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Information System Model for Educational Management in Supply Chain for Thai Higher Education Institutions

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Abstract

The research about information system model for educational management in supply chain for Thai higher education institutions. The objectives of research to design and to assess the suitability of an information system model for educational management in supply chain for Thai higher education institutions. The sample group consisted of ten experts in the field of information system, supply chain and curriculum. The data is analyzed by means and standardized deviations statistically. The research result shows that information system model for educational management in supply chain for Thai higher education institutions is consisted of 7 key elements which are 1) main elements, 2) raw materials, 3) suppliers, 4) manufacturer, 5) service provider, 6) finished product, and 7) customers. The results from experts' agreement information system model for educational management in supply chain for Thai higher education institutions was a high level. It showed that information system model for educational management in supply chain for Thai higher education institutions could be used to develop information system.

Keywords: Information system model, Educational management in supply chain, Thai higher education institutions.

1 | Introduction

According to national education act of B.E.1999, the main specifications are the quality of education which is the ultimate goal of educational reforms at all level. Today, the education system have significant, the Thai government has realized the importance of adjusting the country to increase its capability to compete with other countries in every aspect. Especially in educational development that leads to development of quality of the product , the government has formulated the following policy: To develop quality of people, as the people are human resource of the country and the key element in all aspects of development, to reform the whole system of education, to expand education and modify educational structure, to decentralize educational

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administration to the provinces so that educational management becomes more thorough and responsive to the local needs [10]. This policy also includes the establishment of private and public higher education institutions to meet the needs for national development, especially the people who are knowledgeable and skillful in various vocations, to work in private and public organizations. Higher education institutions need to produce graduates to meet the needs of the country for joining the ASEAN Economic Community in B.E. 2015. As such, the government has formulated an important policy that "The creation of a stable knowledge-based economy and environmental factors must support Thail and to be a center of goods and service production in the region based on creative thinking, creation of innovations, and extension of the body of knowledge in order to support the adjustment of the structure of production and service sector in every stage of supply chain. This is to enable the creative economy to be a new mobilizing power that leads toward the balanced and sustainable economy in the long run, together with the creation of the assurance system and the supply chain system, the management of economic risks, and the creation of the free and just atmosphere to facilitate the production, commerce and investment inclusive of the development of new entrepreneurs, the creation of infrastructure and internal logistics networks that connect with other countries in the region." Based on this policy, the 11th national plan for social and economic development was formulated [10]. The researcher has realized the importance of education development in order to cope with economic changes. Industrial sector the changes have included the movement toward more and more application of the concept of supply chain management information system. This is because industrial sector needs to be highly competitive due to increasingly high competitions from both within and outside the country. In order to be highly competitive, organizations in the sector need to have personnel with knowledge, ability who can work efficiently to increase output and products. The organizations, therefore, need to have sufficient information and resources to increase their values and respond to the demand of their clients. From the status declared directly above; consequently, the researchers are concerned in emerging a supply chain management model for educational management. The researchers understand the perception of supply chain management in order to realize the work's effectiveness educational. This prototypical will have an original for model expansion of information system for educational management in supply chain for Thai higher education institutions. The objectives of research to design and to assess the suitability of information system model for educational management in supply chain for Thai higher education institutions.

2 | Literature Review

Kham [8] said that education supply chain management needs to consider various elements, which has a relationship between various organizations with a clear goal of reducing the operational process of the system increase service levels leading to efficiency meet the needs of customers in general, the supply chain consists of important points, namely 1) suppliers mean those who send raw materials to service units such as producing quality graduates to society etc., 2) manufacturer means the person who is responsible for transforming the raw materials received from the supplier to have higher value, 3) distribution center means the point that serves to distribute products to the consumer or the customer at the center. One product distribution may have products from many agencies, such as higher education institutions. There will be graduates graduating from many institutions. 4) retailers or customers means the end of the supply chain, which is where the products or services must be used until the value is exhausted and without adding value to that product or service.

Lambert and Enz [5] Supply Chain Management is the integration of key business processes from end user through original suppliers that provides products, services, and information that add value for customers and other stakeholders.

Verma and Boyer [11] pointed out that business organizations in the supply chain will work together to turn raw materials into products and deliver to customers. between organizations which will be linked in both physical, data.

3 | Conceptual Framework

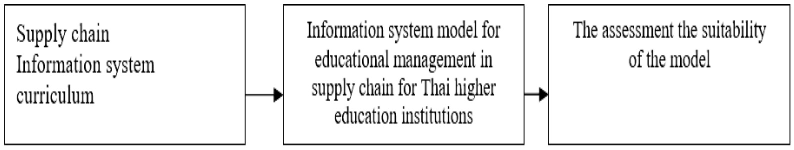

Fig. 1. Conceptual framework about information system model for educational management in supply chain for Thai higher education institutions.

4 | Research Methodology

The methodology comprised 6 steps, as follows:

- *Step 1: To analyse and synthesize related documents and research to the components of information system model for educational management in supply chain for Thai higher education institutions.*
- *Step 2: Defind research framework of information system model for educational management in supply chain for Thai higher education institutions setting.*
- *Step 3: To design information system model for educational management in supply chain for Thai higher education institutions using data collected from studies and analysis of relevant documents and research.*
- *Step 4: To propose the models to consultants and experts for consideration by in-depth interviews.*
- *Step 5: Create tools for assessing the suitability of models.*
- *Step 6: Data collection and develop questionnaire are sent to the experts in order to ask their opinions on appropriateness of information system model for educational management in supply chain for Thai higher education institutions using the arithmetic mean and standard deviation as the following criteria: 4.51-5.00 at highest of appropriate suitability 3.51-4.50 at a high of appropriate suitability 2.51-3.50 at moderately of appropriate suitability 1.51-2.50 at a low of appropriate suitability 0.00-1.50 at lowest of appropriate suitability [2] and [3].*
- *Step 6: to improve model based on suggestions from the experts.*

5 | Research Results

Research results on the information system model for educational management in supply chain for Thai higher education institutions were presented in *Fig. 1*.

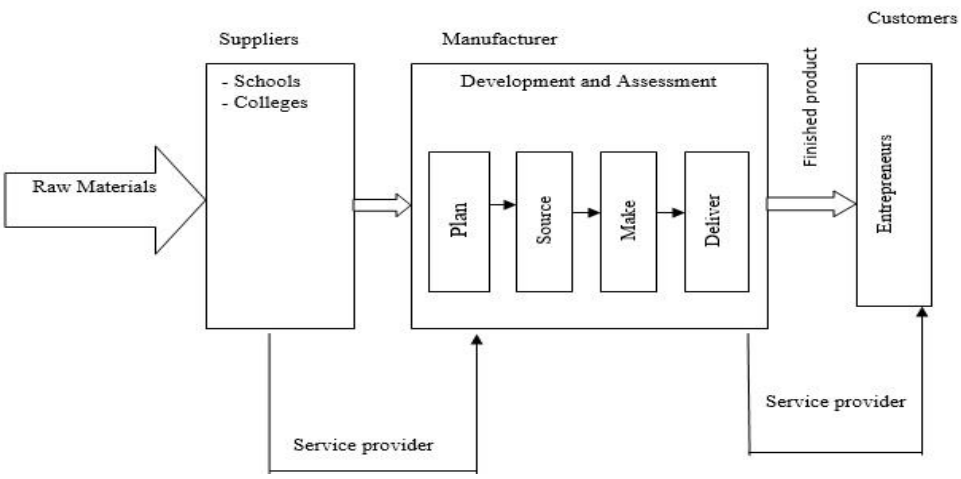

Fig. 2. Information system model for educational management in supply chain for Thai higher education institution.

Table 1. Stakeholders each process is related to the following activities.

Stakeholders	Activities	Needs for information
Suppliers: 1. School 2. College	- Sending graduated high school students. - Self-supporting students. - Preparing the fund support for study. - giving educational scholarships.	- To quick responses and reduction of time wastage.
Steps of manufacturer development and assessment: 1. Plan 2. Source 3. Make	-Forecasting future staffing requirement and student planning. -Planning for and development of the curriculum including its objectives, scope, structure, contents, course description, evaluation system, and required resources. -Development of curriculum documents, instructional media and certificates. -Provision of course materials including texts, study guides, printed and electronic learning media, and equipment. -Provision of fieldwork experience training including training places, supervising staff and mentors, training handbook and materials, and seminars on training experience. - Reporting of learning outcomes in each domain including the knowledge, intellectual skill, attitude, morality, interpersonal relationship, numerical analysis, information technology usage, etc. - Reporting of fieldwork. - experience training outcomes in terms of performance based on the rating scores of 1 – 5 according to evaluation criteria.	- To be easy to monitor. - To be easy to follow up. - Correct and complete evaluation reports. - To facilitate speedy.
4. Deliver	-Reporting of curriculum implementation outcomes in terms of the following: - Number of students who graduate before the completion of curriculum implementation. - Number of students who graduate according to the curriculum. - Number of students who graduate in each major field of study. - Occupations taken up by graduate students from the university. - Level of satisfaction of final year students with the curriculum.	- To be easy to follow up.

Stakeholders	Activities	Needs for information
Customers Entrepreneurs	Employer's satisfaction on graduate evaluate questionnaire with five main components was achieved in the following 1. Ethical and Moral Development: Development of: -Habits of acting ethically and responsibly in personal and public life in ways that are consistent with high moral standards. -Ability to resolve value conflicts through application of a consistent system of values. 2. knowledge, the ability to understand, recall and present information including: -Knowledge of specific facts, -Knowledge of concepts, principles and theories and Employer's satisfaction on graduate evaluate questionnaire with five main components was achieved in the following.	- Questionnaires to assess the employer's satisfaction with the employed graduated student on various aspects of desirable characteristics.
Customers Entrepreneurs	1. Ethical and Moral Development: Development of: -Habits of acting ethically and responsibly in personal and public life in ways that are consistent with high moral standards. -Ability to resolve value conflicts through application of a consistent system of values. 2. Knowledge, the ability to understand, recall and present information including: -Knowledge of specific facts, -Knowledge of concepts, principles and theories and -Knowledge of procedures. 3. Cognitive skills, the ability to Apply knowledge and understanding of concepts, principles, theories and procedures when asked to do so; and -Analyze situations and apply conceptual understanding of principles and theories in critical thinking and creative problem solving when faced with unanticipated new situations. 4. Interpersonal skills and responsibility, the ability to -work effectively in groups, and exercise leadership; -accept personal and social responsibility, and -plan and -plan and take responsibility for their own learning. 5. Analytical and communication skills, the ability to -use basic mathematical and statistical techniques, -communicate effectively in oral and written form, and -use information and communications technology.	- Questionnaires to assess the employer's satisfaction with the employed graduated student on various aspects of desirable characteristics.

5.1 | Principle of the Information System Model for Educational Management in Supply Chain for Thai Higher Education Institutions

5.1.1 | Raw materials

Raw materials mean student who sent from their families or government and private agencies.

5.1.2 | Suppliers

The suppliers mean high school or college that supply raw materials to the manufacturer. Raw materials in this case are students who graduated from high schools or two-year colleges, or students who receive special quotas for admission. They can apply for admission via the computer system that can process data systematically.

5.1.3 | Service provider

Service provider mean support activities that helps the main activities to run smoothly. Support activities consists of organizations infrastructure, human resources management technology development and procurement.

5.1.4 | Manufacturer

The manufacturer means the university that produces graduated students. It performs the duty to transform raw materials into the finished products of qualified graduated students. The university will perform its duty of student development and assessment. It is based on the consideration that all supply chain tasks and activities can be assigned to four fundamental processes - plan, source, make, deliver of each activity, namely. Recruitment of instructors and admission of students, curriculum planning, curriculum development, provision of learning activities for student development, provision of fieldwork experience training, evaluation of learning outcomes, and reporting of curriculum implementation results.

5.1.5 | Finished products

The finished products mean graduated students from the university.

5.1.6 | Customers

The customers mean entrepreneurs or the end-of-process component of the model. They include the society in general and entrepreneurs who receive and/or employ the students who graduated from the university. Finally, the end product of qualified graduated students will add value for customers with supply chain [1], [2], [3], [4], [6], [7], [12], [13], and [15].

6 | Conclusion and Discussion

6.1 | Conclusion

The evaluation result for the information system model for educational management in supply chain for Thai higher education institutions, as shown *Table 2* below:

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Table 2. The assessment of the suitability of information system model for educational management in supply chain for Thai higher education institutions.

No.	Items	$\overline{X}$	S.D.	Suitability
1	Main elements	3.62	0.71	High
2	Raw materials	3.70	1.25	High
3	Suppliers	3.60	0.78	High
4	Manufacturer	3.50	0.70	High
5	Service provider	3.30	0.48	High
6	Finished product	3.60	0.51	High
7	Consumers	3.70	0.48	High
	Total	3.57	0.69	High

From *Table 2*, that experts agree that information system model for educational management in supply chain for Thai higher education institutions is highly appropriate ($\overline{X}$ = 3.52, S.D. = 0.69)that it may develop information system.

6.2 | Discussion

The information system model for educational management in supply chain for Thai higher education institutions is considered to be highly appropriate and the design was according to the review of documents and relevant literature from both within and outside the country on developing research framework.

The efficiency evaluation model was in accord with the related literature from outside the country on supply chain management information system [1]-[4].

6.3 | Suggestions

The limitations of the paper about information system model for educational management in supply chain for Thai higher education institutions, if possible, it should be created database for the developed model in the future.

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Modeling of the Supply Chain of Cooperative Game between Two Tiers of Retailer and Manufacturer under Conditions of Uncertainty

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Abstract

The design of a repurchase agreement related to the amount of goods remaining in the two-echelon supply chain between the retailer and the manufacturer is examined. Two scenarios are considered quite separately; In the first scenario (decentralized) in which the retailer determines the price of the product and the optimal amount of the economic order and the producer is persuaded to follow this method. In the second (centralized) scenario, the goal is to maximize the profit of the whole chain, in which case the price of the product and the amount of the economic order are determined based on the profit of the whole chain. Then, a model of repurchase agreement related to the remaining goods was considered based on cooperative play and contract between two members of the supply chain, in which the goal is to maximize the profit of chain members. Due to the uncertainty of the competitive environment, the demand is considering under uncertain in modeling to determine the optimal level of cooperation in a competitive and cooperative market. The results of the implementation of this contract in a numerical example showed that the profit of the whole chain and the amount of economically optimal order in the centralized state increased compared to the decentralized state and the optimal price of the product decreased. Due to the fact that in the decentralized state the retailer determines the values of the optimal variables, the profit of this member decreases in the centralized state and the producer's profit increases.

Keywords: Leader-follower game, Cooperative game, Stackelberg game .

1 | Introduction

A supply chain consists of independent companies that include all the processes of preparation and conversion of raw materials into final products, their maintenance, distribution, and transfer to end customers as a chain. Today, competition along or across chains is of particular importance among researchers. The supply of products and services by changing the pattern of customer competition has led to a change in the type of market competition from the state of competition between

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independent companies to competition between supply chains. Supply chain design has an extensive literature, most of which is related to the single (exclusive) supply chain, and the existence of competing supply chains and their emergence in the future have not been considered. However, supply chains compete with each other for greater market share. Therefore, supply chains must be prepared for future competitiveness even if there is no competitor at the same time.

In the last two decades, members of scientific societies and business owners have paid more attention to supply chain management. The globalization of trade, increasing competition, and narrowing the gap between products in terms of quality and performance have led all scientists and craftsmen to re-examine how to make business activities more efficient and effective. In fact, they tend to use every means possible to increase their final profit and increase their market share. In particular, in order to achieve the above goals, they seek to establish coordination between members of the supply chain [1]. In supply chain management, the major issue is the management of separate but dependent members of a chain. To achieve an efficient supply chain, chain members must behave as a whole and coherently. But in reality, we often face decentralized supply chains in which each member makes decisions in order to optimize their goals in conflict with the goals of other members [2]. One of the most important issues in the supply chain is coordination and efforts to integrate the operations of the independent units to achieve the maximum possible benefit from the entire chain. Consequently, a key point in supply chain management is the development of mechanisms that align the goals of the independent components and create interaction between their activities and decisions to optimize the performance of the entire system [3].

Today, business units prefer to work with supply chain components to gain a competitive advantage, improve performance, and reduce costs. The supply chain can be divided into three main components, including the upstream part which is responsible for supplying raw materials, the middle part which receives the raw materials from the upstream part and produces the goods, and delivers them to the lower part of the supply chain, i.e., retailers to sell the product to the end customer [4]. There are conflicting goals and activities between parts of the supply chain that may increase the cost of each component of the supply chain. The goal of supply chain management is to coordinate, integrate, and establish mechanisms to build the dependent goals of the independent components of the supply chain [5].

The present study set to examine a two- echelon supply chain network between retailers and manufacturers that both members of the supply chain seek to maximize profits from the sale of their products. Therefore, two completely separate cases are considered. First, in the non-cooperative mode, a Stackelberg game takes place between two members of the supply chain, in which the retailer determines the price of the product and the optimal amount of the economic order, and the producer is persuaded to follow this method, which is called the decentralized scenario. Second, the goal is to maximize the profit of the whole chain, in which the price of the product and the amount of the economic order are determined based on the profit of the chain, which includes the profits of the members of the firms. This is known as a centralized scenario. In the second mode, a model is assumed based on cooperative game and signing a contract between two members of the supply chain, in which the goal is to maximize the profits of chain members by cooperating with each other in pricing and determining the optimal amount of economic order, i.e. the two members of the chain will increase their profits when they enter the contract. Therefore, modeling of such a system in which demand is considered uncertain is examined. The most important part of the proposed model is related to providing a cooperative model between two levels of supply chain network in conditions of demand uncertainty.

2 | Literature Review

In the operations management literature, the relationship between the implementation of single-commodity two-echelon and three-echelon supply chains, including the manufacturer and the retailer, has been discussed. In the traditional supply chain, decision-making power was in the hands of manufacturers, and recently, with the introduction of the concept of supply chain, this decision-making power has been transferred to retailers [5]. Several models have been introduced for manufacturers and retailers. The relationship between producer and retailer is modeled using the Stackelberg game when there is no cooperation. To describe Stackelberg game in the supply chain, it is possible that the producer and retailer are the two actors in this game. Here, decision making is sequential. First, one of the players decides that in this game he is called the leader and the second player gives his best answer according to the behavior of the first player; the second player is called the follower. In several studies, the producer has been modeled as a leader and the retailer as a follower, and in some studies, the retailer has been the leader and the producer the follower [6]. In a cooperative game, the producer and retailer are introduced as the players of the game, and their decision making is such that the collective profit of both is maximized, and their desirability is the profit of each player due to decision making in the cooperative game. The decision making of both players is such that neither the producer nor the retailer, assuming that the other game is constant, has no incentive to change their decision and introduce this point as the Pareto optimal point in this game [7].

Noh et al. [8] played a Stackelberg game between two members of the chain, including retailer and manufacturer. The goal of this game was to increase the profit of the producer and retailer to determine the optimal price for the retailer. Mohagheghian et al. [23] examined a single-product supply chain between several retailers and a manufacturer in which demand is defined as a new nonlinear function. The structure of their model was Stackelberg, the purpose of which was to determine the optimal selling price of each retailer. Numerical examples and sensitivity analysis were performed on the parameters to examine the model. Leng and Parlar [9] examined Stackelberg's game in a chain with several suppliers and a leading producer under return-sell agreements and sharing of lost sales costs, in which the manufacturer first determines the parameters of the contract and then the suppliers decide on a Nash game. Esmaeilzadeh and Taleizadeh [10] determined the optimal pricing for two complementary products in a two-echelon supply chain under two different scenarios. The levels defined in this article included one retailer and two manufacturers. The ultimate goal of this paper was to investigate the effect of different market forces between producers and retailers and demand leakage on optimal wholesale and retail pricing as well as chain profits. In this modeling, manufacturers are considered as leaders and retailers are considered as followers. Hou et al. [11] considered a three-stage supply chain model consisting of one manufacturer, one distributor, and one retailer for a single sales cycle. In this paper, the coordination of a decentralized supply chain is analyzed with a simultaneous move game or a leader-follower game, based on a revenue sharing agreement. Zhang and Huang [12] reviewed a supply chain consisting of a manufacturer and several suppliers, forming a coalition. There is a common platform for building a family of products with different interchangeable modules. The manufacturer, as the customer and the leader of the chain, first decides on the selection of suppliers and the arrangement of the platform, and then the suppliers decide on the prices and the order size in order to maximize the profit of the whole coalition.

