



## **Assess the Efficiency and Effectiveness Simultaneously in a Three-Stage Process, By Using a Unified Model**

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### **ABSTRACT**

By distinction between efficiency and effectiveness scales, the aim of this paper is to propose a model that can show the different of efficiency and effectiveness. For this purpose, enveloping form of ICCR model, has considered to calculate simultaneously the influences of efficiency and effectiveness. this model, is a linear programming model based on Data envelopment analysis (DEA), that combine the input and output oriented CCR model to investigate the efficiency and effectiveness impressed each other ,in a three-stage process. By applying the model on data of 24 bank branches, the result clarify comprehensive view of the performance of the branches that have been substantially three-stage.

**Keywords:** *Data envelopment analysis, efficiency, effectiveness.*

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## **1. Introduction**

Data envelopment analysis (DEA) is a method to estimate the relative efficiency of a set of decision making units (DMU<sub>s</sub>) that calculates by mathematical programming. The use of relative term is because of it is the result of comparing the efficiency of the units together. In a firm, a number of inputs produce some outputs that should be in line with organizational goals. If we assess the optimal use of inputs in order to produce output, we have measured the efficiency, and if we assess the realization of the goals of generated output, we have measured the effectiveness. Efficiency and effectiveness have different meanings that are sometimes mistakenly used interchangeably. In the past, some DEA application is used in order to demonstrate the differentiation of efficiency and effectiveness of the decision making units (DMU<sub>s</sub>) in various organizations. Yu and Lee [1] evaluate the performance of international tourist hotels in terms of efficiency and effectiveness.

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Herbert et al, [2] survey on a network DEA model to compute the efficiency and effectiveness of the Major league Baseball. Wang et al,[3] has used the Data envelopment analysis(DEA), with respect to the impact of information technology on stages, to modeling the units that contain two-stage, in situation that first stage output, by impact of information technology, is the second stage input. This model compute the efficiency of the first and second stage, and the average efficiency of two intermediate stage, gives the total efficiency of DMUs. They for improving the impact of the IT model on efficiency of company, has used DEA to study the final benefit of IT, according to a two-stage method in banking industry. Chen and Zhu [4], proposed a fundamental DEA model that introduce efficient companies in a two-stage production process, and specified the influence of final benefit of IT on productivity.

Efficiency measurement (conversion of factor to produce) and effectiveness measurement (consumption of production) are the sum for the storable commodities, because these products can be stored until they are consumed. Chiou et al. [5], introduced a nonlinear model of network DEA to show that the efficiency and effectiveness for non-storable commodities, distinct; such as transportation services. These products are used along with their supply and cannot store, and when such products are produced the effectiveness will be less than efficiency if some of them do not consume. So each poor performance of such services, can be related to poor efficiency and effectiveness or both of them, and by no distinction between efficiency and effectiveness scales, it is difficult to assess the precise sources of poor performance. In other words, the system performance evaluation for these products, will be much more useful, if we could analyze the efficiency and effectiveness scales simultaneously. To explain this concept Fielding [6] introduced a network DEA method for a public transit systems. although the ICCR model provided by Chiou et al ,is based on DEA fundamental research, the achieved model is a nonlinear programming model in the multiplier form, that assess efficiency and effectiveness simultaneously during a two-stage process with an additive measure and equal weight. Hosseinzadeh et al [78], apply a linear model of ICCR in an envelope form which considering both efficiency and effectiveness simultaneously in a unified manner and two stage approach in Bank industry. In this proposed model, the input and output oriented CCR model combined, that the first process use in second process, which means that the first process is assessed based on input oriented and second process is evaluated based on output oriented.

In this paper, we use the envelope form of ICCR model for modeling the units that included three stage, such that the first stage output is second stage input and the second stage output is the input of the third stage. Present paper will refer to following sections: second section will remind some preliminaries. In third section, we describe the new model; in the Section 4, we investigate the model on the banks that have three stage nature. And finally, the conclusions and suggestions are mentioned in Section 5.

## **2. Preliminaries**

Data envelopment analysis (DEA) is an application of mathematical programming that assesses the relatively efficiency of each Decision Making Units (DMUs), and provides the possibility of a decision making unit performance comparison during various years.

