frac{2420}{120}\right)$. Then the corresponding dual problem is given by: $Max\,\tilde{W}(y) \approx \left(\frac{9320}{120}, \frac{13530}{120}, \frac{17760}{120}, \frac{25530}{120}, \frac{34530}{120}, \frac{42488}{120}, \frac{46822}{120}, \frac{51120}{120}\right)$ where $\tilde{y}_1 = \left(\frac{16}{120}, \frac{18}{120}, \frac{20}{120}, \frac{26}{120}, \frac{30}{120}, \frac{36}{120}, \frac{38}{120}, \frac{40}{120}\right)$ and

$$\tilde{y}_2 = \left(\frac{-4}{120}, \frac{-2}{120}, 0, \frac{2}{120}, \frac{6}{120}, \frac{8}{120}, \frac{10}{120}, \frac{12}{120}\right).$$

Remark 8. We see that both primal and dual problems have fuzzy optimal solutions and the two fuzzy optimal values are equal. In contrast to most existing approaches [7], the centers of all constraints are saturated and some constraint is saturated.

5 | Advantages of the Proposed Method over the Existing Methods

To be more specific, we will concentrate on showing the advantages of the proposed method over the well-known existing methods existing methods proposed by [1], [2], [3], [4], [6], [7], [18].

The advantages of the new method proposed over the existing methods proposed by [1], [2], [3], [4], [6], [7], [18] can be summarized as follows:

I. The new method improves the existing methods for solving the semi-fully interval and semi-fully fuzzy linear programming problems.

II. The new method improves the existing methods for solving the interval Transportation Problems and Fully Fuzzy Transportation Problems [6] and [15].

III. In contrast to most existing approaches, our method of transforming a fuzzy number into interval numbers is the first. So, our method proposed is the first.

IV. The proposed technique does not use the goal and parametric approaches which are difficult to apply in real life situations. These difficulties (or limitations) are overcome by the new proposed method.

V. To solve the *Eq. (12)* by using the existing method, there is need to use arithmetic operations of generalized fuzzy numbers. While, if the proposed technique is used for the same then there is need to use arithmetic operations of real numbers. This proves that it is much easy to apply the proposed method as compared to the existing method.

VI. In contrast to most existing approaches, which provide an optimal solution using ranking function, the proposed method provides a fuzzy optimal solution without using ranking function. Similarly, to the competing methods in the literature, the proposed method is applicable for all types of fuzzy numbers.

VII. Also, the fuzzy optimal solution, obtained by using the new method mentioned, will always exactly satisfy the centers of all the constraints and some constraints.

6 | Concluding Remarks and Future Research Directions

6.1 | Concluding Remarks

The present paper proposes an alternative solution approach for solving the semi-fully fuzzy linear programming problem where the coefficients in the objective function, the right-hand side vector and the decision variables are a kind of fuzzy numbers, simultaneity. Firstly, the Semi-fully Fuzzy Linear Programming Problem is transformed into equivalent semi-fully interval linear programming problems. After that, the solutions to these interval linear programming problems are then obtained with the help of linear programming technique. The comparisons numerical examples show that in all problems the proposed method provides a better solution than the existing methods [1], [2], [3], [4], [6], [7], [18]. So, the proposed approach can be considered as an alternative approach for solving the Semi-fully Fuzzy Linear Programming Problems if decision maker is interested in finding the fuzzy optimal solution with minimum uncertainty.

6.2 | Future Research Directions

Finally, we feel that, there are many other points of research and should be studied later on interval numbers or fuzzy numbers. Some of these points are below:

- *Linear programming problem with generalized interval-valued fuzzy numbers.*
- *Interval-valued intuitionistic semi-fully fuzzy linear programming problem.*
- *Semi-Fully fuzzy linear fractional programming problems with fuzzy numbers and intuitionistic fuzzy.*

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Conflicts of Interest

All authors have contributed equally in this work. The authors declare that there is no conflict of interest for this publication.

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