Ke et al. [13] examined a supply chain pricing decision problem in a two-echelon supply chain with one manufacturer and two competing retailers. Production costs, sales costs, and market rates are all considered uncertain parameters whose distribution is estimated by experienced experts. Considering the strength of different markets of channel members, three decentralized game models were used to detect equilibrium behaviors in appropriate decision-making conditions. Innovation of this article is how channel members should choose their most profitable pricing strategies in the face of uncertainties. Nematollahi et al. [14] considered a retailer corporate supplying goods. It is possible for the supplier to invest in social responsibility activities of corporates, which can increase the popularity of products and consequently market demand. In the first stage, the decisions of both members are examined under decentralized and centralized decision-making models. Then, a new cooperative model is proposed to determine the optimal order quantity and CSR investment. Mahdiraji et al. [15] modeled and solved a two-echelon supply chain model for retailers and manufacturers in non-cooperative conditions. In this article, social responsibilities play a role as a basic parameter in product pricing. Malekian and Rasti-Barzoki [16] considered increasing prices and national advertising of the producer in a producer-retailer supply chain assuming the effect of consumer reference price. They examined the proposed model in two scenarios: centralized and decentralized. The results indicate that the reference price and customer memory coefficient have profound effects on the profit of supply chain members, the optimal depth of price promotion, and the level of advertising. Ghosh et al. [17] considered a two-echelon supply chain model with one manufacturer and one retailer. They assumed that demand for uncertain goods and part of the demand performance depend on the retailer unit price.

Yadav et al. [18] considered the shortage as a seller's decision variable and demand is receptive to selling price and marketing expenditure of the buyer. In this paper layer's interaction is reviewed and determined as non-cooperative Stackelberg game. Yang et al. [19] designed a two-layer game model to optimize the park integrated energy system internally and externally. Firstly, an upper-level Stackelberg game model of the superior energy network and park system designed to carry out external optimization of the park integrated energy system. Second, a cooperative game model presented for the park users, the gas supply system, and the park integrated energy system to undertake internal optimization of the park integrated energy system. Das et al. [20] suggested a discount mechanism by which companies can coordinate their ordering and pricing strategies throughout a supply chain model with a single manufacturer and single retailer. Also, the demand curve is iso-elastic price sensitive. Hong and Meng [21] used fuzzy set theory to examine the optimal decision of each member of a two-stage supply chain, which includes a manufacturer and a retailer. In this supply chain, the retailer takes the leading position and makes sales efforts. By considering the market demand function, the manufacturer's manufacturing cost and the retailer's operating cost as fuzzy variables, and by employing sequential game, expected value, and opportunity constraint models, the optimal decision-making solutions are resolved.

According to the research background, the articles have designed the supply chain cooperation network. However, very few articles discuss uncertainty in potential demand and pricing in uncertainty conditions, leading to research gaps. In this paper, a new model is presented for cooperation (signing a contract) between two chains of retailer (leader) and manufacturer (follower). Therefore, in general, the research gap can be raised in providing a collaborative model between the two levels of the supply chain to increase profits in the face of demand uncertainty. In this cooperative game, the level of the return product contract is also determined.

3 | Problem Definition and Modeling

A single-cycle supply chain consisting of two manufacturers and a retailer is examined. In this chain, the manufacturer produces the product and sells it to the retailer, and the retailer sells the product to the customer. The main purpose is to maximize the profit of the producer and retailer by determining the optimal amount of order and the price of the product. Also, customer demand is considered uncertain. In this model, the retailer decides on the order quantity Q and the product price p. The manufacturer also produces the product at cost c and sells it to the retailer at a wholesale price w. The main parameters and variables of the modeling are defined as follows:

- *p: Total price.*
- *Q: Order quantity.*
- *w: Wholesale price.*
- *h: Maintenance costs.*
- *c: Production costs.*
- π_r: *Retailer profit.*
- π_m: *Producer profit.*
- π_{sc}: *The profit of the whole chain.*
- $X(\varepsilon,p)$: *Demand function.*
- $D(p)$: *The certain part of the demand function.*
- ε: *Random demand variable.*

– *[A,B]: The lower and upper bound of the uniform distribution function.*

According to the defined symbols, the demand function is displayed as the following model:

$$X(P,\varepsilon)=D(p)+\varepsilon. \quad (1)$$

In the demand function, *D(p)* is the definite part of demand, which is linear and a function of the variable *p*. The value of *ε* is a random variable that has a uniform distribution in the *[A,B]* range [22]. In the initial part of the model, two decentralized and centralized scenarios are presented, and the optimal value of the decision variable of the order quantity *Q* and the product price *p* are obtained. Demand function is a linear and descending function of the final price of the product, which is shown as follows:

$$D(P)=\alpha-\beta p \quad \alpha,\beta\geq 0. \quad (2)$$

Since ε is a random variable with a uniform distribution function, it has the mean μ as below:

$$\varepsilon\sim U(A,B), \qquad \mu=\frac{A+B}{2}. \quad (3)$$

In the following, the decentralized market scenario is first examined, and the optimal quantity of order *Q* and product price *p* in this market are determined.

3.1 | Decentralized Scenario

In this scenario, the members of the chain are considered as separate economic enterprises, each of which determines the decision variables based on their interests and profits. In this model, the retailer is responsible for determining the optimal order quantity *Q* and product price *p*. Therefore, in this scenario, the retailer determines the order quantity based on its profit, and the manufacturer follows the retailer's decisions. The profit function of the retailer, manufacturer, and the profit of the whole supply chain network are expressed as follows:

$$\pi_R=\begin{cases} p.X(p,\varepsilon)-wQ-h[Q-X(p,\varepsilon)] & X(p,\varepsilon)\leq Q \\ pQ-wQ & X(p,\varepsilon)>Q \end{cases}, \quad (4)$$

$$\pi_m=(w-c)Q\,, \quad (5)$$

$$\pi_{sc}=\pi_m+\pi_r. \quad (6)$$

Eq. (4) shows the retailer's profit function; *Eq. (5)* shows the manufacturer's profit function, and *Eq. (6)* shows the profit function of the whole supply chain network. In the following, the retailer's profit function is examined, and the optimal quantity of order and product price are determined. By placing the *X(p,ε)* value of *Eq. (1)* in the retailer profit function, *Eq. (4)* is corrected as follows:

$$\pi_R=\begin{cases} (p-w)Q-(h+p)(Q-\alpha+\beta p-\varepsilon) & Q\geq\alpha-\beta p+\varepsilon \\ (p-w)Q & Q<\alpha-\beta p+\varepsilon \end{cases}. \quad (7)$$

Therefore, the expected amount of the retailer's profit function is equal to the following:

$$E[\pi_r]=(p-w)Q-(h+p)\int_A^{Q-\alpha+\beta p}(Q-\alpha+\beta p-\varepsilon)\, f(\varepsilon)d\varepsilon, \quad (8)$$

$$\rightarrow E[\pi_r]=Q(p-w)+\frac{(h+p)(A-Q+\alpha-p\beta)^2}{2(A-B)}. \quad (9)$$

To determine the optimal quantity of economic order, first-order derivative should be obtained from the profit function of the retailer with respect to the economic order quantity variable, and the value of the obtained function should be zero.

$$\frac{\partial E[\pi_r]}{\partial Q}=p-w-\frac{(h+p)(A-Q+\alpha-p\beta)}{A-B}. \tag{10}$$

Also, to determine the optimal quantity of the product price, first-order derivative should be obtained from the profit function of the retailer with respect to the product price variable, and the value of the obtained function should be zero.

$$\frac{\partial E[\pi_r]}{\partial p}=Q-\frac{(h+p)\beta(A-Q+\alpha-p\beta)}{A-B}+\frac{(A-Q+\alpha-p\beta)^2}{2(A-B)}. \tag{11}$$

In order to determine the economic order quantity and the optimal amount of product price, the concavity of the retailer's profit function must be proved. Therefore, the Hessian function matrix should be defined as follows:

$$H=\begin{bmatrix} \frac{\partial^2 E[\pi_r]}{\partial Q^2} & \frac{\partial E[\pi_r]}{\partial Q\partial p} \\ \frac{\partial E[\pi_r]}{\partial p\partial Q} & \frac{\partial^2 E[\pi_r]}{\partial p^2} \end{bmatrix}=\begin{bmatrix} \frac{h+p}{A-B} & \frac{-B+Q-\alpha+h\beta+2p\beta}{A-B} \\ \frac{-B+Q-\alpha+h\beta+2p\beta}{A-B} & \frac{\beta(-2A+2Q-2\alpha+h\beta+3p\beta)}{A-B} \end{bmatrix}. \tag{12}$$

Theorem 1. To calculate the optimal quantity of economic order and the price of the final product as follows, the $\frac{h+p}{A-B}$ value must be negative and the $\frac{(-B+Q-a+h\beta+2p\beta)^2}{(A-B)^2}$ value must be positive. Therefore, considering the positive value of price and maintenance cost, as well as the higher value of B compared to A, it can be shown that $\frac{h+p}{A-B}<0$. Also, the value of $\frac{(-B+Q-a+h\beta+2p\beta)^2}{(A-B)^2}$ can be easily shown to be strictly positive. Therefore, the above function is concave, and the quantity of economic order and the optimal price can be determined by solving the following equations.

$$\frac{\partial E[\pi_r]}{\partial Q}=0\to Q^*_{dc}=\frac{B(p-w)+A(h+w)+(h+p)(\alpha-p\beta)}{h+p}. \tag{13}$$

$$\frac{\partial E[\pi_r]}{\partial p}=0\to p^*_{dc}=\frac{2A\beta-2Q\beta+2\alpha\beta-h\beta^2+\sqrt{\beta^2(A^2+6BQ+2A(-4Q+\alpha+h\beta)+(-Q+\alpha+h\beta)^2)}}{3\beta^2}. \tag{14}$$

3.2 | Centralized Scenario

In the centralized scenario, the supply chain is considered by a central decision maker, and the goal is to maximize the profit of the whole chain. Therefore, the value of the decision variable of the economic order quantity is obtained from the total profit function:

$$\pi_{sc}=\begin{cases} p.X(p,\varepsilon)-cQ-h[Q-X(p,\varepsilon)] & X(p,\varepsilon)\le Q \\ pQ-cQ & X(p,\varepsilon)>Q' \end{cases} \tag{15}$$

$$\pi_m=(w-c)Q, \tag{16}$$

$$\pi_r=\pi_{sc}-\pi_m. \tag{17}$$

Eq. (15) shows the total profit of the supply chain network; *Eq. (16)* shows the total profit of the manufacturer, and *Eq. (17)* shows the profit of the retailer. By placing the value of $X(p,\varepsilon)$ from *Eq. (1)* in the supply chain profit function, *Eq. (15)* is corrected as follows:

$$\pi_{sc}=\begin{cases}(p-c)Q-(h+p)(Q-\alpha+\beta p-\varepsilon) & Q\geq\alpha-\beta p+\varepsilon\\ (p-c)Q & Q<\alpha-\beta p+\varepsilon\end{cases}. \tag{18}$$

Therefore, the expected value of the whole chain profit function in this case is equal to:

$$E[\pi_{sc}]=(p-c)Q-(h+p)\int_{A}^{Q-\alpha+\beta p}(Q-\alpha+\beta p-\varepsilon)\,f(\varepsilon)d\varepsilon, \tag{19}$$

$$\rightarrow E[\pi_{sc}]=(-c+p)Q+\frac{(h+p)(A-Q+\alpha-p\beta)^2}{2(A-B)}. \tag{20}$$

Hence, the profit of the whole chain becomes more than the decentralized scenario, but one of the members suffers loss. In this scenario, in order to determine the optimal quantity of economic order, the first-order derivative of the profit function of the whole supply chain network must be set equal to zero in relation to the quantity of economic order.

$$\frac{\partial E[\pi_{sc}]}{\partial Q}=p-\frac{(h+p)(A-Q+\alpha-p\beta)}{A-B}-c. \tag{21}$$

Also, to determine the optimal value of the product price, the first-order derivative of the retailer's profit function must be taken relative to the product price variable, and the value of the obtained function must be set equal to zero.

$$\frac{\partial E[\pi_{sc}]}{\partial p}=\frac{2(A-B)Q+(A-Q+\alpha-p\beta)^2+2(h+p)\beta(-A+Q-\alpha+p\beta)}{2(A-B)}. \tag{22}$$

In order to determine the quantity of economic order and the optimal amount of product price, the concavity of the retailer's profit function must be proved. Therefore, the Hessian function matrix should be defined as follows.

$$H=\begin{bmatrix}\frac{\partial^2 E[\pi_{sc}]}{\partial Q^2} & \frac{\partial E[\pi_{sc}]}{\partial Q\partial p}\\ \frac{\partial E[\pi_{sc}]}{\partial p\partial Q} & \frac{\partial^2 E[\pi_{sc}]}{\partial p^2}\end{bmatrix}=\begin{bmatrix}\frac{h+p}{A-B} & \frac{-B+Q-\alpha+h\beta+2p\beta}{A-B}\\ \frac{-B+Q-\alpha+h\beta+2p\beta}{A-B} & \frac{\beta(-2A+2Q-2\alpha+h\beta+3p\beta)}{A-B}\end{bmatrix}. \tag{23}$$

Theorem 2. To calculate the optimal quantity of economic order and the price of the final product as follows, the $\frac{h+p}{A-B}$ value must be negative and the $\frac{(-B+Q-\alpha+h\beta+2p\beta)^2}{(A-B)^2}$ value must be positive.

Therefore, considering the positive value of price and maintenance cost, as well as the higher value of B compared to A, it can be shown that $\frac{h+p}{A-B}<0$. Also, the value of $\frac{(-B+Q-\alpha+h\beta+2p\beta)^2}{(A-B)^2}$ can be easily shown to be strictly positive. Therefore, the above function is concave, and the quantity of economic order and the optimal price can be determined by solving the following equations.

$$\frac{\partial E[\pi_{sc}]}{\partial Q}=0\rightarrow Q_c^*=\frac{A(c+h)+B(-c+p)+(h+p)(\alpha-p\beta)}{h+p}, \tag{24}$$

$$\frac{\partial E[\pi_{sc}]}{\partial p}=0\to p_c^*=\frac{2A\beta-2Q\beta+2\alpha\beta-h\beta^2+\sqrt{\begin{array}{c}\beta^2(A^2+6BQ+2A(-4Q+\alpha+h\beta)+\\(-Q+\alpha+h\beta)^2)\end{array}}}{3\beta^2}. \tag{25}$$

3.3 | Contract Model

In this section, the contract related to the remaining products is evaluated. The value of the retailer's profit, taking into account the contract, is as follows:

$$\pi_r=\begin{cases}(p-w_b)Q-(p+h-b)\big(Q-X(p,\varepsilon)\big) & Q-X(p,\varepsilon)>\gamma\\(p-w_b)Q-(p+h)\big(Q-X(p,\varepsilon)\big) & 0<Q-X(p,\varepsilon)<\gamma\\(p-w_b)Q & Q-X(p,\varepsilon)<0\end{cases}, \tag{26}$$

$$\pi_r=\begin{cases}(p-w_b)Q-(p+h-b)(Q-\alpha+\beta p-\varepsilon) & Q-\alpha+\beta p-\gamma>\varepsilon\\(p-w_b)Q-(p+h)(Q-\alpha+\beta p-\varepsilon) & \begin{array}{c}Q-\alpha+\beta p-\gamma<\varepsilon\\\varepsilon<Q-\alpha+\beta p\end{array}\\(p-w_b)Q & Q-\alpha+\beta p<\varepsilon\end{cases}, \tag{27}$$

$$E[\pi_r]=(p-w_b)Q-(p+h)\int_A^{Q-\alpha+\beta p}(Q-\alpha+\beta p-\varepsilon)f(\varepsilon)d\varepsilon+b\int_A^{Q-\alpha+\beta p-\gamma}(Q-\alpha+\beta p-\varepsilon)f(\varepsilon)d\varepsilon. \tag{28}$$

The value of producer's profit can also be calculated from the following equation.

$$\pi_m=\begin{cases}(w_b-c)Q-b\big(Q-X(p,\varepsilon)\big) & Q-X(p,\varepsilon)>\gamma\\(p-w_b)Q & Q-X(p,\varepsilon)<\gamma\end{cases}, \tag{29}$$

$$\pi_m=\begin{cases}(w_b-c)Q-b(Q-\alpha+\beta p-\varepsilon) & Q-\alpha+\beta p-\gamma>\varepsilon\\(w_b-c)Q & Q-\alpha+\beta p-\gamma<\varepsilon\end{cases}, \tag{30}$$

$$E[\pi_m]=(w_b-c)Q-b\int_A^{Q-\alpha+\beta p-\gamma}(Q-\alpha+\beta p-\varepsilon)f(\varepsilon)d\varepsilon. \tag{31}$$

As a result, the profit of the whole supply chain network is equal to:

$$\pi_{sc}=\pi_m+\pi_r. \tag{32}$$

In the above relations, w_b, b, and γ are the contract parameters. By integrating the profit function of the retailer and production, the ultimate profit of these two functions is as follows:

$$\pi_r=-\frac{\begin{array}{c}b(A-Q+\alpha-2p\beta-p\beta\gamma)(A-Q+\alpha+p\beta\gamma)+\\(A-Q+\alpha-p\beta)^2(-h+p(-1+Q)-Qw_b)\end{array}}{2(A-B)}. \tag{33}$$

$$\pi_m=\frac{b(A-Q+\alpha-2p\beta-p\beta\gamma)(A-Q+\alpha+p\beta\gamma)}{2(A-B)}+Q(-c+w_b). \tag{34}$$

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To determine the quantity of economic order and the optimal value of product price, derivative from the retailer's profit function must be taken with respect to the value of the decision variables Q and p and set equal to zero.

$$\frac{\partial E[\pi_r]}{\partial Q} - \frac{(A-Q+\alpha-p\beta)(-2b+2h+2p+Ap-3pQ+p\alpha-p^2\beta-(A-3Q+\alpha-p\beta)w_b)}{2(A-B)}. \tag{35}$$

$$\frac{\partial E[\pi_r]}{\partial p} = -\frac{1}{2(A-B)}((-1+Q)(A-Q+\alpha-p\beta)^2+b\beta\gamma(A-Q+\alpha-2p\beta-p\beta\gamma)-b\beta(2+\gamma)(A-Q+\alpha+p\beta\gamma)$$
$$-2\beta(A-Q+\alpha-p\beta)(-h+p(-1+Q)-Qw_b)). \tag{36}$$

$$Q^*_{co}(w_b,b) = \frac{-2b+2h+2p+Ap+p\alpha-p^2\beta-(A+\alpha-p\beta)w_b}{3(p-w_b)}. \tag{37}$$

$$p^*_{co}(w_b,b,\gamma) = \frac{1}{3(-1+Q)\beta^2}(-2A\beta+2Q\beta+2AQ\beta-2Q^2\beta-2\alpha\beta+2Q\alpha\beta+h\beta^2$$
$$+2b\beta^2\gamma+b\beta^2\gamma^2+Q\beta^2w_b+\sqrt{(\beta^2((2A(-1+Q)+2Q-2Q^2-2\alpha+2Q\alpha+h\beta+2b\beta\gamma+b\beta\gamma^2+Q\beta w_b)^2}$$
$$-3(-1+Q)(A-Q+\alpha)(A(-1+Q)+Q-Q^2-\alpha+Q\alpha-2b\beta+2h\beta+2Q\beta w_b)))). \tag{38}$$