There are some standard models in data envelopment analysis that help us to evaluate an organization performance. The CCR model, is offered by Charnes, Cooper and Rhodes(1978), under the constant returns to scale (CRS) assumption and the BCC model, is given by Banker, Charnes and Cooper(1984) under the variable returns to scale (VRS) assumption, have been proposed as the basic models of data envelopment analysis. Chiou et al(2010), by introducing the ICCR and IBCC models, surveyed the efficiency and effectiveness comparison by measuring efficiency and effectiveness simultaneously in an additive manner, under constant and variable return to scale technologies for non-storable commodities. The ICCR model, provided with Chiou et al,'s formulate as follows:

$$\begin{aligned}
 \text{[ICCR]} \quad \text{Max } H_k &= \left( \frac{\sum_{r=1}^R u_r y_{kr}}{\sum_{j=1}^J v_j x_{kj}} \right) + \left( \frac{\sum_{s=1}^S w_s z_{ks}}{\sum_{r=1}^R u_r y_{kr}} \right) \\
 \text{s.t.} \quad \sum_{r=1}^R u_r y_{ir} &\leq \sum_{j=1}^J v_j x_{ij} \quad , \quad i = 1, 2, \dots, I \\
 \sum_{s=1}^S w_s z_{is} &\leq \sum_{r=1}^R u_r y_{ir} \quad , \quad i = 1, 2, \dots, I \\
 v_j &\geq 0 \quad j = 1, \dots, J \\
 w_s &\geq 0 \quad s = 1, \dots, S \\
 u_r &\geq 0 \quad r = 1, \dots, R
 \end{aligned} \tag{2.1}$$

By implementing this model on the data, the total efficiency number  $H_k \in [0, 2]$  is achieved for  $DMU_k$ . If  $H_k$  be equal to 2, the  $DMU_k$  is defined as the relative efficiency, and is relatively inefficient otherwise. In this model, the weights  $v_j, u_r$  and  $w_s$  are determined associated with factors, production and consumption under constant returns to scale (CRS) technology, so that the total efficiency is maximized. IBCC model is formulated as follows by adding the variable corresponding convexity restriction to ICCR model:

$$\begin{aligned}
 \text{[IBCC]} \quad \text{Max}_{u,v,w} H_k &= \left( \frac{\sum_{r=1}^R u_r y_{kr} - u_0}{\sum_{j=1}^J v_j x_{kj}} \right) + \left( \frac{\sum_{s=1}^S w_s z_{ks} - u_1}{\sum_{r=1}^R u_r y_{kr} - u_0} \right) \\
 \text{s.t.} \quad \sum_{r=1}^R u_r y_{ir} - u_0 &\leq \sum_{j=1}^J v_j (x_{ij} - s_{ij}), \quad i = 1, \dots, I \\
 \sum_{s=1}^S w_s (z_{is} + s_{is}) - u_1 &\leq \sum_{r=1}^R u_r y_{ir} - u_0, \quad i = 1, \dots, I \\
 v_j &\geq 0 \quad j = 1, \dots, J \\
 w_s &\geq 0 \quad s = 1, \dots, S \\
 u_r &\geq 0 \quad r = 1, \dots, R
 \end{aligned} \tag{2.2}$$

The enveloping form of ICCR model, that was introduced by Hosseinzadeh et al, is as follows:

$$\begin{aligned}
 & \max \phi - \theta \\
 \text{s.t. } & \sum_{j=1}^J \lambda_j x_{ij} - s_i^- = \theta x_{ip}, & i \in I_1, \\
 & \sum_{j=1}^J \lambda_j x_{ij} - s_i^- = \theta x_{ip}, & i \in I_2, \\
 & \sum_{j=1}^J \lambda_j y_{rj} = \sum_{j=1}^J \mu_j y_{rj}, & r = 1, \dots, R, \\
 & \sum_{j=1}^J \lambda_j y_{rj} + n_r - p_r = y_{rp}, & r = 1, \dots, R, \\
 & \sum_{j=1}^J \mu_j y_{rj} + a_r - b_r = y_{rp}, & r = 1, \dots, R, \\
 & \sum_{j=1}^J \mu_j z_{sj} \geq \phi z_{sp}, & s = 1, \dots, S, \quad (2.3) \\
 & \theta x_{ip} + s_i^- = t_i, & i \in I_2, \\
 & t_i \in Z^+, & i \in I_2, \\
 & \theta \leq 1, \quad \phi \geq 1, \\
 & \lambda_j \geq 0, & j = 1, \dots, J, \\
 & n_r \geq 0, \quad p_r \geq 0, \quad a_r \geq 0, \quad b_r \geq 0, & j = 1, \dots, J.
 \end{aligned}$$

It should be noted that this enveloping form is not dual of multiplier form provided by Chiu et al, and the advantage of this model compared to the multiplier models provided by Chiu et al is that the model is linear and gives us the bench mark unit in integer.

Also, the IBCC model enveloping form obtained by adding the  $\sum_{j=1}^J \lambda_j = 1$  restriction.

### 3. A three stage model for unified approach

For distinction between efficiency and effectiveness, Chiu et al (2010), used network DEA and offered the ICCR and IBCC models that are been multiplier form. To explain the same concept, Hosseinzadeh et al, provided models with enveloping form for the units that contain two stages. In this paper, we use DEA to formulate the units with three stages.

In general DEA models , for each DMU<sub>s</sub> number of inputs enter and number of outputs leave, such that there is no intermediate stages of production. In three stages models, we have three SUB-DMU<sub>s</sub> that should pay attention to their behavior. In three stages DEA, the output of SUB-DMU<sub>1</sub> is the SUB-DMU<sub>2</sub> input and the output of SUB-DMU<sub>2</sub> is the SUB-DMU<sub>3</sub> input. The advantage of this model is that by solving a model, it calculates simultaneously first and second stages efficiency and also third stage effectiveness. Figure 1 shows an overall three-stage process which in the first stage applies the inputs  $x_i (i=1,2,\dots,I)$  to produce outputs  $y_r (r=1,2,\dots,R)$  and then uses these  $y_r$  as the second stage inputs to generate  $w_t (t=1,2,\dots,t)$  outputs and finally by applying  $w_t$  as the inputs of third stage, bring  $z_s (s=1,2,\dots,S)$  in to being as the third stage outputs.

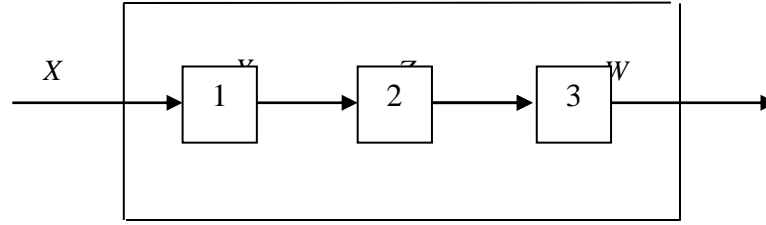


Figure 1: A DMU with three processes.

The presented model for this three stage process is as follows:

$$\begin{aligned}
 & \max \quad \phi - \theta \\
 & \text{s.t.} \quad \sum_{j=1}^J \lambda_j x_{ij} \leq \theta x_{ip}, \quad i \in 1, 2, \dots, I, \\
 & \quad \sum_{j=1}^J \lambda_j y_{rj} + a_r - b_r = y_{rp}, \quad r = 1, \dots, R, \\
 & \quad \sum_{j=1}^J \mu_j y_{rj} + n_r - p_r = y_{rp}, \quad r = 1, \dots, R, \\
 & \quad \sum_{j=1}^J \lambda_j y_{rj} = \sum_{j=1}^J \mu_j y_{rj}, \quad r = 1, \dots, R, \\
 & \quad \sum_{j=1}^J \mu_j w_{tj} = \sum_{j=1}^J \gamma_j w_{tj}, \quad t = 1, \dots, T, \quad (3.1)
 \end{aligned}$$

$$\begin{aligned}
 \sum_{j=1}^J \mu_j w_{tj} + nn_t - pp_t &= w_{tp}, & t = 1, \dots, T, \\
 \sum_{j=1}^J \gamma_j w_{tj} + aa_t - bb_t &= w_{tp}, & t = 1, \dots, T, \\
 \sum_{j=1}^J \gamma_j z_{sj} &\geq \phi z_{sp}, & s = 1, \dots, S, \\
 \theta &\leq 1, \quad \phi \geq 1, \\
 \lambda_j &\geq 0, & j = 1, \dots, J, \\
 n_r \geq 0, \quad p_r \geq 0, \quad a_r \geq 0, \quad b_r \geq 0, & & r = 1, \dots, R. \\
 nn_t \geq 0, \quad pp_t \geq 0, \quad aa_t \geq 0, \quad bb_t \geq 0, & & t = 1, \dots, T.
 \end{aligned}$$