Then, the optimal quantity of the order and the value of the product price in the contract must be equal to the optimal quantity of the order and the value of the product price in the centralized state before the contract, i.e.,

$$Q^*_{co}(w_b,b) = Q^*_c, \tag{39}$$

$$p^*_{co}(w_b,b,\gamma) = p^*_c. \tag{40}$$

Therefore, by equating the two expressions above, the value of the parameter w_b and γ based on a function of $f(b)$ is obtained as described in the following relation:

$$w_b = \frac{-2h^2+2b(h+p)+2hp(-2+A+\alpha-p\beta)-p(3B(c-p)+A(-3c+p)+2p(1-\alpha+p\beta))}{-3B(c-p)+A(3c+2h-p)+2(h+p)(\alpha-p\beta)}, \tag{41}$$

$$b = \frac{1}{2(h+p)}(2h^2+2hp(2-A-\alpha+p\beta)+p(3B(c-p)+A(-3c+p)+2p(1-\alpha+p\beta))$$
$$+(-3B(c-p)+A(3c+2h-p)+2(h+p)(\alpha-p\beta))w_b). \tag{42}$$

$$\gamma = -((A^2b-2bBQ+bQ^2+2Ab\alpha-2bQ\alpha+b\alpha^2-2Abh\beta+$$
$$2bhQ\beta-2bh\alpha\beta+\sqrt{(-\frac{1}{\beta^2}b(A^2-2BQ+2A(\alpha-h\beta)+(Q-\alpha)(Q-\alpha+2h\beta((}$$
$$)-2A^2Q\beta+2ABQ\beta+2AQ^2\beta+2A^2Q^2\beta-2BQ^2\beta-2ABQ^2\beta-2AQ^3\beta+2BQ^3\beta-$$
$$2AQ\alpha\beta+2BQ\alpha\beta+2AQ^2\alpha\beta-2BQ^2\alpha\beta+A^2b\beta^2-4AbQ\beta^2+2bBQ\beta^2+AhQ\beta^2-$$
$$A^2hQ\beta^2-BhQ\beta^2+bQ^2\beta^2+3AhQ^2\beta^2-BhQ^2\beta^2-hQ^3\beta^2+2Ab\alpha\beta^2-2bQ\alpha\beta^2-$$

$$
\begin{aligned}
&2AhQ\alpha\beta^2+2hQ^2\alpha\beta^2+b\alpha^2\beta^2-hQ\alpha^2\beta^2+Abh\beta^3-bhQ\beta^3-Ah^2Q\beta^3+h^2Q^2\beta^3+\\
&bh\alpha\beta^3-h^2Q\alpha\beta^3+AQ\sqrt{\beta^2(A^2+6BQ+2A(-4Q+\alpha+h\beta)+(-Q+\alpha+h\beta)^2)}-\\
&BQ\sqrt{\beta^2\left(A^2+6BQ+2A(-4Q+\alpha+h\beta)+(-Q+\alpha+h\beta)^2\right)}-\\
&AQ^2\sqrt{\beta^2(A^2+6BQ+2A(-4Q+\alpha+h\beta)+(-Q+\alpha+h\beta)^2)}+\\
&BQ^2\sqrt{\beta^2\left(A^2+6BQ+2A(-4Q+\alpha+h\beta)+(-Q+\alpha+h\beta)^2\right)}-\\
&Ab\beta\sqrt{\beta^2(A^2+6BQ+2A(-4Q+\alpha+h\beta)+(-Q+\alpha+h\beta)^2)}+\\
&bQ\beta\sqrt{\beta^2(A^2+6BQ+2A(-4Q+\alpha+h\beta)+(-Q+\alpha+h\beta)^2)}+\\
&AhQ\beta\sqrt{\beta^2(A^2+6BQ+2A(-4Q+\alpha+h\beta)+(-Q+\alpha+h\beta)^2)}-\\
&hQ^2\beta\sqrt{\beta^2\left(A^2+6BQ+2A(-4Q+\alpha+h\beta)+(-Q+\alpha+h\beta)^2\right)}-\\
&b\alpha\beta\sqrt{\beta^2\left(A^2+6BQ+2A(-4Q+\alpha+h\beta)+(-Q+\alpha+h\beta)^2\right)}+\\
&hQ\alpha\beta\sqrt{\beta^2\left(A^2+6BQ+2A(-4Q+\alpha+h\beta)+(-Q+\alpha+h\beta)^2\right)}-A^2Q\beta^2w_b+\\
&4AQ^2\beta^2w_b-2BQ^2\beta^2w_b-Q^3\beta^2w_b-2AQ\alpha\beta^2w_b+2Q^2\alpha\beta^2w_b-Q\alpha^2\beta^2w_b\\
&-AhQ\beta^3w_b+hQ^2\beta^3w_b-\\
&hQ\alpha\beta^3w_b+AQ\beta\sqrt{\beta^2\left(A^2+6BQ+2A(-4Q+\alpha+h\beta)+(-Q+\alpha+h\beta)^2\right)}w_b-\\
&Q^2\beta\sqrt{\beta^2\left(A^2+6BQ+2A(-4Q+\alpha+h\beta)+(-Q+\alpha+h\beta)^2\right)}w_b+\\
&Q\alpha\beta\sqrt{\beta^2\left(A^2+6BQ+2A(-4Q+\alpha+h\beta)+(-Q+\alpha+h\beta)^2\right)}w_b(((/\\
&(b(A^2-2BQ+2A(\alpha-h\beta)+(Q-\alpha)(Q-\alpha+2h\beta))).
\end{aligned}
\tag{43}
$$

Finally, the following relation must be solved to determine the effective parameter range in the contract, i.e. *b*. That is, the profit of the retailer and the producer after the contract is higher than the profit of the retailer and wholesaler in the decentralized state before the contract, i.e.,

$$\pi_r^{co}(w_b,b,\gamma)\geq\pi_r^{dc}\text{-}\to\pi_r^{co}(b)\geq\pi_r^{dc}, \tag{44}$$

$$\pi_m^{co}(b,\gamma)\geq\pi_m^{dc}\text{-}\to\pi_m^{co}(b)\geq\pi_m^{dc}. \tag{45}$$

After presenting a mathematical model for a single-cycle supply chain problem in centralized and decentralized mode and modeling the contract between the retailer and the manufacturer, this chapter deals with problem solving and analysis. Thus, after presenting different numerical examples, the sensitivity of each sample problem in the pre-contract and post-contract states is analyzed and compared.

4 | Solving Sample Problems in the Pre-Contract State

To assess the modeling results, five sample problems with different parameters presented in *Table 1* are discussed. Therefore, at the beginning of this section, the values of decision variables, the optimal quantity of economic order, and product price are shown in the centralized and decentralized scenarios. In the second section, the sensitivity of the sample problems under variation of its various parameters is analyzed.

4.1 | Numerical Results of Modeling in the Pre-Contract State

Several numerical examples are provided to examine the profit value of the retailer, producer, and the optimal quantity of economic order in the centralized and decentralized scenarios. *Table 1* shows five numerical examples for examining centralized and decentralized scenarios.

Table 1. Numerical parameters of sample problems in centralized and decentralized scenarios.

Instance	h	c	B	A	β	α	w
1	5	10	20	10	1.2	30	20
2	10	12	20	5	1.2	30	20
3	6	12	30	10	0.8	20	20
4	6	12	30	15	0.6	25	24
5	10	15	40	15	0.6	25	25

Table 2 shows the optimal quantity of economic order, product prices in the centralized and decentralized scenarios as well as firm profits in these two scenarios.

Table 2. The quantity of economic order, product prices, and profits of firms in pre-contract centralized and decentralized scenarios.

Instance	Decentralized					Centralized				
	p_{dc}	Q_{dc}	π_m^{dc}	π_r^{dc}	π_{sc}^{dc}	p_c	Q_c	π_m^c	π_r^c	π_{sc}^c
1	27.51	9.29	92.89	61.15	154.05	23.15	16.88	168.82	22.58	191.40
2	25.47	6.74	53.97	30.59	84.56	22.25	13.06	104.50	5	109.50
3	32.08	10.67	85.41	90.70	176.11	29.35	16.31	130.55	67.55	198.27
4	49.77	17.06	204.80	350.56	555.36	44.79	22.80	273.66	315.50	589.17
5	53.03	19.29	192.96	385.15	578.12	49.40	24.83	248.35	357.04	605.40

The total profit of the chain and the order quantity in all examples increased in the centralized mode compared to the decentralized mode, and the price of the product decreased. Due to the fact that in the decentralized state, the retailer determines the values of the optimal variables, the profit of this member decreases in the centralized state and the producer's profit increases.

4.2 | Sensitivity Analysis of the Problem in Pre-Contract State

First, the sensitivity of the problem parameters on the sample problem 1 is analyzed. Hence, the value of problem decision variables against parameters variations is examined. First, the variations of decision-making variables of the problem are examined in relation to the variations of maintenance costs. Therefore, the value of decision variables is as follows.

Table 3. Sensitivity analysis of the values of the objective functions for the maintenance cost parameter.

Parameter variations	Centralized					decentralized				
	π_{sc}^{c}	π_{r}^{c}	π_{m}^{c}	Q_c	P_c	π_{sc}^{dc}	π_{r}^{dc}	π_{m}^{dc}	Q_{dc}	P_{dc}
2	195.09	21.92	173.16	17.31	23.28	156.52	62.04	94.48	9.44	27.59
4	192.54	22.39	170.15	17.01	23.19	154.81	61.43	93.38	9.33	27.54
5	191.40	22.58	168.82	16.88	23.15	154.05	61.15	92.89	9.29	27.51
6	190.35	22.75	167.60	16.76	23.12	153.33	60.89	92.44	9.24	27.49
8	188.46	23.03	165.43	16.54	23.05	152.03	60.42	91.61	9.16	27.45
10	186.80	23.25	163.55	16.35	22.98	150.89	60.00	90.88	9.08	27.41

According to *Table 3*, with the increase of maintenance costs, the quantity of economic order decreased and in turn the price of the product decreased in the centralized and decentralized chain. Since the maintenance cost has a positive effect on the profit of the centralized and decentralized supply chain network regarding the value of the two objective functions, the profit of the whole network increases with its increase and the profit of the producer decreases due to decrease in product price. *Fig. 1* shows the trend of variations in decision variables.

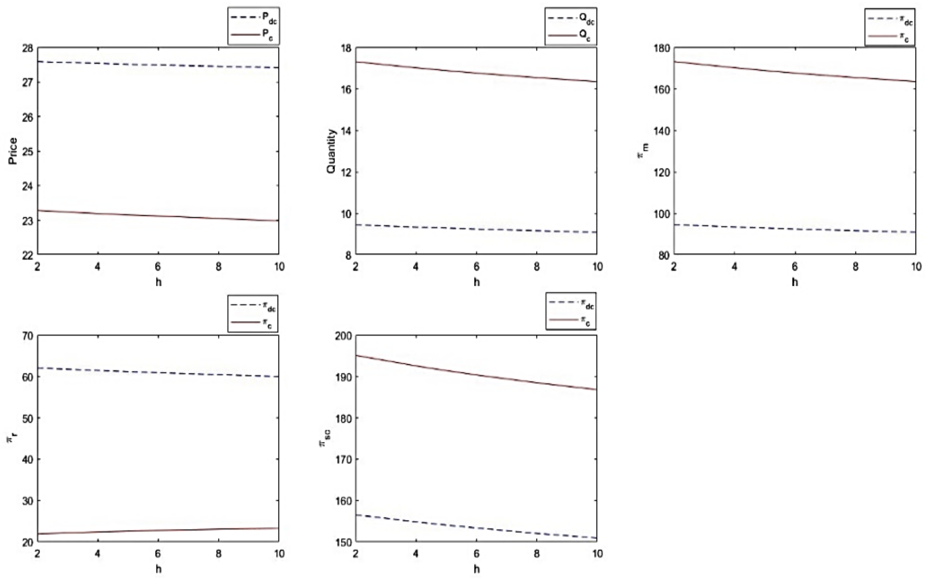

Fig. 1. The trend of variations in decision variables due to variations of maintenance costs.

In the following, production cost sensitivity analysis is examined. Therefore, the value of decision variables is below.

Table 4. Sensitivity analysis of the values of the objective functions for the production cost parameter.

Parameter variations	Decentralized					Centralized				
	P_{dc}	Q_{dc}	π_{m}^{dc}	π_{r}^{dc}	π_{sc}^{dc}	P_c	Q_c	π_{m}^{c}	π_{r}^{c}	π_{sc}^{c}
8	27.51	9.29	111.47	61.15	172.63	22.27	18.50	222.02	4.76	226.78
10	27.51	9.29	92.89	61.15	154.05	23.15	16.88	168.82	22.58	191.40
12	27.51	9.29	74.31	61.15	135.47	24.03	15.30	122.41	36.81	159.22
15	27.51	9.29	46.44	61.15	107.60	25.34	12.99	64.97	51.82	116.80
20	27.51	9.29	0	61.15	61.15	27.51	9.29	0	61.15	61.15

According to the results, with the increase of production cost, there has been no change in the optimal quantity of order and retailer price in the decentralized supply chain. However, with the increase of production costs, the price of the product increased in the centralized scenario and the optimal quantity of order decreased. Therefore, the total profit of the producer decreased and the retailer profit increased due to increase in production costs. *Fig. 2* shows the trend of variations in this parameter.

Fig. 2. The trend of variations in decision variables due to variations of production costs.

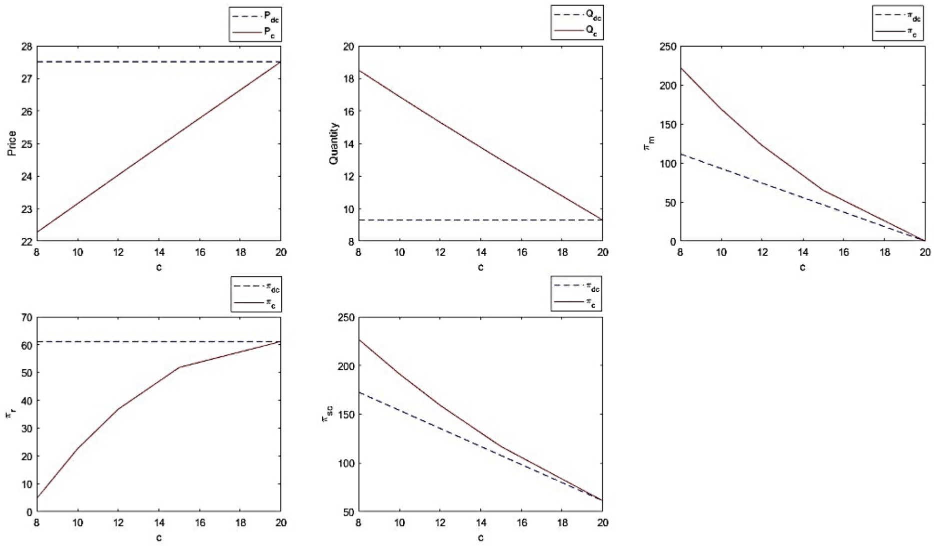

Table 5 and *Fig. 3* show the variations of decision variables by wholesale price variations.

Table 5. Sensitivity analysis of the objective functions' values for the wholesale price parameter.

Parameter variations	Decentralized P_{dc}	Q_{dc}	π_m^{dc}	π_r^{dc}	π_{sc}^{dc}	Centralized P_c	Q_c	π_m^c	π_r^c	π_{sc}^c
10	23.15	16.88	0	191.40	191.40	23.15	16.88	0	191.40	191.40
12	24.03	15.30	30.60	159.22	189.83	23.15	16.88	33.76	157.64	191.40
15	25.34	12.99	64.97	116.80	181.77	23.15	16.88	84.41	106.99	191.40
18	26.65	10.75	86.02	81.19	167.21	23.15	16.88	135.06	56.34	191.40
20	27.51	9.29	92.89	61.15	154.05	23.15	16.88	168.82	22.58	191.40

According to the results, with increase in the wholesale price, the profit of the whole supply chain network did not change. In the decentralized scenario, however, the retailer profit increased due to the increase in the wholesale price and decreased in the centralized scenario. *Fig. 3* shows the trend of variations in this parameter.

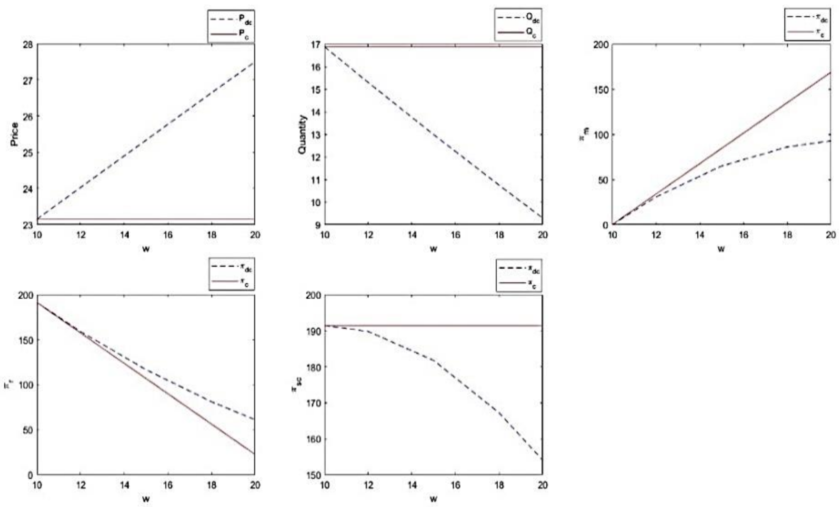

Fig. 3. The trend of variations in decision variables due to wholesale prices variations.

Table 6 and *Fig. 4* show the variations in decision variables by parameter α variations.

Table 6. Sensitivity analysis of the objective functions' values for parameter α.

Parameter variations	Decentralized					Centralized				
	P_{dc}	Q_{dc}	π_m^{dc}	π_r^{dc}	π_{sc}^{dc}	P_c	Q_c	π_m^c	π_r^c	π_{sc}^c
30	27.51	9.29	92.89	61.15	154.05	23.15	16.88	168.82	22.58	191.40
32	28.41	10.41	104.18	77.09	181.27	24.02	18	180	38.59	218.59
35	29.75	12.1	121.00	104.35	225.35	25.32	19.66	196.65	65.97	262.62
38	31.08	13.77	137.71	135.61	273.33	26.61	21.31	213.18	97.34	310.53
40	31.96	14.88	148.80	158.66	307.46	27.47	22.41	224.14	120.47	344.61

According to the results, with increase in parameter α, the profit of retailer, producer, and the whole supply chain network increased. This occurred due to the increase in product price and the quantity of economic order because of increase in the value of parameter α. *Fig. 4* shows the trend of variations in this parameter.

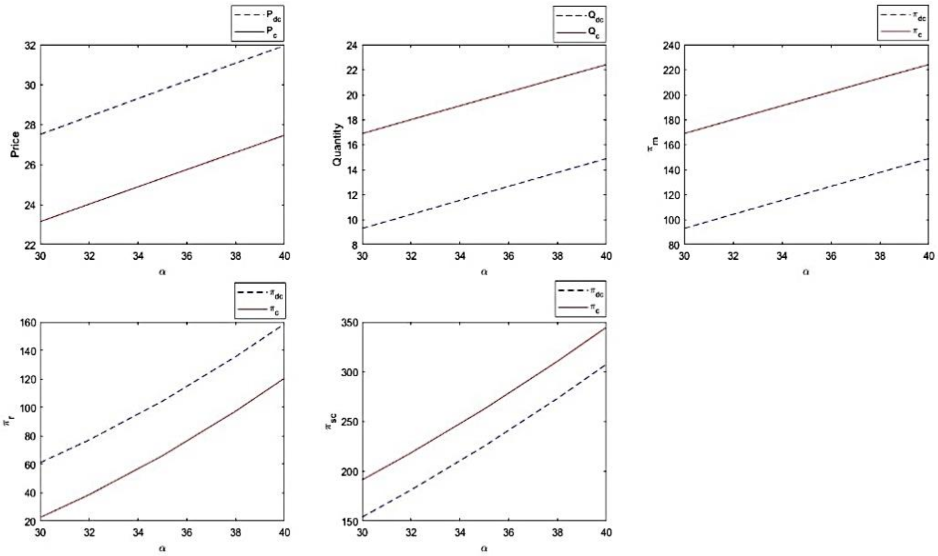

Figure 4. The trend of variations in decision variables due to variations in parameter α.