That the effectiveness for DMU<sub>k</sub> is defined by  $\phi - \theta$ ; And optimal solutions  $\theta^*$  and  $\phi^*$  show efficiency of attracting resource, efficiency of resource in this model respectively. Also  $\lambda_j, \mu_j$  and  $\gamma_j$  indicate variables corresponding to each stage. The positive variables  $n, p, a, b$ , apply to correspond the changes of  $y$  when it is the output of the first stage process and the change of  $y$  when it is the input of the second stage process. And also the positive variable  $nn, pp, aa, bb$ , are applied to correspond the changes between  $W$  when it is the output of the second stage and the changes between  $W$  when it is the input of the third stage process.  $\theta \leq 1$  and  $\phi \geq 1$  are enforced in the model to achieve a pareto-efficient unit by decreasing the inputs and increasing the outputs.

All of the above are true for the variable returns to scale technology, and similarly the envelope form of IBCC model is introduced by adding the convexity restriction to ICCR model, for a three-stage process:

$$\begin{aligned}
 \max \quad & \phi - \theta \\
 \text{s.t.} \quad & \sum_{j=1}^J \lambda_j x_{ij} \leq \theta x_{ip}, & i \in 1, 2, \dots, I, \\
 & \sum_{j=1}^J \lambda_j y_{rj} + a_r - b_r = y_{rp}, & r = 1, \dots, R, \\
 & \sum_{j=1}^J \mu_j y_{rj} + n_r - p_r = y_{rp}, & r = 1, \dots, R, \\
 & \sum_{j=1}^J \lambda_j y_{rj} = \sum_{j=1}^J \mu_j y_{rj}, & r = 1, \dots, R, \\
 & \sum_{j=1}^J \mu_j w_{tj} = \sum_{j=1}^J \gamma_j w_{tj}, & t = 1, \dots, T,
 \end{aligned} \tag{3.2}$$

$$\begin{aligned} \sum_{j=1}^J \mu_j w_{tj} + nn_t - pp_t &= w_{tp}, & t = 1, \dots, T, \\ \sum_{j=1}^J \gamma_j w_{tj} + aa_t - bb_t &= w_{tp}, & t = 1, \dots, T, \\ \sum_{j=1}^J \gamma_j z_{sj} &\geq \phi z_{sp}, & s = 1, \dots, S, \\ \sum_{j=1}^J \lambda_j &= 1, \\ \theta &\leq 1, \quad \phi \geq 1, \\ \lambda_j &\geq 0, & j = 1, \dots, J, \\ n_r \geq 0, \quad p_r \geq 0, \quad a_r \geq 0, \quad b_r \geq 0, & r = 1, \dots, R. \\ nn_t \geq 0, \quad pp_t \geq 0, \quad aa_t \geq 0, \quad bb_t \geq 0, & t = 1, \dots, T. \end{aligned}$$

#### 4. Application

In this section, we study our model on 24 Iran bank branches that all have three phase nature. Figure 2 shows a bank performance in a three stage process; That the first stage, displays the efficiency of attracting resource, the second stage shows the efficiency of resource allocation and the third stage present effectiveness.

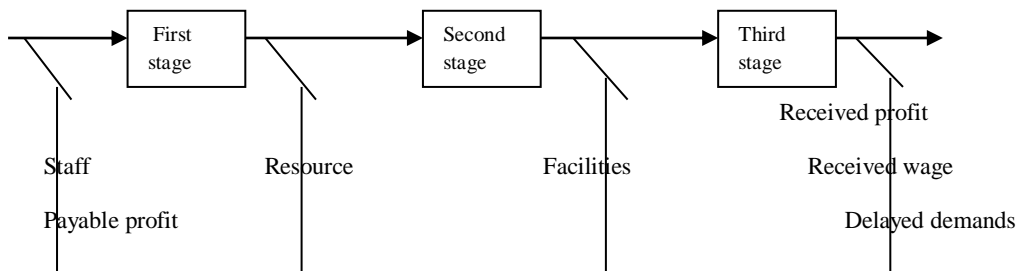


Figure 2: A Bank performance with three-stage.

In the first stage, banks apply number of staff and payable profit as the inputs to make the resources for intermediate size. In the second stage, the loans allocated to resources (Facilities); Facilities are used as outputs for the second stage. The third stage indicates that these facilities have profit, received wage and some when loss (demands) for the bank.

Data set includes 24 DMU<sub>s</sub> are shown completely in the Tables1.