Table 7 and *Fig. 5* illustrate the variations in decision variables by parameter β variations.

Table 7. Sensitivity analysis of the values of objective functions for parameter β.

Parameter variations	Decentralized					Centralized				
	P_{dc}	Q_{dc}	π_m^{dc}	π_r^{dc}	π_{sc}^{dc}	P_c	Q_c	π_m^c	π_r^c	π_{sc}^c
0.8	37.01	14.43	144.35	211.20	355.55	32.62	19.91	199.10	183.40	382.51
0.9	33.85	13.1	131.00	156.74	287.74	29.47	19.12	191.22	126.16	317.38
1	31.31	11.80	118.00	115.89	233.90	26.94	18.35	183.56	82.59	266.16
1.1	29.24	10.53	105.31	84.87	190.19	24.88	17.61	176.10	48.91	225.01
1.2	27.51	9.29	92.89	61.15	154.05	23.15	16.88	168.82	22.58	191.40

It is observed that with increasing parameter β, the profit of the retailer, producer, and the whole supply chain network decreased, which is due to the increase in product price and the quantity of economic order due to the decrease in the value of parameter β. *Fig. 5* shows the trend of variations in this parameter.

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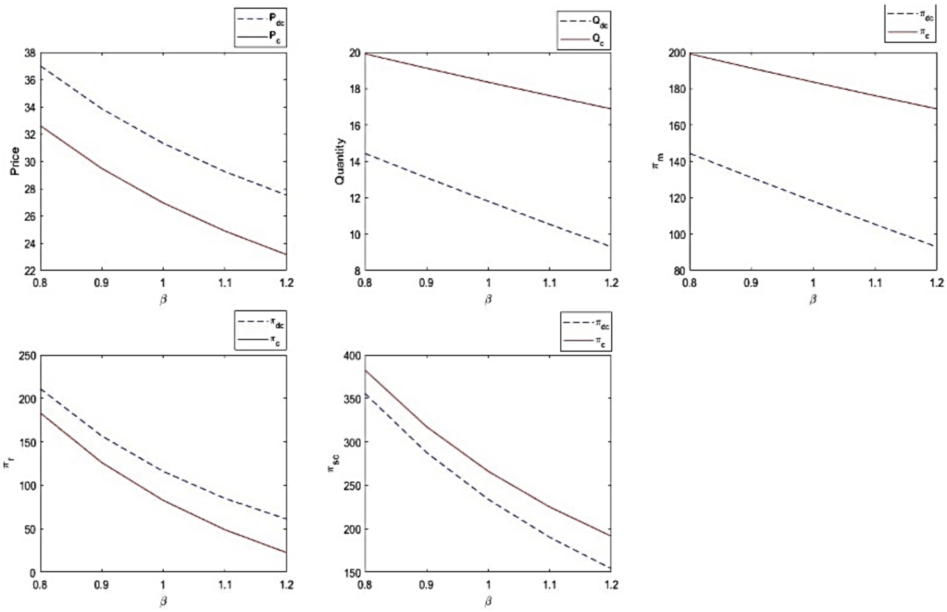

Fig. 5. The trend of variations in decision variables due to variations in parameter β.

5 | Solving Sample Problems in the Post-Contract Mode

After evaluation of the sample problem for centralized and decentralized scenarios and examining output variables and carrying out sensitivity analysis on all input parameters, the model is examined and solved in the post-contract state. In this section, the sensitivity of the parameters is analyzed and the exact range of the contract b parameter is determined after solving the sample problem.

5.1 | Results of the Numerical Example After Signing the Contract

In this section, the results of output variables are examined, and also the exact range of contract *b* parameter for sample problem 1 is determined. *Table 8* shows the output variables including product price, optimal quantity of economic order, and profit of all firms before and after the contract.

Table 8. The quantity of economic order, product price, and profit of firms' post-contract.

Scenario	Contract b parameter	P	Q	π_m	π_r	π_{sc}
Centralized	-	23.15	16.88	168.82	22.58	191.40
Decentralized	-	27.51	9.29	92.89	61.15	154.05
Signing a contract	370.34	23.15	16.88	160.26	61.49	221.42
	485.71			92.89	137.21	230.11

According to *Table 8*, the product price and the quantity of the economic order in the centralized scenario are equal to the post-contract sate. However, the profit of the retailer, producer, and whole supply chain at the time of signing contract is more than the decentralized scenario. *Fig. 6* shows the trend of profit variations of firms in sample problem 1 in the range of contract *b* parameter.

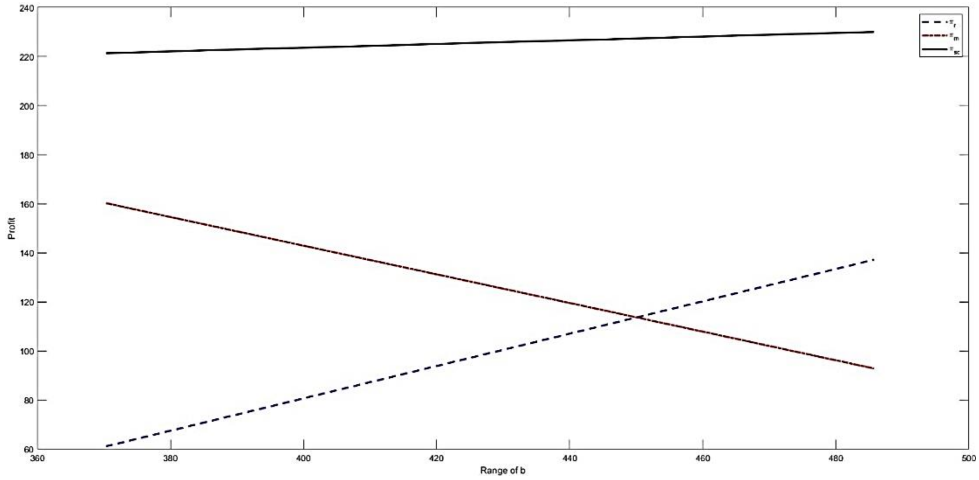

Fig. 6. The trend of variations of firms' profit after signing the contract in the range of contract b parameter.

Fig 6 displays when two members of the supply chain are in the range of contract b parameter, the profit of both firms increases compare with the decentralized state. Also, the profit of the whole chain is higher than the centralized and decentralized scenarios.

5.2 | Sensitivity Analysis of the Problem After Signing the Contract

In this section, the problem sensitivity is analyzed separately for post-contract state for the input parameters of the problem. The purpose of this analysis is to assess the effect of input parameters on the range of contract *b* parameter. First, the sensitivity of the sample problem 1 is analyzed under the maintenance cost parameter. *Table 9* shows the lower and upper bounds of the contract *b* parameter and the profits of firms. For further analysis, *Fig. 7* shows the trend of variations of firms' profit and the range of contract *b* parameter by variations of maintenance costs.

Table 9. Variations in firms' profits due to changes in maintenance costs post-contract.

Parameter variations	Contract b parameter		π_{sc}	π_r	π_m
8	Lower bound	296.79	230.44	62.04	168.40
	Upper bound	359.61	241.34	146.86	94.48
10	Lower bound	370.34	221.42	61.15	160.26
	Upper bound	485.71	230.11	137.21	92.89
12	Lower bound	386.88	206.64	60.00	146.64
	Upper bound	762.77	212.36	121.48	90.88

Accordingly, with increasing maintenance costs, the bound of contract *b* parameter ranges increased and also the total profit of firms including the profit of retailer, producer, and supply chain network decreased. However, the profit of the retailer, producer, and the total profit is higher compared to the total profit of the centralized and decentralized scenarios. As a result, the profit of firms decreases by increasing the mantanence cost. The following figure shows the trend of variations of firms' profit in different values of maintenance costs for post-contract.

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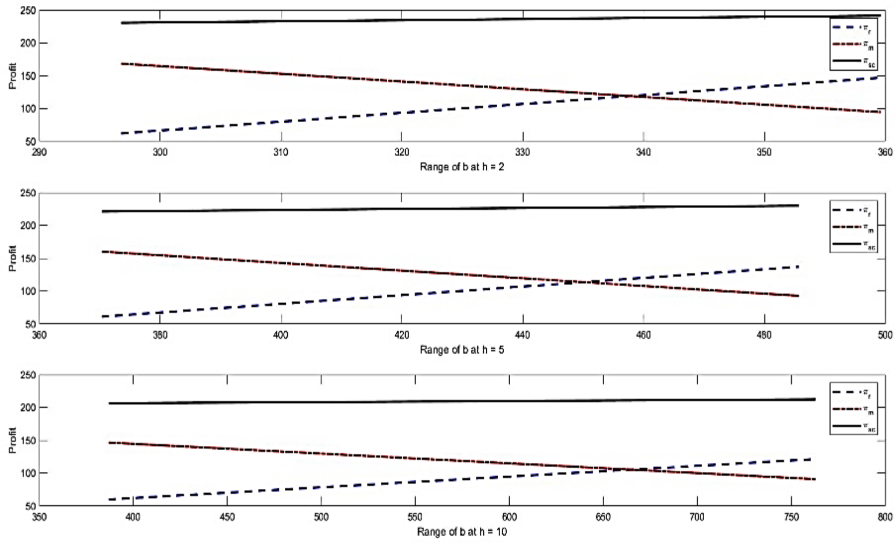

Fig. 7. The trend of variations in firms' profits due to variations in maintenance costs after signing the contract.

The following show the variations in the profits of supply chain members and the entire network post contract in exchange for changes in production costs. *Table 10* and *Fig. 8* show the variations trend for sample problem 1 for different production costs.

Table 10. The trend of variations in firms' profits due to variations in production costs post-contract.

Parameter variations	Contract b parameter		π_{sc}	π_r	π_m
8	Lower bound	254.15	234.14	61.15	172.99
	Upper bound	364.15	248.71	131.24	117.47
10	Lower bound	370.34	221.42	61.15	160.26
	Upper bound	485.71	230.11	137.21	92.89
12	Lower bound	479.38	204.69	61.15	143.54
	Upper bound	552.03	218.96	144.65	74.31

According to the table above, with increasing production costs, the retailer's profit increases, while the producer's profit decreases. However, the profit of both firms is higher in the decentralized scenario, while the price of the product and the quantity of production are the same as in the centralized scenario. The profit of the whole network in post contract also decreased with the increase of production costs.

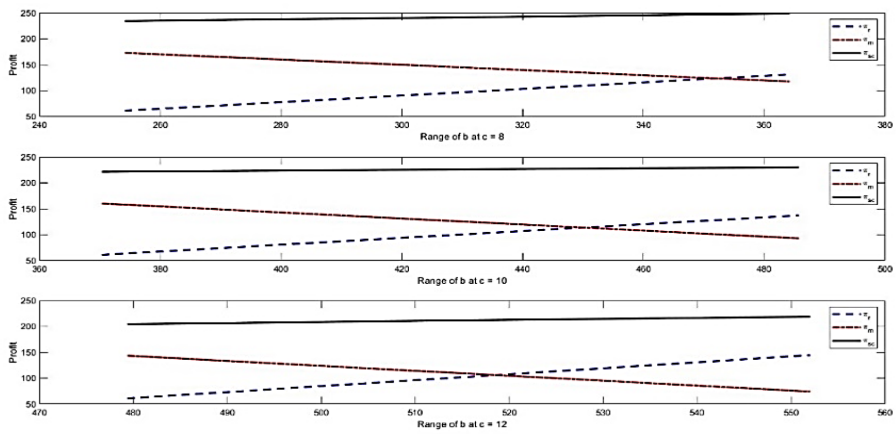

Fig. 8. The trend of variations in firms' profits due to variations in production costs post-contract.

Table 11 examines the profits of chain members and the total profits of firms in exchange for variations in the wholesale price. This table also specifies the contract parameter interval as *b*. *Fig. 9* also shows the variations trend of the profits of chain members and the firm in exchange for changes in the wholesale price in the post-contract period.

Table 11. The trend of variations in firms' profits due to variations in wholesale prices post-contract.

Parameter variations	Contract b parameter		π_{sc}	π_r	π_m
12	Lower bound	249.76	187.63	159.22	28.41
	Upper bound	368.46	218.63	188.03	30.60
15	Lower bound	310.12	203.47	116.80	86.67
	Upper bound	423.10	224.36	159.39	64.97
20	Lower bound	370.34	221.42	61.15	160.26
	Upper bound	485.71	230.11	137.21	92.89

Based on the results of the analysis obtained from the wholesale price variations on the firms 'profits post contract, it is observed that the profit of the whole network and the profit of each member of the chain in the contract state is higher than the members' profit in the decentralized state. Also, with the increase in maintenance costs, the variations trend of the profit of the entire supply chain network decreases.

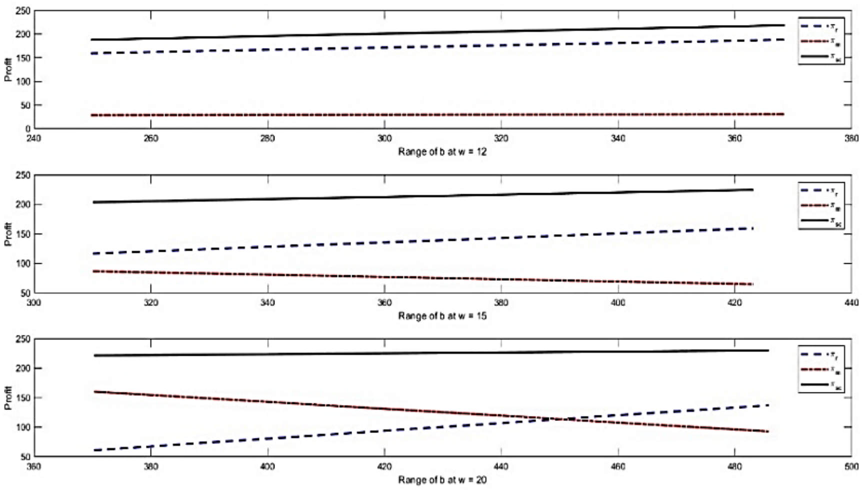

Fig. 9.

The trend of variations in firms' profits due to variations in wholesale prices post-contract.

In the following, the problem sensitivity is analyzed under the input parameter α. The profits of the chain members by variations in the value of a are shown in *Table 12*, including the profits of the retailer, producer, and the whole chain.

Table 12. The trend of variations in firms' profits due to variations in parameter α post-contract.

Parameter variations	Contract b parameter		π_{sc}	π_r	π_m
30	Lower bound	370.34	221.42	61.15	160.26
	Upper bound	485.71	230.11	137.21	92.89
35	Lower bound	420.38	243.57	104.35	139.22
	Upper bound	586.14	276.45	155.45	121.00
40	Lower bound	624.81	268.94	158.66	110.28
	Upper bound	971.36	312.48	163.68	148.80

Fig. 10 also shows the variations trend in firms' profits due to wholesale price changes after signing the contract for the sample problem 1.

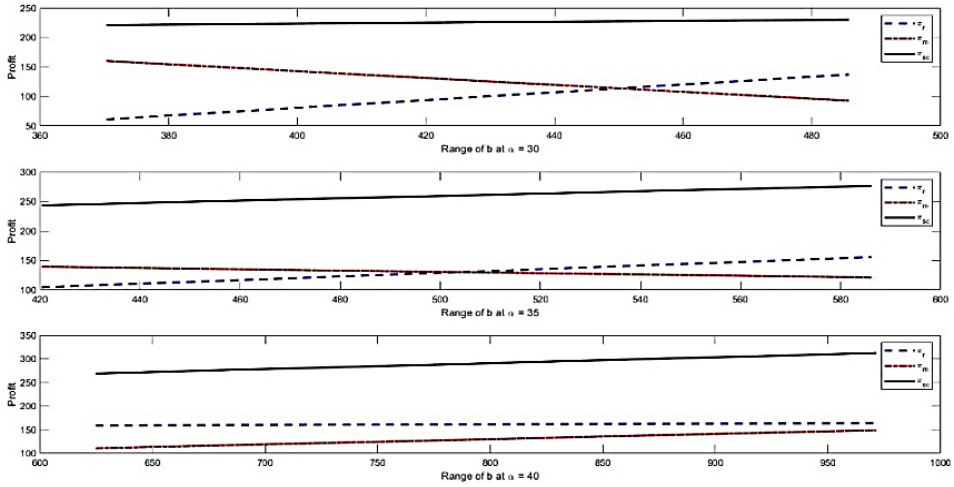

Fig. 10. The trend of variations in firms' profits due to variations in parameter α post-contract.

Analyses revealed that with increasing the coefficient of parameter *a*, the profit of all members in the contract increased, and also with increasing this coefficient, the profit of the whole network increased in the contract. *Table 13* shows the trend of variations in firms' profits due to variations in parameter β post-contract for sample problem 1.

Table 13. The trend of variations in firms' profits due to variations in parameter β post-contract.

Parameter variations	Contract b parameter		π_{sc}	π_r	π_m
12	Lower bound	635.82	312.14	211.20	100.94
	Upper bound	763.58	324.69	180.34	144.35
15	Lower bound	420.36	255.12	115.89	139.23
	Upper bound	512.36	286.23	168.23	118.00
20	Lower bound	370.34	221.42	61.15	160.26
	Upper bound	485.71	230.11	137.21	92.89

According to the analysis of *Table 13*, with increase in the input parameter β, the profit of the whole chain decreased. Nevertheless, the profit of members increased compared to the decentralized scenario. *Fig. 11* also shows the trend of variations in firms' profits due to variations in parameter β post-contract for sample problem 1.

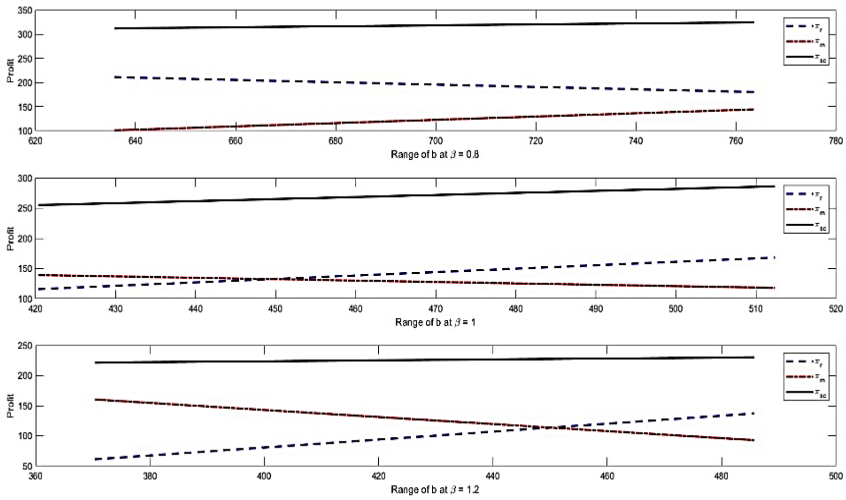

Fig. 11. The trend of variations in firms' profits due to variations in parameter β after signing the contract.

6 | Conclusion

In this paper, the design of a repurchase agreement related to the amount of goods remaining in the two-echelon supply chain between the retailer and the manufacturer is examined, in which both members of the supply chain seek to maximize the profit from the sale of their products. Two scenarios are considered quite separately; In the first scenario (decentralized) in which the retailer determines the price of the product and the optimal amount of the economic order and the producer is persuaded to follow this method. In the second (centralized) scenario, the goal is to maximize the profit of the whole chain, in which case the price of the product and the amount of the economic order are determined based on the profit of the whole chain. Then, a model of repurchase agreement related to the remaining goods was considered based on cooperative play and contract between two members of the supply chain, in which the goal is to maximize the profit of chain members.

Due to the uncertainty of the competitive environment, in this paper, the demand parameter is uncertain and the uniform distribution function is used in modeling to determine the optimal level of cooperation in a competitive and cooperative market. After presenting a mathematical model for the pre-contract state in both centralized and decentralized scenarios and providing numerical examples, the profit of whole chain and the order quantity increased in all examples in the centralized scenario and the product price decreased, compared to the decentralized scenario. As in the decentralized scenario, the retailer determines the values of the optimal variables, the profit of this member decreases in the centralized scenario and the producer's profit increases. After performing the sensitivity analysis on the input parameters and on all numerical examples, the quantity of economic order decreased with increase on the maintenance cost, and consequently, the product price in the centralized and decentralized chain decreased. Since the maintenance cost has a positive effect on the profit of the centralized and decentralized supply chain network regarding the value of the two objective functions, the profit of the whole network increased with its increase, and the producer profit decreased due to decline in the product price. Also, with the increase in production costs, no change was observed in the optimal order quantity and retailer price in the decentralized supply chain.

Nevertheless, with the increase of production costs, the product price in the centralized scenario increased and the optimal quantity of the order decreased. Therefore, due to the increase in production costs, the total profit of the producer decreased and the profit of the retailer increased. It was further found that with the increase in the wholesale price, the profit of the whole supply chain network did not change. In the decentralized scenario, however, the retailer profit increased due to the increase in the wholesale price and decreased in the centralized scenario. Finally, by performing sensitivity analysis on parameters a and β, it was found that with increasing parameter a, the profit of the retailer, producer, and the whole supply chain network increased. This occurred due to the increase in product price and the quantity of economic order because of increase in the value of parameter a. Also, with increase in parameter β, the profit of the retailer, producer, and the whole supply chain network decreased. This is due to the increase in product price and the quantity of economic order due to the decrease in the value of parameter β. Then, a cooperative model was designed between the two members of supply chain, and numerical examples were designed under the input parameters of the contract and executed on the model. The result was to determine the contract parameter range on the problem. Their profit increased compared to the decentralized scenario by creating a cooperative relationship between two parties of the supply chain. Sensitivity analysis was also performed on all parameters in a cooperative manner.

To improve the work, it is suggested that the proposed model be designed in a three-echelon supply chain network. Also, due to the uncertainty in real environments, it is suggested that other parameters in the model be considered uncertain. Finally, to be as close as possible to the real world, considering the discount in the model can be considered as one of the future studies. The results of the models presented in this paper can be used to improve the business of small and medium units as well as their cooperation to gain more profit from the market in conditions of demand uncertainty.

Conflict of Interest

No potential conflict of interest was reported by the authors.

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Paper Type: Original Article

Optimal Value Determination for a Shape Changeable Furniture Design Parameters Using Full Factorial Design of Experiment Analysis

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Abstract

Optimization and selecting effective design parameters are a big issue in the field of product design and development. The optimal value determination for design parameters is a challenge for the furniture industry. Therefore, an appropriate statistical approach is required. In this research the full factorial two-level four-factor design of experiment method was used to determine the optimal values for a shape changeable furniture. For the m-chair's- which can be used as a chair, floor bed, and table, four design parameters- bending axis material surface roughness, density of body material, width of the surface plane, and the number of bending axis were evaluated on basis of the performance time of shape changing where the optimal value of influential parameters were determined for a minimum reshaping time using Minitab. This method can be implemented for unique product design. But for further study, different form changing times should be considered individually.

Keywords: Full factorial DOE analysis, Two-level four-factor DOE, Minimization optimization model, Shape-changeable furniture design.

1 | Introduction

Product design is a dynamic cycle which use arithmetic, fundamental science, design methodologies for addressing required issues. The process consists of several investigations, testing and assessment to bring out the result align with ideal and desired target [1].

In recent years with the development in product design, the use of multipurpose furniture is increased significantly. The major reason for converting single furniture into multiple functional furniture is preferred by the users as it offers optimal use in a confined living space. Therefore,

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allowing using the space in an optimal condition by maximizing space utilization and improving the livelihood condition of the user. If a single piece of furniture can be used for multiple functional units it will also reduce the user expense of buying multiple furniture units [2].

One of the new trends for furniture design development is the shape-changing furniture that can change form from one to another and materials characteristics and geometry is directly related to the reshaping capability of the furniture [3]. For furniture design, user satisfaction is directly related to the space-saving aspect and user-centered design [4]. The furniture must have easy-touse quality and aesthetic aspects [2]. Most of the product development is carried on the market survey and customer requirements. For suppose designing a multifunctional laptop table design house of quality was used and contemplating the customer need and market similar product the design parameters were determined, where a few new features were added with the basic features [5]. Rajan et al. [6] showed a multi-functional table design based on customer demand and software modeling parameters, however, the table was only for multiple types of table use. But reshape-able products are based on the integration of different furniture units. Although the market has data related to an individual unit, in most cases data is not available for the integrated furniture. As a result, possible customer feedback is also unpredictable. The only way does a statical study on this type of product is to choose the parameters which can provide an optimized result for a major user interface. On the other hand, design can be only implied as user-oriented if the major feature of the design is easily assessable and understandable by the user. For multi-functional furniture, the reshaping feature has the most significant purpose [7]. Hence, user satisfaction will only be obtained when the reshaping could be done with ease and if the reshaping is an easy process the user would be able to fluently implement the feature and reform the furniture in an alternative shape. On that account, the unique product development design of the experiment can easily pave the way.

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Design of Experiments (DoE) is a statistical tool now and then used for optimization and robust design [8]. The full factorial DoE is one of the most used statistical analyses, where the interactions of different levels of factors screening process been conducted. If the number of factors is k and 2 is the level number used, then the possible combination for full factorial DoE will be n^k [9].

Research with DoE method can be used to improve the existing product's quality and optimization of the performance parameter, as well as for developing a whole new product. Some of the recent applications of DoE method were for improving the thermal performance of gas ovens by adjusting the parameters and interacting with those parameters [10], pharmaceutical drug development for improving quality and posit an optimal product [11], optimization of Electrophoretic Deposition (EPD) process parameters for PEEK base coatings [12], metabolomic related studies [13], for energy performance measurement [14] and so on. In practice is it common to find the use of DoE for optimization [15] as well as for product and process improvement [16].

In this paper, the full factorial DoE design was conducted and the influential parameters were determined. Furthermore, the optimal solutions for the influential parameter, for which the best response output obtained had been defined. In end, cost estimation for the prototype model was also demonstrated. This paper would pave a way to establish a multi-functional and reshape-able furniture design without customer survey and market benchmarking. The objective of this paper is to selecting optimal parameter values for unique multi-functional furniture design using full factorial DoE, to determining parameters that show an effect on reshaping time.

2 | Methodology

In this paper, a full factorial DoE method was used to determine the best possible parameters for the key performance selection for product design. In this article, 4 independent parameters were considered as factors with one dependent key performance evaluating parameter. Minitab 2020 version was used to determine the combinations, regression analysis, cube plot diagram and model solution establishment.

2.1 | Selection of Factors and Levels

The key performance indicator for the reshape able furniture was how easily it was be changed from one shape to another and the much easy the process the less time is likely to take to carry the process. So here the time needed (*T*) for reshaping was taken as a dependent variable and the parameters that were directly related to this performance were taken as independent variables. The four independent variables were bending axis material surface roughness (*R*), the density of body material (*D*), the width of the surface plane (*W*), and the number of bending axis (*N*) as they have a direct ergonomic effect on material handling.

Two levels (low and high) were assigned for all four factors, as shown in *Table 1*. The axis was a cylindrical metal pipe shape and the low rough surface was for Stretched steel (0.015* 10-3 m), high value for weld steel (0.045* 10-3 m) [17]. The furniture is a wooden base and two different types of wood material density were taken. The martial density was determined by measuring the weight/mass of 1 cubic unit if material using standard weight scale. Other measured value which was determined using slide-calipers was width of surface plane. The number of bending axis or joint axis is shown in *Fig. 1*.

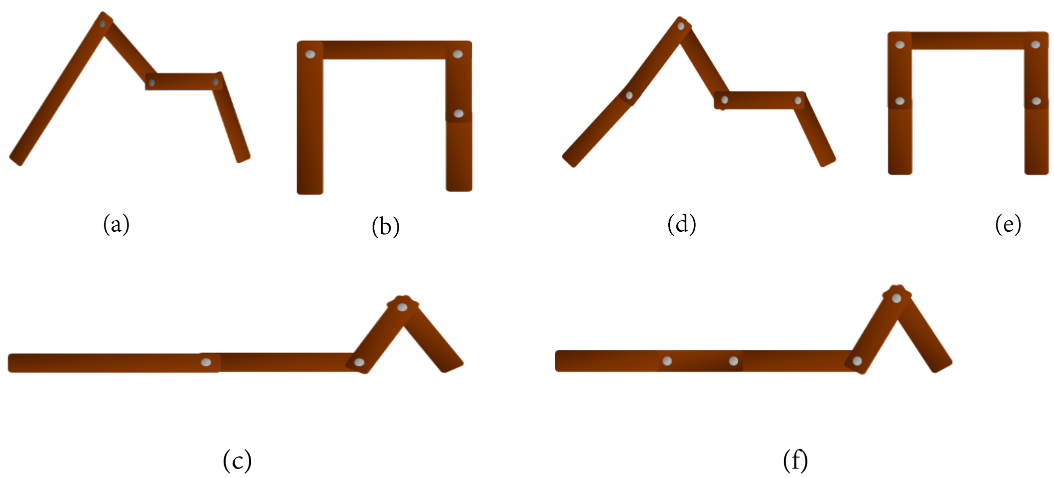

Fig. 1. Three joint chair-table-floor bed form and four joint chair-table-floor bed form respectively (side view).

Table 1. Factor's symbols and levels.

Factors	Symbols	Low	High
Bending axis material absolute surface roughness (ε in mm)	R	0.015 [17]	0.045 [17]
Density of body material (kg/m^3)	D	1200	2000
Width of the surface plane (m)	W	0.4	0.45
Number of the bending axis	N	3	4

2.2 | Response Determination for Possible Combinations

There were 4 factors, hence, the number of combinations was 2^4 or 16, (*Table 2*) which was taken randomly arranging the parameters [9].

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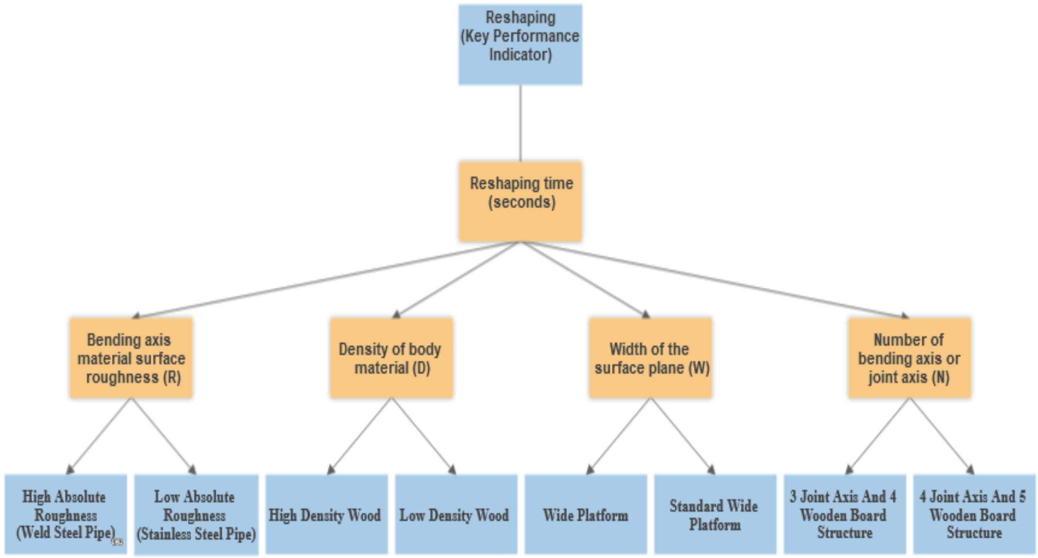


Fig. 2. Diagram showing parameters for DoE.

Table 2. Design summary.

Factors:	**4**
Runs:	16
Blocks:	1
Base Design:	4, 16
Replicates:	1
Center pts (total):	0

The combination was made with all possible factor interactions when all terms were free from aliasing. For two-level full factorial DoE total of 16 possible combinations were possible without an additional center point combination. Total 48 trials were conducted combining the time estimation for bed to chair, bed to table, chair to table, chair to bed, table to chair, and table to bed shape-changing, az shown in *Table 3*.

Table 3. Trial data.

StdOrder	RunOrder	CenterPt	Blocks	R [17]	D	W	N	T 1	T 2	T 3
1	1	1	1	0.015	1200	0.40	3	50.1	49.3	51.60
12	2	1	1	0.045	2000	0.40	4	64.7	65.3	66.00
15	3	1	1	0.015	2000	0.45	4	58.2	59.1	57.60
9	4	1	1	0.015	1200	0.40	4	57.0	58.5	58.40
16	5	1	1	0.045	2000	0.45	4	66.1	68.1	63.90
13	6	1	1	0.015	1200	0.45	4	59.0	56.0	56.10
10	7	1	1	0.045	1200	0.40	4	59.0	60.7	57.30
4	8	1	1	0.045	2000	0.40	3	54.9	57.0	53.00
7	9	1	1	0.015	2000	0.45	3	53.5	53.4	53.70
2	10	1	1	0.045	1200	0.40	3	52.8	51.1	55.10
8	11	1	1	0.045	2000	0.45	3	56.1	56.0	56.00
14	12	1	1	0.045	1200	0.45	4	59.9	58.3	59.00
5	13	1	1	0.015	1200	0.45	3	50.3	51.0	49.50
11	14	1	1	0.015	2000	0.40	4	59.0	59.0	59.00
3	15	1	1	0.015	2000	0.40	3	53.2	51.5	53.24
6	16	1	1	0.045	1200	0.45	3	54.0	52.9	55.00

2.3 | Determine Standard Deviation and Mean for All Combinations

As the trails for combinations were showing different values, for standardizing the response in factorial analysis the Standard Deviations (SD) and means were employed.

Table 4. Mean and SD of trails.

StdOrder	RunOrder	CenterPt	Blocks	R [17]	D	W	N	Mean	SD
1	1	1	1	0.015	1200	0.40	3	50.3333	1.16762
12	2	1	1	0.045	2000	0.40	4	65.3333	0.65064
15	3	1	1	0.015	2000	0.45	4	58.3000	0.75498
9	4	1	1	0.015	1200	0.40	4	57.9667	0.83865
16	5	1	1	0.045	2000	0.45	4	66.0333	2.10079
13	6	1	1	0.015	1200	0.45	4	57.0333	1.70392
10	7	1	1	0.045	1200	0.40	4	59.0000	1.70000
4	8	1	1	0.045	2000	0.40	3	54.9667	2.00083
7	9	1	1	0.015	2000	0.45	3	53.5333	0.15275
2	10	1	1	0.045	1200	0.40	3	53.0000	2.00749
8	11	1	1	0.045	2000	0.45	3	56.0333	0.05774
14	12	1	1	0.045	1200	0.45	4	59.0667	0.80208
5	13	1	1	0.015	1200	0.45	3	50.2667	0.75056
11	14	1	1	0.015	2000	0.40	4	59.0000	0.00000
3	15	1	1	0.015	2000	0.40	3	52.6467	0.99324
6	16	1	1	0.045	1200	0.45	3	53.9667	1.05040

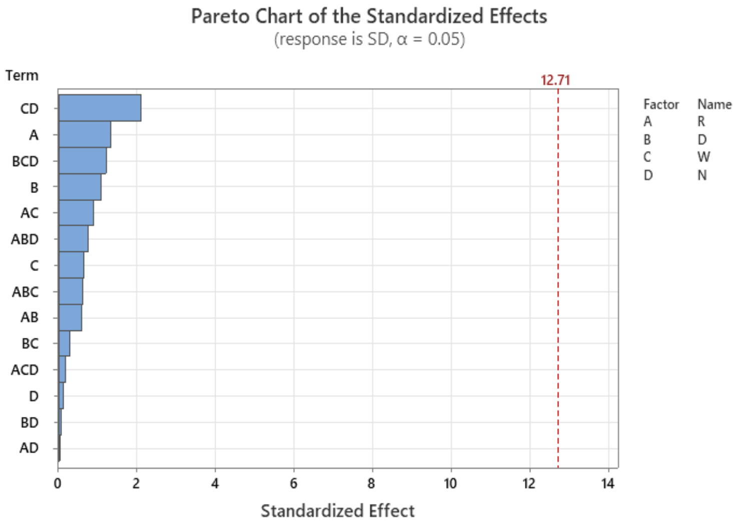
2.4 | Factorial Regression Analysis and Interpretation

The regression was enacted for SD and mean with *Eq. (1)* and *Eq. (2)*. The regression equations are in uncoded units.

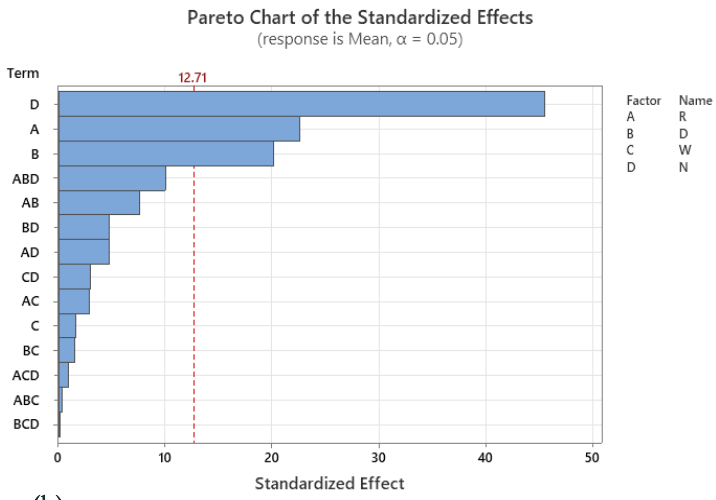
$$SD=-92+126R+0.0793D+203W+22.2N-0.482R*D-2386R*-158*N-0.178D*W-0.0207D*N-47.0W*N+0.79R*D*W+0.0472R*D*N+192R*W*N+0.0456D*W*N, \quad (1)$$

$$Mean =-23.8 + 1455\,R + 0.0109\,D + 52\,W + 26.6\,N - 0.736\,R*D - 480\,R*W - 537\,R*N + 0.0258\,D*W- 0.00498\,D*N - 26.7\,W*N - 0.189\,R*D*W + 0.2614\,R*D*N + 396\,R*W*N -0.0023\,D*W*N. \quad (2)$$

For DoE, SD and mean values were considered as a response (independent variable); the degree of freedom 3. *Fig. 3(a)* shows the Pareto chart for SD and none of the factors are affecting the time. Interpretation could be done the same for p-values (p-value > 0.05) as shown in *Table 5*. In *Fig. 3(b)*, bending axis material solute surface roughness (*R*), the density of principle material (*D*), and the number of joint axes (*N*) affected the mean values and for *R*, *D*, and *N* p-value is less than 0.05 as shown in *Table 6*.



(a)



(b)

Fig. 3. Pareto chart for standard effect on SD and mean of reshaping time respectively.

Table 5. Coded coefficient for factorial regression: SD versus R, D, W, N.