**Table.1** Inputs and Outputs

DMU	Inputs		intermediate size1	intermediate size2	outputs		
	Payable profit	staff	Resource	Facilities	Received profit	Received wage	Delayed demands
1	1838.88	113	393998	384795	273.88	1656.58	33467
2	1393.47	112	209171.51	134204.23	2471.58	124.79	11337.76
3	1040.88	101	186221.89	248038.6	2106.21	67.48	14263.4
4	66.52	119	244917	664811	9283.09	2380.11	8978.5
5	2400.57	112	273851.83	669040.83	23751.97	87.85	20814.83
6	1344.64	132	197872.4	73446.28	1385.84	49.52	6323
7	602.52	121	228141.49	91374.83	1432.22	12.44	5741.33
8	499.47	115	153285	310363	14835.11	662.28	82756
9	615.42	126	80994	13895	251.45	7.01	3841
10	1698.81	129	244411	100082	5488.6	62.41	2237
11	618.35	117	114865.5	214697.5	1693.87	43.36	12677
12	385.55	106	84059	25323	189.61	10.16	21743
13	890.64	118	122102.84	108202.71	1478.75	31.16	9450.57
14	661.27	122	91972	75537	424.84	12.54	19617
15	728.25	105	311790	172312	677.27	976.22	12168
16	546.48	145	112085	95125	788.19	9.63	3912
17	524.39	141	149825.33	99800	736.34	321.13	12077.33
18	237.85	118	88078	33611	419.85	4.98	7673
19	552.1	153	883207	96373	787.32	23.39	221
20	28.59	118	137968	36189	739.8	8.23	5321
21	606.83	127	1120107	72190	823.9	35.29	2024
22	841.67	107	130345.99	84861	2139.03	38.02	42455
23	589.3	114	380212	649752	2439.86	302.48	10127
24	480.19	101	92170.99	110414	1288.49	31.49	377.33

There two possible ways to talk about intermediate sizes. When we consider 2,3,6, and 7 restriction on assessment, the results are shown in Table 2.

**Table.2** Results

DMU	ICCR model		
	Efficiency of attracting resource	Efficiency of resource	of effectiveness
1	1.00000	9.68974	8.68974
2	1.00000	98.87803	97.87803
3	1.00000	1.801 E+2	1.791 E+2
4	1.00000	6.80165	5.80165
5	1.00000	19.85035	18.85035
6	1.00000	2.104 E+2	2.094 E+2
7	1.00000	2.545 E+2	2.535 E+2
8	1.00000	24.06314	23.06314
9	1.00000	3.951 E+2	3.941 E+2
10	1.00000	1.312 E+2	1.302 E+2
11	1.00000	2.860 E+2	2.850 E+2
12	1.00000	7.762 E+2	7.752 E+2
13	1.00000	3.188 E+2	3.178 E+2
14	1.00000	6.408 E+2	6.398 E+2
15	1.00000	18.58724	17.58724
16	1.00000	3.201 E+2	3.191 E+2
17	1.00000	47.04747	46.04747
18	1.00000	3.860 E+2	3.850 E+2
19	1.00000	24.87633	23.87633
20	1.00000	54.40813	53.40813
21	1.00000	1.838 E+2	1.828 E+2
22	1.00000	3.096 E+2	3.086 E+2
23	1.00000	53.25225	52.25225
24	1.00000	43.99562	42.99562



The advantage of this model is that by solving a model, we calculate, simultaneously, first and second stages efficiency and also in the third stage, the effectiveness.

The other model is that to eliminate the 2,3,6 and 7 restrictions of the model and do not consider them in the assessment; Then the assessment results will be same as before and it seems that the removal of these restrictions does not make any change on the results.

## 5. Conclusion

In general DEA, the efficiency and effectiveness of a DMU can be calculate, but in rather to better performance assessment and show the efficiency and effectiveness distinction, it's needed to apply a model that can handling simultaneously efficiency and effectiveness. Chiou et al (2010) could show this distinction by providing ICCR and IBCC models. Hosseinzadeh et al, also by presenting a two stage unified model for simultaneously assessing both efficiency and effectiveness, could survey this distinction. In this paper by keeping the same idea of that model,a model presented to use the DMU<sub>s</sub> with three stage nature. Though both models provided by Hosseinzadeh et al, are based on fundamental studies of DEA, but there is a bug that they do not define the total performance.

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