Term	Effect	Coef	SE Coef	T-Value	P-Value	VIF
Constant		1.046	0.189	5.54	0.114	
R	0.501	0.251	0.189	1.33	0.411	1.00
D	-0.414	-0.207	0.189	-1.10	0.471	1.00
W	-0.248	-0.124	0.189	-0.66	0.630	1.00
N	0.046	0.023	0.189	0.12	0.922	1.00
R*D	0.226	0.113	0.189	0.60	0.656	1.00
R*W	-0.339	-0.169	0.189	-0.90	0.534	1.00
R*N	-0.012	-0.006	0.189	-0.03	0.980	1.00
D*W	0.104	0.052	0.189	0.27	0.830	1.00
D*N	0.029	0.015	0.189	0.08	0.951	1.00
W*N	0.791	0.396	0.189	2.10	0.283	1.00
R*D*W	0.237	0.118	0.189	0.63	0.643	1.00
R*D*N	0.283	0.142	0.189	0.75	0.591	1.00
R*W*N	0.072	0.036	0.189	0.19	0.880	1.00
D*W*N	0.456	0.228	0.189	1.21	0.440	1.00

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Table 6. Coded coefficient for factorial regression: means versus R, D, W, N.

Term	Effect	Coef	SE Coef	T-Value	P-Value	VIF
Constant		56.6550	0.0783	723.26	0.001	
R	3.5400	1.7700	0.0783	22.60	0.028	1.00
D	3.1517	1.5758	0.0783	20.12	0.032	1.00
W	0.2483	0.1242	0.0783	1.59	0.358	1.00
N	7.1233	3.5617	0.0783	45.47	0.014	1.00
R*D	1.1817	0.5908	0.0783	7.54	0.084	1.00
R*W	0.4517	0.2258	0.0783	2.88	0.213	1.00
R*N	0.7433	0.3717	0.0783	4.74	0.132	1.00
D*W	0.2400	0.1200	0.0783	1.53	0.368	1.00
D*N	0.7483	0.3742	0.0783	4.78	0.131	1.00
W*N	-0.4650	-0.2325	0.0783	-2.97	0.207	1.00
R*D*W	-0.0567	-0.0283	0.0783	-0.36	0.779	1.00
R*D*N	1.5683	0.7842	0.0783	10.01	0.063	1.00
R*W*N	0.1483	0.0742	0.0783	0.95	0.517	1.00
D*W*N	-0.0233	-0.0117	0.0783	-0.15	0.906	1.00

2.5 | Multiple Regression Analysis with of Influential Parameters

Furthermore, *R*, *D*, and *N* were employed as factors for modeling and the simplified multi-linear regression equation for mean values were:

$$\text{Mean}=21.88 + 118.0\ R + 0.003940\ D+ 7.123\ N. \tag{3}$$

The coefficient of the factors showed significance on the response (p-value<0.05, R^2 =92.94%).

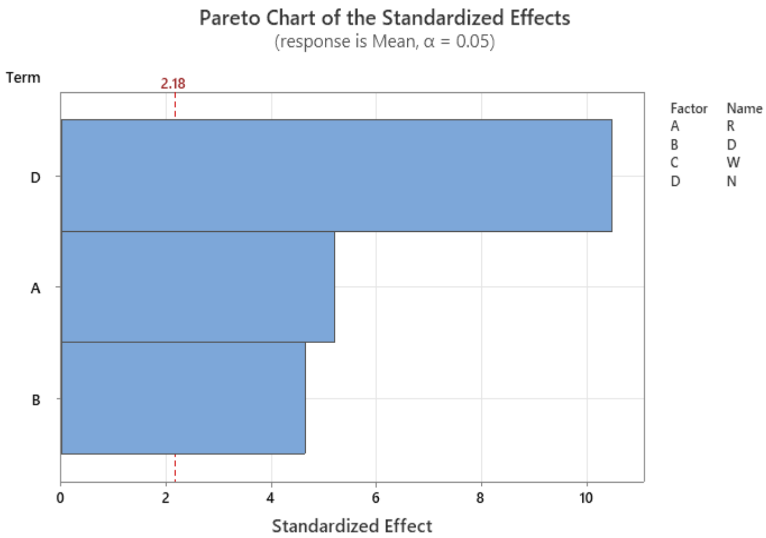


Fig. 4. Pareto chart for mean for D and N.

Table 6. Coded coefficient for factorial regression: means versus D and N.

Term	Effect	Coef	SE Coef	T-Value	P-Value	VIF
Constant		56.655	0.340	166.49	0.000	
R	3.540	1.770	0.340	5.20	0.000	1.00
D	3.152	1.576	0.340	4.63	0.001	1.00
N	7.123	3.562	0.340	10.47	0.000	1.00

Fig. 4 showed the Pareto chart of standard effect on mean of reshaping time in terms of *R*, *D*, and *N*. And from the Pareto chart all three factors affected the mean value of the trials.

3 | Result

Optimal values were defined for *R*, *D*, and *N*, as *W* did not have a particular effect on reshaping time. Form the Minitab model, for reshaping time minimization approach was taken and the upper bound was 1 minute, which means the reshaping time could not be more than 1 minute.

3.1 | Cube Plot for (Fitted Means) for Mean Values

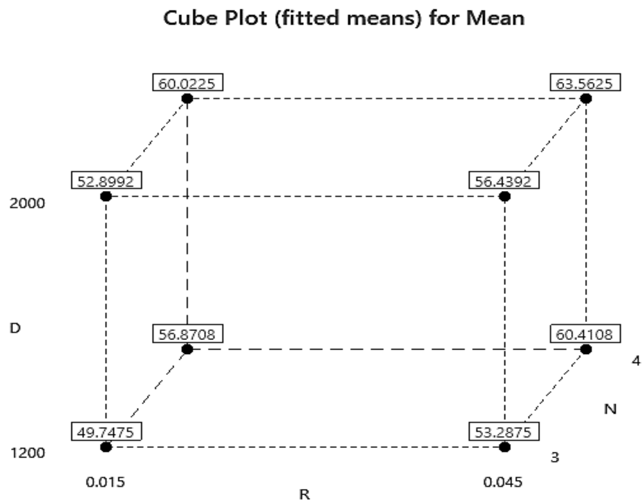
3D cube plot (fitted means), showed for the low-level values of *R*, *D*, and *N*, the minimum value could be invenit. From there in the point split pattern the time increased. Cube plot drawn from minitab showed for which level change of factor which minimum value to expect.

3.2 | The Optimal Values of Influential Factors for Time Minimization

3.2.1 | Model parameters

Target value= 50.2667 seconds.

Upper bound = 60.0 seconds.



Factors= R, D, N.

Fig. 5. Fitted means Cube plot for mean values vs R [17], D, and N.

3.2.2 | Model solution

Bending axis material surface roughness, *R*= 0.015 ε in mm [17].

The density of body material, *D*= 1200 kg/m^3.

Number of bending axis, *N*= 3.

In *Fig. 6*, Mean minimum time (mean fit) = 49.7475 seconds.

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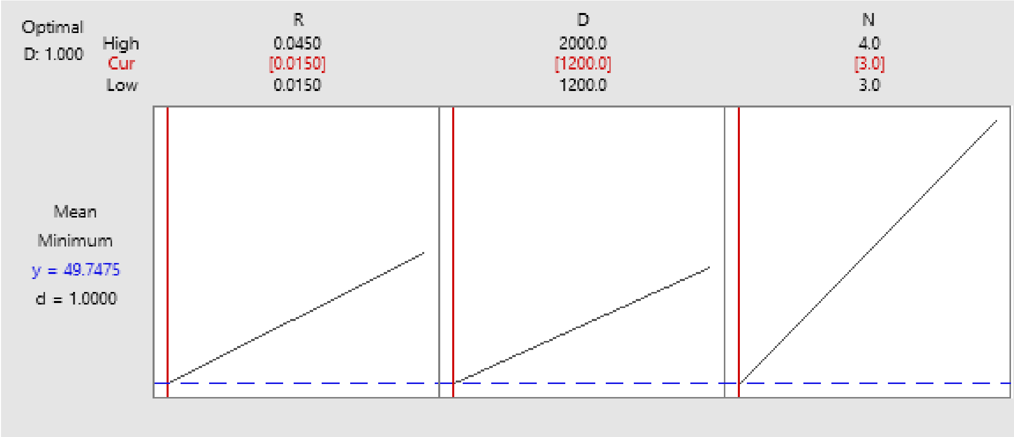

Fig. 6. Optimal solution graph [17].

3.2.3 | Multiple response prediction

Standard error, SE fit= 0.681.

For confidence interval 95% CI range = (48.265, 51.230).

For prediction interval 95% PI range = (46.432, 53.063).

3.3 | Cost Estimation

The mainframe was a wooden structure and 3 joint parts were added using a weld steel pipe.

Fig. 7. Multi-functional furniture side view in the folded position (1,2,3 joint position).

Table 7. Cost estimation for the prototype is given below.

Cost of labor= (3*200) BDT [200 BDT / day] =	600 BDT
Wood price=	800 BDT
Metal pipe price=	180 BDT
Utility cost=	100 BDT
Total cost=	1680 BDT

4| Conclusion

In this article, a multi-functional furniture's design parameters were determined using DoE full factorial two levels four-factor analysis. The furniture was convertible into chair to table to bed and vice-versa. Hence the reshaping time was taken as the independent variable and factors considered for performance enhancement were bending axis material surface roughness, the density of body material, the width of the surface plane, and the number of bending axis. From model established in minitab it is clear that the width of the surface plane did not show an effect in the performance time, whereas, bending axis material surface roughness, the density of body material, and the number of bending axis had a significant effect. Thereafter optimal solution was determined based on the factors for the minimum time needed to reshape the furniture form. To accustom this minimum time of approximately 49 seconds design should contain 3 joint axes and, bending axis material surface roughness 0.015 ε in mm the wood material density 1200 kg per cubic meter [17]. For Wooden frame width up to 0.45-meter, width did not have any effect on the reshaping time. A cubic plot also showed the effect of the factor value on reshaping mean fit time.

4.1| Future Research Scope and Drawbacks

For the early stage of this study, only a two-level full factorial design was carried on without additional center point combination, which remains the condition for further study with parameter combination with corner point replication and multiple level factorial analysis. Also, this study did not consider the table-chair, chair-bed, and bed- table reshaping time individually. A thorough analysis regarding this would remain for future development.

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A New Model to Measuring Efficiency and Returns to Scale on Data Envelopment Analysis

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Abstract

We extend the concept of returns to scale in Data Envelopment Analysis (DEA) to the weight restriction environments. By adding weight restrictions, the status of returns to scale, i.e. increasing, constant, and decreasing, may need a change. We first define "returns to scale" underweight restrictions and propose a method for identifying the status of returns to scale. Then, we demonstrated that this addition would usually narrow the region of the Most Productive Scale Size (MPSS). Finally, for an inefficient Decision-Making Unit (DMU), we will present a simple rule for determining the status of returns to the scale of its projected DMU. Here, we carry out an empirical study to compare the proposed method's results with the BCC model. In addition, we demonstrate the change in the MPSS for both models. We have presented different models of DEA to determine returns to scale. Here, we suggested a model that determines the whole status to scale in decision-making units.

Keywords: Data envelopment analysis, Decision-making units, Most productive scale size, Returns to scale.

1 | Introduction

One of the concepts that have gained significant attention from economic and management-science research is the concept of Returns To Scale (RTS). There are several types of research on the right and left RTS behavior, as two specific directions of RTS, based on Data Envelopment Analysis (DEA) models. However, the main weakness of the majority of these methods is that researchers have based them on the defined parameters. Then we demonstrate that leads to the high sensitivity of the models to variations in the magnitudes of the parameters. Thus, unreliable results. Mirbolouki & Allahyar [11], In a paper, proposed a simple procedure for detecting the right and left RTS classification with an important feature that is independent of any predetermined parameters. Besides the RTS type, they suggested a method to determine the right and left RTS value corresponding to each of the efficient DMUs [11].

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The analyses introduced by DEA has redefined the concept of returns to scale. It is now possible to evaluate the returns-to-scale characteristics of each observation in the data set, in contrast to the Cobb-Douglas type production function approaches that usually estimate the characteristics as a whole. Furthermore, Banker and Thrall [3] introduced the concept of Most Productive Scale Size (MPSS) which scholars widely used for analyzing the optimal level of the scale size associated with the set of observations concerned. Meanwhile, weight restrictions have been recognized as an essential factor when applying DEA to actual situations, and scholars have developed several models for this purpose. These include the Assurance Region (AR) model and the Conratio Approach [3] and [4]. Since researchers regard weights for inputs/outputs as associated with costs of inputs and prices of outputs, constraints on weights should preferably reflect the actual costs/prices information. However, exact knowledge of prices and costs is often unavailable or unstable, so we use some substitutes in these models. For example, in the AR model, upper and lower bounds are imposed on the ratio of weights for specific pairs of inputs or outputs. These weight restrictions contribute to avoiding the occurrence of frequently observed zero optimal weights to some inputs/outputs caused by the optimization mechanism of DEA. Hence the results of analysis using weight restrictions are more persuasive than those without restrictions. This paper aims to develop the returns-to-scale concept when we impose weight restrictions and to demonstrate that the status of returns to scale may suffer a change when we add these restrictions. Therefore, the research subject must investigate returns to scale under weight restrictions theoretically and empirically [1].

The knowledge of DEA evaluates the effectiveness of decision-making units. One of the problems of DEA is that if the number of units with the same efficiency equal to one was more than one, they could not select the best between them. It means that we cannot rank them. Therefore, we consider the need for ranking these units by the managers. Different methods are scale proposed in this context. DEA models imitate most of these methods [14].

There are many methods to determine returns to scale. Here, we have not discussed the details of these methods. However, we will discuss the suggested model more thoroughly. This model is a linear programming question. Its solution gives us the returns to scale of the evaluated unit [3].

Despite the massive use of DEA models for efficiency estimations in scientific applications, no paper cared about identifying the DEA model, providing the most accurate efficiency estimates so far. Develop an established method based on a Monte Carlo data generation process to create artificial data. As a user, the trans-log production function, instead of the commonly utilized Cobb Douglas production function, can construct meaningful scenarios for constant returns to scale. Then, we use the decision-making units resulting from the generated data to calculate DEA estimators using different DEA models. Today, the use of data envelopment analysis techniques is expanding rapidly. The researchers use it to evaluate various organizations and industries such as banks, hospitals, training centers, Etc. In real-world problems, the values observed from input and output data are often ambiguous and random. researchers have proposed data envelopment analysis in a stochastic fuzzy environment to solve this problem [12] and [13].

With this procedure, can postulate general statements on parameters that influence the quality of DEA studies in a positive/negative way and determine which DEA model operates in the most accurate way for a range of scenarios. Here, we show that the Assurance Region and Slacks-Based-Measurement models outperform the CCR (Charnes–Cooper–Rhodes) model in constant returns to scale scenarios [15].

The concepts of RTS and Scale Economies (SE) have a crucial position in economics and production theory. Scholars use these concepts to provide valuable information on the optimal size of the firms [22]. In DEA, DMUs are classified into three categories based on their type of RTS: Constant RTS (CRS), Decreasing RTS (DRS), and Increasing RTS (IRS). RTS is applied to recognize whether an efficient production activity can enhance its productivity by changing the scale of its operations [6] and [11].

Eslami and Khoveyni [8] studied determining the type and measuring value right and left returns to scales in data envelopment analysis. Taeb et al. [16] studied to determine the efficiency of time depended on units using data envelopment analysis. This study identifies types and values of right and left RTSs of efficient decision-making units (DMUs) in DEA [7]. Alireza et al.[2] studied Objective identification of technological returns to scale for data envelopment analysis models. This paper considered one of the most critical problems for setting up a data envelopment analysis model: identifying suitable RTS for the data. Referred to it as the Technological Returns To Scale (TRTS) to completely separate the technology's RTS from the DMU's RTS [2].Abri [1] considered investigating the sensitivity and stability radius of returns to scale and efficiency in data envelopment analysis. This paper will study the sensitivity of the RTS classifications in data envelopment analysis using linear programming problems. It is surprising since RTS classifications provide essential information for improving an individual DMU's performance when scale inefficiencies are detected [1].

We should note that the subject of RTS mainly has a clear interpretation only if the DMU under evaluation is efficient. RTS is a characteristic of the frontier at a specific point, so RTS is discussed only for efficient DMUs in this study.

Reedy [17] assessed Thirupati Reddy Comparison and Correlation Coefficient between CRS and VRS models of OC Mines [17]. Hatami-Marbini et al. [8] considered the measurement of returns-to-scale where they used interval data envelopment analysis, models. In this paper, researchers have studied the economic concept of RTS intensively in the context of DEA. The conventional DEA models that researchers use for RTS classification require well-defined and accurate data, whereas in reality, observations gathered from production systems may be characterized by intervals. For instance, the heat losses of the Combined Production of Heat and Power (CHP) systems may be within a specific range, hinging on a wide variety of factors such as external temperature and real-time energy demand [8].

Khodadadi and Haghighi [10] studied Two Methods for Measuring the Environmental Returns to Scale Using Data Envelopment Analysis Approach. Sueyoshi and Wang [19] studied measuring scale efficiency on sizeable commercial rooftop photovoltaic systems in California. They examined managerial sources of operational efficiency or inefficiency on 855 large commercial rooftop PV power systems in California by examining both scale efficiency and RTS. For the research purpose, this study utilizes DEA as a methodology to assess the scale measures. Furthermore, by Paying attention to the effects of those uncontrollable factors, this study discusses how to measure scale efficiency and RTS within the framework of DEA.

Toloo and Allahyar [18], investigated a generalized simplification returns to scale approach for selecting performance measures in data envelopment analysis. Toloo and Tichý [20], to hold the rule of thumb in data envelopment analysis, developed a pair of models that optimally chooses some inputs and outputs among selective measures under variable returns to scale assumption. Their approach involves a lower bound for the input and output weights in the multiplier model and a penalty term in the objective function of the envelopment model [20].

Ghasemi et al. [9], a case study with DEA for Estimate Efficiency and Ranking operating rooms. Czyżewski et al. [5] Assessed the impact of environmental policy on eco-efficiency in country districts in Poland: How does the decreasing return to scale change perspectives? in this study, authors show how changing CRS assumption affects environmental policy effectiveness based on Polish example. The problem revealed in the conducted analysis is in many countries, where the local perspective may efface global threats. The empirical objective of this paper is to assess the cost-effectiveness of environmental policies at the county level under various RTS scenarios [5].

Bernstein [6] investigated an updated assessment of technical efficiency and returned to scale for U.S. electric power plants. This paper utilizes cutting-edge panel stochastic frontier electricity production models to measure the impact of state and federal regulations on United States (U.S.) natural gas-fired power plants from 1994 to 2016. Deploying a trans-log functional form, extract firm-specific information on RTS [6].

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We have organized the rest of this manuscript as follows. We will also study the BCC model to determine returns to scale in section 2 and present our suggested model in section 3.

2 | BCC Model in Determining Returns to Scale

Farrel [21] defines the measurement of efficiency and its function by Charnes et al. [4]. It led to the establishment of tools of important methods DEA for evaluating the efficiency. The standard method DEA, which measures technical efficiency with return hypothesis to variable scale in input essence, is done by Banker, Charnes, and Cooper (BCC) [3]:

$$
\begin{aligned}
&\text{Miximize} \quad \theta = \sum_{r=1}^{s} u_r y_{ro} + u_0 \\
&\text{Subject} \quad \text{to.} \\
&\qquad \sum_{i=1}^{m} v_i x_{io} = 1 \\
&\qquad \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} + u_0 \le 0 \qquad j = 1,\dots,n \\
&\qquad u_r \ge \varepsilon \qquad\qquad r = 1,\dots,s \\
&\qquad v_i \ge \varepsilon \qquad\qquad i = 1,\dots,m \\
&\qquad u_0 \qquad\qquad\quad \text{free}
\end{aligned}
\tag{1}
$$

In this problem, θ_o is the efficiency level of the evaluated decision-making unit. y_{rj}, x_{ij} are, respectively introduced as ith levels of input of output DMU_j. v_i, u_r are weights related to inputs and outputs, comparable with model variables. It can be interpreted as shadow price; therefore, the price of input and output DMU which will be shown, is the best possible price. ε is a small non archimedes value. It guarantees that all inputs and outputs will be used in calculations for efficient evaluation.

Definition: DMU_o is completely efficient, if and only if the condition $\theta_o = 1$ available in its evaluation by *Model 1*. Banker and Thrall [3] proved this to identify returns to scale with u_o.

3 | Model Generating Returns to Scale GRS

While $L \le 1$ and $U \ge 1$ the resulting model have a Generating Returns to Scale (GRS), this model allows the expansion and limitation of size over a while. *Fig. 1* shows the difference between the GRS boundary and the BCC and CCR boundaries.

Theorem 1. These conditions identify this situation for returns to efficient unit scale in the BCC model.

1. Returns to scale DMU_o is increasing if and only if $u_o \succ 0$ for all optimal solutions.

2. Returns to scale DMU_o is decreasing if and only if $u_0 \prec 0$ for all optimal solutions.

3. Returns to scale DMU_o is constant (MPSS), if and only if $u_o = 0$ for some optimal solutions.

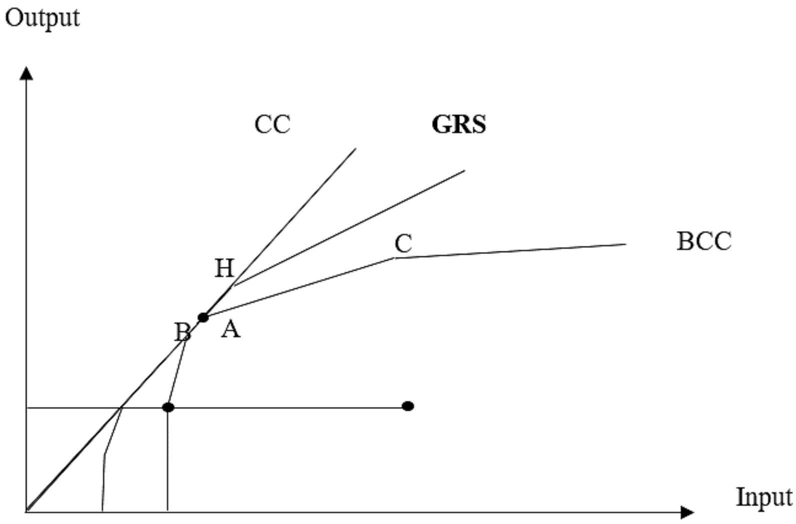

Fig 1. The GRC.

However, Banker and Thrall [3] presented a way to abandon the need to find all the optimal solutions. We are not going to discuss it here.

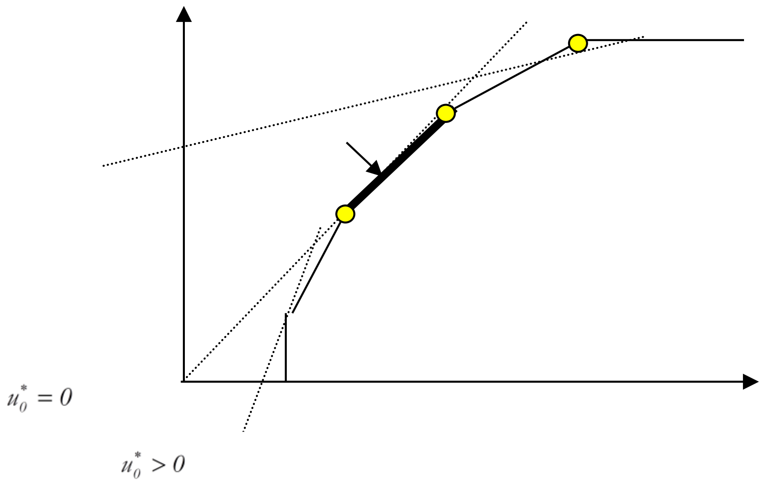

Fig. 2. The identification of returns to scale with u_o

4 | Proposed Model

The additive model which has been provided by Charnes et al. [4] to evaluate decision-making units is defined as follows [2].

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$$\text{Maximize} \quad \sum_{i=1}^{m} S_i^- + \sum_{r=1}^{s} S_r^+$$

Subject to.

$$\begin{aligned} &\sum_{j=1}^{n} \lambda_j x_{ij} + S_i^- = x_{io} \qquad \forall i, \\ &\sum_{j=1}^{n} \lambda_j y_{rj} - S_r^+ = y_{ro} \qquad \forall r, \\ &\sum_{j=1}^{n} \lambda_j = 1 \\ &S_i^-, S_r^+ \geq 0 \qquad \forall i, r, \\ &\lambda_j \geq 0 \qquad \forall j, \end{aligned} \tag{2}$$

The objective function of the above model is linear which indicates the L_1 norm.

In this model, the objective function calculates the farthest distance of the evaluated unit on dominant efficient units.

Theorem 2. In the integrative model, the evaluated decision-making unit is efficient if and only if the objective function is zero in optimality.

Theorem 3. Suppose that $DMU_o = (x_o, y_o)$ is efficient. then:

1) $0 \prec \xi \prec 1$ so that $(\xi x_0, \xi y_0)$ is the possible inefficient production in series, if and only if (xo, yo) has decreasing returns to scale.

2) $\xi > 1$ so that $(\xi x_0, \xi y_0)$ is the possible inefficient production in series, if and only if (xo, yo) has increasing returns to scale.

3) For every $\xi > 0$ so that $(\xi x_0, \xi y_0)$ is the possible efficient production, if and only if (xo, yo) has constant returns to scale (MPSS).

Proof 1. If $0 \prec \xi \prec 1$ so that $(\xi x_0, \xi y_0)$ is the possible inefficient production in series. Since (xo, yo) is an efficient unit, there is a supporting hyperplane $U^* y - V^* x + u_0^* = 0$ in possible production series so that it is active on (xo, yo) (because it is supposed that (U^*, V^*, u_0^*) is the optimal solution for the BCC model in (xo, yo) evaluation. Since $(\xi x_0, \xi y_0)$ is inefficient in possible production series, we have

$$U^* \xi y_o - V^* \xi x_o + u_0^* < 0 \Rightarrow \xi (U^* y_o - V^* x_o + u_0) - \xi u_0^* + u_0^* < 0 \Rightarrow u_0^* (1 - \xi) < 0 \Rightarrow u_0^* < 0.$$

Hence, *(xo, yo)* has decreasing returns to scale according to *Theorem 1*.

On the contrary, suppose that (xo, yo) has decreasing returns to scale. Therefore, we should show that $\xi \varepsilon$ (0,1) so that $(\xi x_0, \xi y_0)$ is the possible inefficient production in series.

Since (xo, yo) has decreasing returns to scale, for each optimal solution for the BCC model in its evaluation $u_0 \prec 0$ according to therem1: Supposition breach for every $\xi \varepsilon (0,1)$ so that it $(\xi x_0, \xi y_0)$ is the possible production, it is efficient. Therefore, every convex combination of $(\xi x_0, \xi y_0)$ and (xo, yo) is in the possible production of series and is available on the efficient symbol. Therefore, the supporting hyperplane $\bar{U}y - \bar{V}x + \bar{U}o = o$ can be taken on possible production series so that it passes the connecting line $(\xi x_0, \xi y_0)$ and (xo, yo). If $\bar{V}x = a$, $(U^*, V^*, u_0^*) = (a^{-1}\bar{U}, a^{-1}\bar{V}, a^{-1}u_0)$ is an optimal solution in the evaluation of (xo, yo) so that it is active on (xo, yo) and $(\xi x_0, \xi y_0)$ then

$$U^* y_o - V^* x_o + u_0^* = 0.$$

$$U^* \xi y_o - V^* \xi x_o + u_0^* = 0.$$

In the two above equations, if we subtract ε the first equality from the second equality, there is:

$$u_0^*(1-\xi) = 0 \Rightarrow u_0^* = 0.$$

It means that (x_o, y_o) has constant returns to scale. This is in contrast with our hypothesis. Therefore, the supposition breach is invalid and the axiom is true.

The proof of the conditions 2) and 3) can be done similarly. We tried to make $(\xi x_0, \xi y_0)$ inefficient in our model to determine returns to scale so that the value of ε can be determined in this way. Therefore, if the inefficiency $(\xi x_0, \xi y_0)$ leads to an increase of ε from level, a return to scale is increasing. If the inefficiency of $(\xi x_0, \xi y_0)$ leads to a decrease of ε from leve1, returns to scale are decreasing. If $(\xi x_0, \xi y_0)$ cannot be the possible inefficient production, there will be constant returns to scale. The suggested model is:

$$\begin{aligned}
\text{Maximize} \quad & \sum_{i=1}^{m} S_i^- + \sum_{r=1}^{s} S_r^+ \\
\text{Subject to.} \quad & \\
& \sum_{j=1}^{n} \lambda_j x_{ij} + S_i^- = \xi x_{io} \qquad \forall i, \\
& \sum_{j=1}^{n} \lambda_j y_{rj} - S_r^+ = \xi y_{ro} \qquad \forall r, \\
& \sum_{j=1}^{n} \lambda_j = 1, \\
& S_i^-, S_r^+ \geq 0 \qquad \forall i, r, \\
& \lambda_j \geq 0 \qquad \forall j, \\
& \xi \geq 0.
\end{aligned} \tag{3}$$

We present the following theorem to identify returns to scale with *Model 3*.

Theorem 4. The following conditions determine returns to scale for DMUo in *Model 3*.

1) The optimal value of the objective function in optimality is non – zero and, $\varepsilon^* \succ 1$ if and only if returns to scale DMUo is increasing.

2) The optimal value of the objective function in optimality is non zero an and, $\varepsilon^* \prec 1$ if and only if returns to scale DMUo decreases.

3) The optimal value of the objective function in optimality is zero, if and only if returns to scale DMUo are constant (MPSS).

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The proof of condition 1. We suppose that the optimal value of the objective function in optimality is nonzero and $\varepsilon^* \succ 1$, therefore $(\xi x_0, \xi y_0)$ is the possible production in series, and it is inefficient. Returns to scale DMUo are increasing according to *Theorem 3*.

On the contrary, if returns to scale of DMUo are increasing $\varepsilon^* \succ 1$ according to *Theorem 3* so that $(\xi x_0, \xi y_0)$ is the possible inefficient production. The inefficiency of $(\xi x_0, \xi y_0)$ in *Model 3* requires the level of the objective function to be above zero. Because this function is a kind of maximizing, function in optimality should be non-zero.

It means that $(\xi x_0, \xi y_0)$ is the possible inefficient production in series.

Returns to scale are increasing, therefore, $\varepsilon^* \succ 1$.

we can prove conditions 2 and 3 similarly.

It means that is the possible inefficient production in series.

Returns to scale are increasing. Therefore,

Example 1. *Table 1* presents a model of the characteristics of three decision-making units to determine returns to scale.

Table 1. RTS data.

RTS	ξ^*	$S_1^{+^*}$	$S_1^{-^*}$	y_1	x_1	DMU#
Increasing	2	0	1	1	1	1
Constant	1	0	0	2	2	2
Decreasing	0/65	0	1	5	4	3

Model 3 to determine returns to scale in decision-making unit in *Example 1* is:

$$
\begin{aligned}
&\text{Maximize} \quad S_1^- + S_1^+ \\
&\text{Subject to.} \\
&\qquad 1\lambda_1 + 2\lambda_2 + 4\lambda_3 + S_1^- - 1\xi = 0, \\
&\qquad 1\lambda_1 + 2\lambda_2 + 5\lambda_3 - S_r^+ - 1\xi = 0, \\
&\qquad \lambda_1 + \lambda_2 + \lambda_3 = 1, \\
&\qquad \lambda_1, \lambda_2, \lambda_3 \geq 0, \\
&\qquad \xi \geq 0.
\end{aligned}
$$

The optimal response to this question is:

$$(\lambda_1^*, \lambda_2^*, \lambda_3^*, S_1^{-^*}, S_2^{+^*}, \xi^*) = (0,1,0,1,0,2).$$

$$\xi^* = 2 > 1.$$

The value of the objective function in optimality is 1, which indicates that returns to scale of this decision-making unit cannot be constant. On the other hand, $\varepsilon^* = 2 \succ 1$ therefore, returns to scale are increasing.

Another characteristic of the suggested model is that we can identify both the returns to scale of the evaluated decision-making unit and MPSS as a management objective with its solution. The following theorem indicates this subject.

5 | Conclusions

In this manuscript, we used the DEA method to find the weight restriction environments. Our empirical study compares the results obtained through the proposed method with those of the BCC model. It further demonstrates the change in the MPSS for both models. We have presented different data analysis models to determine returns to scale and have suggested a model here. The suggested model determines the constant returns to scale, increasing returns to scale, and decreasing returns to scale in decision-making units.

This manuscript studies the new method detects both the type and the value of returns to scale.

The advantage of the new method is that it uses no parameter in model formulation.

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Development of a Multi-Period Multi-Attribute Group Decision-Making Method Using Type- 2 Fuzzy Set of Linguistic Variables

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Abstract

Recently, fuzzy linguistic variables have gained a great deal of attention from researchers in many decision-making problems. In this problems, type-2 fuzzy sets have been used to better cover linguistic data uncertainty. However, many of research works in this regard have been performed in type-2 fuzzy domain and in static mode. Since the decision-making problems in the real world usually fluctuate over time, so it needs to use decision making models in multi period of time. In the present research, a Multi-Period (Dynamic) Multi-Attribute Group Decision-Making (MPMADM) method is presented based on type-2 fuzzy sets where decision-making attributes are first expressed in linguistic terms and then incorporated, as interval type-2 fuzzy numbers, into problem solving where a new integrating operator called Multi-Period Trapezoidal Interval Type-2 Fuzzy Number Weighted Arithmetic averaging (MPTIT2FNWA) is defined on type-2 interval fuzzy numbers to integrate decision-making information in multiple periods of time. Once finished with explaining the proposed method, a numerical example is given to evaluate the proposed method in terms of effectiveness and applicability, with the results compared to those of other methods.

Keywords: Interval type-2 fuzzy sets, Linguistic variables, Multi-period trapezoidal interval type-2 fuzzy number weighted arithmetic operator, Multi-period multi-attribute group decision-making.

1 | Introduction

Today, Multi-Attribute Decision-Making (MADM) methods have found numerous applications in solving different management problems. However, in many real-world problems, decision-making process has its basis on decision information of several time periods, and one should include the trend of change in the values induced over the course of time in the decision-making process because information is subject to change with time and these changes can seriously affect the trend of decision-making process and prioritization of alternatives; in this case, Multi-Period Multi-Attribute Decision-Making (MPMADM) comes into play.

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Even though MPMADM problems are more complicated than MADM, their application can be associated with better results. Numerous research works have been done in this respect. In a study, Xu [1] presented a MPMADM method where Simple Additive Weighting (SAW) was used to undertake MPMADM. He used distance data to develop a model where a Multi-Period Weighted Averaging (DWA) operator was used to integrate values of the attributes in different periods of time, and further evaluated an investment method selection problem. Lin et al. [2] proposed a MPMADM method wherein TOPSIS method was used to rank alternatives in different periods of time. In this study, values of attributes were considered as Triangular Fuzzy Numbers (TriFN) and Minkowski Distance was used to compare these numbers. They finally implemented the proposed method for contractor selection. In a MPMADM problem, Xu and Yager [3] first introduced Multi-Period Intuitionist Fuzzy Weighted Averaging (DIFWA) and Uncertain Multi-Period Intuitionist Fuzzy Weighted Averaging (UDIFWA) operators. In this study, first, these operators were employed to integrate decision matrixes for all periods of time. Then TOPSIS technique was used to have available alternatives ranked. Yong et al. [4] proposed TOPSIS technique to solve MPMADM on gray numbers. The began with obtaining weighted Minkowski distance function of gray number by using the Euclidian distance between two gray numbers and the concept of weighted Minkowski distance function. They then implemented different steps of TOPSIS technique. Chen and Li [5] proposed a MPMADM method based on Triangular Intuitionist Fuzzy Numbers (TriIFN) where fuzzy entropy method and averaging operator were used to determine weights of attributes. They obtained a general ranking of alternatives by integrating, using TOPSIS, the results obtained from different periods of time, and finally analyzed an investment problem. Zhu and Hipel [6] proposed a MPMADM method based on linguistic variables and then evaluated performance of vendors of an electronic navigation system in terms of economic and production development of aviation industry in China, in this study, decision information was used as 2-tuple linguistic terms and solved MPMADM based them. Hu and Yang [7] proposed a MPMADM method on the basis of probability theory where attributes had their values expressed in terms of discrete random variables. They used this method for risk evaluation. Sadeghian and Forootan [8] proposed a MPMADM method using a regression model. In their study, they presented a regression model for each element in the decision-making matrix and then used TOPSIS technique to rank alternatives. They used this method to rank investments in textile industry. Park et al. [9] used VIKOR method to develop a MPMADM approach. In this study, Interval Triangular Fuzzy Numbers (ITFN) to express decision information and used the proposed method to evaluate university professors for promotions. Liu et al. [10] used extendible interval numbers in MPMADM method. In this study, they introduced a distance function for extendible interval numbers, and following the determination of comparative value of each alternative in each time period, TOPSIS technique was used to rank the alternatives to select one city among three candid cities for investment on public transportation development. Li et al. [11] presented a MPMADM method where used data was expressed in terms of TriFNs. In this research, the authors used mathematical programming to determine weights of different attributes in different periods of time, followed by ranking the alternatives utilizing TOPSIS technique. Bai et al. [12] presented a MPMADM method proposed a MPMADM based on TOPSIS technique and representation of available data in terms of Trapezoidal Fuzzy Numbers (TraFNs); the proposed method was then implemented it to rank a set of suppliers. Bera et al. [13] presented a two phase MPMADM Approach for supplier evaluation and order allocation considering multi-objective, multi-product and multi-period. In this research, in first phase, the ranking of supplier was performed by using fuzzy MULTIMOORA method with regard to the important criteria. In the second phase, multi-objective linear programming (MOLP) method in fuzzy environment was proposed to allocate orders to the preferred suppliers. Li et al. [14] presented a MPMADM method for supplier selection problem. In this research, data of decision problem was expressed in generalized fuzzy numbers and the weights of different periods are determined by a mathematical programming method. Fei and Feng [15] proposed a novel framework for dynamic MADM in Pythagorean fuzzy environments based on Dempster–Shafer Theory (DST). In this research, the period weight from the dual dimensions of information and consistency were determined and the period weight from the dual dimensions of information and consistency were determined and the dynamic MADM was completed by combining the decision information of all attributes in each period, fusing the decision information of different periods, and calculating the ranking index of each alternative.

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Decision-making information, which are expressed by group of experts, are usually inaccurate and ambiguous because of, for example, lack of accurate data, shortage of time, or failure to pay adequate attention or inadequate knowledge of the members of expert group. In many problems, it is difficult to access accurate values of data for decision-making [16]. In such situations, opinions of the expert group (originally expressed in words) serve as criteria for valuation. However, words are always associated with ambiguities, so that researchers have tended to use fuzzy sets theory to address this ambiguity, where values of variables are specified with a membership degree. However, complexities in some of decision-making problems have made it difficult to determine exact value of membership degree [17]. The fuzzy linguistic approach is an approximate technique appropriate to deal with the qualitative aspects of decision-making problems [6]. Zadeh [18] introduced Type-2 Fuzzy Sets (T2FS) as an extension to Type 1 Fuzzy Sets (T1FS). T2FSs tend to exhibit better performance in reducing the effect of uncertainty in fuzzy rules. Due to fuzzy nature of membership functions in T2FSs, the possibility to model linguistic uncertainties is effectively improved. Turksen [19] introduced the application of T2FSs to support word calculation reasoning. Based on the reasoning that words are associated with more complex uncertainties than that of T1FSs, Mandel [20] recognized the use of T1FSs for word modelling as being inappropriate and believed that T2FSs can better model word uncertainties. Since computation in type-2 fuzzy domain has numerous complexities, the use of Interval Type-2 Fuzzy Sets (IT2FS), for which numerous functions and operators have been proposed, has recently gain a large deal of attention, so that it is now developed as an efficient theory in domains of high uncertainty [21] and [22].

In MADM problems, there are typically approaches to dealing with linguistic models as, T1FS [11], [12], [23]-[25], hesitant fuzzy [26]-[28], intuitionistic fuzzy [3], [5], [9], type-2 fuzzy [17], [29]-[36], [41]-[43], fuzzy 2- tuple [6], Neutrosophic Sets (NS) [37] and etc.

Even though many research works have been performed on MPMADM, this method is yet to be addressed in type-2 fuzzy environment and its application to solve of different problems in domain of management can have interesting results. As such, the present research aims to present MPMADM in type-2 fuzzy domain. In this method, a new operator Multi-Period Trapezoidal Interval Type-2 Fuzzy Number Weighted Arithmetic Averaging (MPTIT2FNWA) based on Basic Unit-interval Monotonic (BUM) probability distribution function is defined to integrate decision information in multiple time periods. This operator was found to provide required flexibly to select any trend of time series for the specified weights of time periods depending on the problem characteristics. To determine the efficiency of the proposed method, we will solve a numerical example in [6] and compare the results with their method. Accordingly, in Section 2, T2FSs are defined together with respective functions and operators. Then in Section 3 we proceed to introduce MPTIT2FNWA operator. Section 4 explains the framework of MPMADM in type-2 fuzzy domain, and Section 5 gives a numerical example to better understand the proposed method. Finally, conclusions are drawn in Section 6.

2 | Type-2 Fuzzy Sets

First introduced by Zadeh in 1965, fuzzy sets theory serves as a modeling tool for complicated systems [38] and [39]. Original concepts within the scope of fuzzy sets theory were formulated under the name of Type 1 Fuzzy Sets (T1FSs). These then found numerous applications, particularly in MADM problems. In T1FSs, each set is determined by its elements and their membership function which gives a real number between 0 and 1 for each member in the set. Zadeh [18] introduced T2FS as an extension to T1FSs. In T2FSs, membership function of the elements in the set is itself a fuzzy set. Mandel and Wu [21] presented a new concept of T2FSs with a simple calculation process where superior and inferior limits are considered for the membership functions, with each of these membership functions resembling a membership function in T1FSs. Later on, Mandel et al. [22] further proposed a new concept called IT2FS, where membership function of each element was a fuzzy set in the interval of [0, 1]. In the following sections, definitions of some of concept and operators related to IT2FS are given.

2.1 | Type-2 Fuzzy Sets

Explained in this section are definitions of some of the concept and operators related to IT2FS, followed by a presentation of the IT2FS method along with the required operators.

Definition 1. [30] and [31]. If $\tilde{A}$ is a T2FS on the universe of discourse X, it can be defined as follows:

$$\tilde{A}=\left\{\left((x,u),\mu_{\tilde{A}}(x,u)\right);\forall x\in X,\forall u\in J_x\subseteq[0,1]\right\}. \tag{1}$$

where $0\geq\mu_{\tilde{A}}(x, u)\geq 1$, and we have:

$$\tilde{A}=\int_{x\in X}\int_{u\in J_x}\mu_{\tilde{A}}(x,u)/(x,u)=\int_{x\in X}\left(\int_{u\in J_x}\mu_{\tilde{A}}(x,u)/u\right)/x. \tag{2}$$

Where $\int$ represents the sum of all combinations of (x, u), x is the primary variable with its membership function being $J_x\subseteq[0,1]$, and u is the secondary variable with the membership function $\int_{u\in J_x}\mu_{\tilde{A}}(x,u)/u$ on X.

Definition 2. [30], [31]. Let $\tilde{A}$ is a T2FS where in all $\mu_{\tilde{A}}(x,u)$ are equal to 1, then $\tilde{A}$ is referred to as an IT2FS, in which case we have:

$$\tilde{A}=\int_{x\in X}\int_{u\in J_x}1/(x,u)=\int_{x\in X}\left(\int_{u\in J_x}1/u\right)/x\,. \tag{3}$$

where x is the primary variable with its membership function being $J_x\subseteq[0,1]$, and u is the secondary variable with the membership function $\int_{u\in J_x}1/u$ and foot print of uncertainty in the set $\tilde{A}$ is defined as follows:

$$FOU(\tilde{A})=\cup_{x\in X}J_x\,. \tag{4}$$

which includes the sum of primary membership function over the reference set X.

Definition 3. [21]. In general case, a Trapezoidal Interval Type-2 Fuzzy Number (TIT2FN) is defined as $\tilde{A}=\left(\tilde{A}^U,\tilde{A}^L\right)=\left(\left(a_1^U,a_2^U,a_3^U,a_4^U;H_1\left(\tilde{A}^U\right),H_2\left(\tilde{A}^U\right)\right),\left(a_1^L,a_2^L,a_3^L,a_4^L;H_1\left(\tilde{A}^L\right),H_2\left(\tilde{A}^L\right)\right)\right)$ where $\tilde{A}^U$ and $\tilde{A}^L$ are type-1 fuzzy numbers (T1FN) and, $a_1^U,a_2^U,a_3^U,a_4^U;H_1\left(\tilde{A}^U\right),H_2\left(\tilde{A}^U\right),a_1^L,a_2^L,a_3^L,a_4^L;H_1\left(\tilde{A}^L\right),H_2\left(\tilde{A}^L\right)$ are real numbers and is establishing the inequality $a_1^U\leq a_2^U\leq a_3^U\leq a_4^U, a_1^L\leq a_2^L\leq a_3^L\leq a_4^L$, as can be seen on *Fig. 1*, $H_i\left(\tilde{A}^U\right)$ is the membership value of the element a_{i+1}^U in the upper trapezoidal membership function (UMF) and $H_i\left(\tilde{A}^L\right)$ is membership value of the element a_{i+1}^L in the lower trapezoidal membership function (LMF) where, $0\leq H_i\left(\tilde{A}^U\right)\leq 1$, $0\leq H_i\left(\tilde{A}^L\right)\leq 1$ and $1\leq i\leq 2$.

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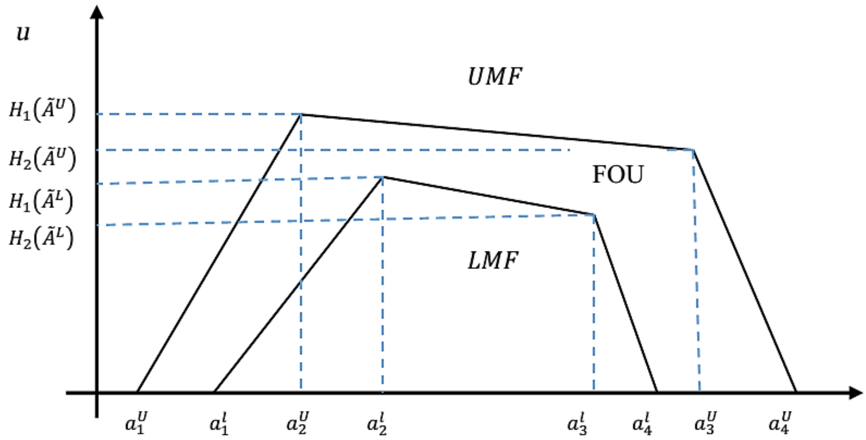

Fig. 1. A trapezoidal interval type-2 fuzzy number.

Definition 4. [30] and [31]. Let $\tilde{A}_1$ and $\tilde{A}_2$ be two TIT2FNs defined as follows:

$$\tilde{A}_1=\left(\tilde{A}_1^U,\tilde{A}_1^L\right)=$$

$$\left(\left(a_{11}^U,a_{12}^U,a_{13}^U,a_{14}^U;H_1\left(\tilde{A}_1^U\right),H_2\left(\tilde{A}_1^U\right)\right),\left(a_{11}^L,a_{12}^L,a_{13}^L,a_{14}^L;H_1\left(\tilde{A}_1^L\right),H_2\left(\tilde{A}_1^L\right)\right)\right),$$

$$\tilde{A}_2=\left(\tilde{A}_2^U,\tilde{A}_2^L\right)=$$

$$\left(\left(a_{21}^U,a_{22}^U,a_{23}^U,a_{24}^U;H_1\left(\tilde{A}_2^U\right),H_2\left(\tilde{A}_2^U\right)\right),\left(a_{21}^L,a_{22}^L,a_{23}^L,a_{24}^L;H_1\left(\tilde{A}_2^L\right),H_2\left(\tilde{A}_2^L\right)\right)\right).$$

Then, summation operator on these numbers can be defined as follows:

$$\tilde{A}_1\oplus\tilde{A}_2=\left(\tilde{A}_1^U,\tilde{A}_1^L\right)\oplus\left(\tilde{A}_2^U,\tilde{A}_2^L\right)=$$

$$\left(\left(a_{11}^U,a_{12}^U,a_{13}^U,a_{14}^U;H_1\left(\tilde{A}_1^U\right),H_2\left(\tilde{A}_1^U\right)\right),\left(a_{11}^L,a_{12}^L,a_{13}^L,a_{14}^L;H_1\left(\tilde{A}_1^L\right),H_2\left(\tilde{A}_1^L\right)\right)\right)\oplus$$

$$\left(\left(a_{21}^U,a_{22}^U,a_{23}^U,a_{24}^U;H_1\left(\tilde{A}_2^U\right),H_2\left(\tilde{A}_2^U\right)\right),\left(a_{21}^L,a_{22}^L,a_{23}^L,a_{24}^L;H_1\left(\tilde{A}_2^L\right),H_2\left(\tilde{A}_2^L\right)\right)\right)=$$

$$\left(\left(a_{11}^U+a_{21}^U,a_{12}^U+a_{22}^U,a_{13}^U+a_{23}^U,a_{14}^U+a_{24}^U;\min\left(H_1\left(\tilde{A}_1^U\right),H_1\left(\tilde{A}_2^U\right)\right),\right)\right) \tag{5}$$

$$\min\left(H_2\left(\tilde{A}_1^U\right),H_2\left(\tilde{A}_2^U\right)\right)a_{11}^L+a_{21}^L,a_{12}^L+a_{22}^L,a_{13}^L+a_{23}^L,a_{14}^L+a_{24}^L;$$

$$\min\left(H_1\left(\tilde{A}_1^L\right),H_1\left(\tilde{A}_2^L\right)\right),\min\left(H_2\left(\tilde{A}_1^L\right),H_2\left(\tilde{A}_2^L\right)\right).$$

Definition 5. [30] and [31]. Let $\tilde{A}$ be a TIT2FN and

$$\tilde{A}=\left(\tilde{A}^U,\tilde{A}^L\right)=\left(\left(a_1^U,a_2^U,a_3^U,a_4^U;H_1\left(\tilde{A}^U\right),H_2\left(\tilde{A}^U\right)\right),\left(a_1^l,a_2^l,a_3^l,a_4^l;H_1\left(\tilde{A}^L\right),H_2\left(\tilde{A}^L\right)\right)\right).$$

Then we will have:

$$\lambda\times\tilde{A}=\left(\lambda\times\tilde{A}^U,\lambda\times\tilde{A}^L\right)$$

$$\left(\left(\lambda\times a_1^U,\lambda\times a_2^U,\lambda\times a_3^U,\lambda\times a_4^U;H_1\left(\tilde{A}^U\right),H_2\left(\tilde{A}^U\right)\right),\left(\lambda\times a_1^l,\lambda\times a_2^l,\lambda\times a_3^l,\lambda\times a_4^l;H_1\left(\tilde{A}^L\right),\right)\right) \quad (6)$$

$$H_2\left(\tilde{A}^L\right)).$$

$$\frac{\tilde{A}}{\lambda}=\left(\frac{\tilde{A}^U}{\lambda},\frac{\tilde{A}^L}{\lambda}\right)=$$

$$\left(\left(\frac{a_1^U}{\lambda},\frac{a_2^U}{\lambda},\frac{a_3^U}{\lambda},\frac{a_4^U}{\lambda};H_1\left(\tilde{A}^U\right),H_2\left(\tilde{A}^U\right)\right),\left(\frac{a_1^l}{\lambda},\frac{a_2^l}{\lambda},\frac{a_3^l}{\lambda},\frac{a_4^l}{\lambda};H_1\left(\tilde{A}^L\right),H_2\left(\tilde{A}^L\right)\right)\right). \quad (7)$$

and

$$(\tilde{A})^\lambda=\left((a_1^U)^\lambda,(a_2^U)^\lambda,(a_3^U)^\lambda,(a_4^U)^\lambda;H_1\left(\tilde{A}^U\right),H_2\left(\tilde{A}^U\right)\right),$$

$$\left((a_1^l)^\lambda,(a_2^l)^\lambda,(a_3^l)^\lambda,(a_4^l)^\lambda;H_1\left(\tilde{A}^L\right),H_2\left(\tilde{A}^L\right)\right). \quad (8)$$

Definition 6. [30]. If $\tilde{A}$ is a TIT2FN, magnitude of the rank of $\tilde{A}$, $Rank(\tilde{A})$ is defined as follows:

$$Rank(\tilde{A})=M_1\left(\tilde{A}^U\right)+M_1\left(\tilde{A}^L\right)+M_2\left(\tilde{A}^U\right)+M_2\left(\tilde{A}^L\right)+M_3\left(\tilde{A}^U\right)+M_3\left(\tilde{A}^L\right)-1/4(S_1\left(\tilde{A}^U\right)$$

$$+S_1\left(\tilde{A}^L\right)+S_2\left(\tilde{A}^U\right)+S_2\left(\tilde{A}^L\right)+S_3\left(\tilde{A}^U\right)+S_3\left(\tilde{A}^L\right)+S_4\left(\tilde{A}^U\right)+S_4\left(\tilde{A}^L\right))+H_1\left(\tilde{A}^U\right) \quad (9)$$

$$+H_1\left(\tilde{A}^L\right)+H_2\left(\tilde{A}^U\right)+H_2\left(\tilde{A}^L\right).$$

where $M_p(\tilde{A}^q)$ is the average between the elements a_p^q and a_{p+1}^q, that is $M_p(\tilde{A}^q)=\frac{\left(a_p^q+a_{p+1}^q\right)}{2}$, $S_p(\tilde{A}^q)$ represents the standard deviation of the elements a_p^q and a_{p+1}^q, that is $S_p(\tilde{A}^q)=\sqrt{\frac{1}{2}\sum_{k=p}^{p+1}\left(a_k^q-\frac{1}{2}\sum_{k=p}^{p+1}a_k^q\right)^2}$, and $S_4(\tilde{A}^q)$ represents standard deviation of the elements a_1^q, a_2^q, a_3^q, and a_4^q, that is $S_4(\tilde{A}^q)=\sqrt{\frac{1}{4}\sum_{k=1}^{4}\left(a_k^q-\frac{1}{4}\sum_{k=1}^{4}a_k^q\right)^2}$. Moreover, $H_p(\tilde{A}^q)$ denotes membership degree of the element a_{p+1}^q where $1\le p\le 3$ and $q\in\{U,L\}$.

For example, let $\tilde{A}=\{(0.34, 0.4, 0.42, 0.48; 1, 1),(0.36, 0.38, 0.4, 0.44;0.95, 0.95)\}$, then we have:

$$Rank(\tilde{A})=M_1\left(\tilde{A}^U\right)+M_1\left(\tilde{A}^L\right)+M_2\left(\tilde{A}^U\right)+M_2\left(\tilde{A}^L\right)+M_3\left(\tilde{A}^U\right)+M_3\left(\tilde{A}^L\right)-1/4$$

$$\left[S_1\left(\tilde{A}^U\right)+S_1\left(\tilde{A}^L\right)+S_2\left(\tilde{A}^U\right)+S_2\left(\tilde{A}^L\right)+S_3\left(\tilde{A}^U\right)+S_3\left(\tilde{A}^L\right)+S_4\left(\tilde{A}^U\right)+S_4\left(\tilde{A}^L\right)\right]$$

$$H_1\left(\tilde{A}^U\right)+H_1\left(\tilde{A}^L\right)+H_2\left(\tilde{A}^U\right)+H_2\left(\tilde{A}^L\right)=0.37+0.37+0.41+0.39+0.45+0.42-1/4[0.03$$

$$+0.01+0.01+0.012247+0.03+0.02+0.05+0.02958]+1+0.95+1+0.95=6.2620$$

In decision making problems where data is expressed in terms of linguistic variables by qualitative terms such as very poor (VP), poor (P), poor to medium (PM), fair (F), medium to good (MG), good, (G), and very good (VG), according to *Table 1*, one can implement corresponding TIT2FNs in the problem [17].

Table 1. Linguistic variables and corresponding TIT2FNs [17].

Linguistic variables	TIT2FN
Very poor (VP)	((0, 0, 0, 0.1; 1, 1), (0, 0, 0, 0.05; 0.95, 0.95))
Poor (P)	((0, 0.01, 0.15, 0.3; 1, 1), (0.05, 0.1, 0.1, 0.2; 0.95, 0.95))
Poor to medium (PM)	((0.15, 0.3, 0.35, 0.5; 1, 1), (0.2, 0.25, 0.3, 0.4; 0.95, 0.95))
Fair (F)	((0.3, 0. 5, 0.55, 0.7; 1, 1), (0.4, 0.45, 0.5, 0.6; 0.95, 0.95))
Medium to good (MG)	((0.5, 0.7, 0.75, 0.9; 1, 1), (0.6, 0.65, 0.7, 0.8; 0.95, 0.95))
Good (G)	((0.7, 0.9, 0.95, 1; 1, 1), (0.8, 0.85, 0.9, 0.95; 0.95, 0.95))
Very good (VG)	((0.9, 1, 1, 1; 1, 1), (0.95,1, 1, 1;0.95,0.95))

3 | MPTIT2FNWA Operator

Integration of the information collected from different sources is a necessary and important process in multi-period problems, representing a highlighted research topic [9]. Previous studies have proposed operators for integrating TriT1FNs [9], [11]-[12], intuitionistic fuzzy data [3] and [5], and interval data [10]. Therefore, this section defines MPTIT2FNWA operator for integrating MPMAGDM information.

Definition 7. Let $\widetilde{A}_t=\left(\widetilde{A}_t^U,\widetilde{A}_t^L\right)$ be a set of variables at p periods ($t=t_1,t_2,\dots,t_p$), with $\eta_t=\left(\eta_{t_1},\eta_{t_2},\dots,\eta_{t_p}\right)^T$ being the function for evaluating its weights. Then, MPTIT2FNWA operator is defined as follows:

$$\mathrm{MPTIT2FNWA}_{\eta_t}\left(\widetilde{A}_{t_1},\widetilde{A}_{t_2},\widetilde{A}_{t_3},\dots,\widetilde{A}_{t_p}\right)=\frac{1}{\sum_{k=1}^{p}\eta_{t_k}}\left(\eta_{t_1}\left(\widetilde{A}_{t_1}\right)\oplus\eta_{t_2}\left(\widetilde{A}_{t_2}\right)\oplus\dots\oplus\eta_{t_p}\left(\widetilde{A}_{t_p}\right)\right). \tag{10}$$

Where,

$$\sum_{k=1}^{p}\eta_{t_k}=1\ ,\ \eta_{t_k}\geq 0 \text{ for } k=1,2,\dots,p. \tag{11}$$

Therefore, using *Eqs. (6), (9), (10)* and *(11)* one can write:

$$\mathrm{MPTIT2FNWA}_{\eta_t}\left(\widetilde{A}_{t_1},\widetilde{A}_{t_2},\widetilde{A}_{t_3},\dots,\widetilde{A}_{t_p}\right)=\left(\sum_{k=1}^{p}\eta_{t_k}\left(\widetilde{A}_{t_k}^U\right),\sum_{k=1}^{p}\eta_{t_k}\left(\widetilde{A}_{t_k}^L\right)\right)=$$

$$\left(\begin{array}{c}\left(\begin{array}{c}\sum_{k=1}^{p}\eta_{t_k}\left(a_{1t_k}^U\right),\sum_{k=1}^{p}\eta_{t_k}\left(a_{2t_k}^U\right),\sum_{k=1}^{p}\eta_{t_k}\left(a_{3t_k}^U\right),\sum_{k=1}^{p}\eta_{t_k}\left(a_{4t_k}^U\right);\\ \min_k\left(H_1\left(\widetilde{A}_{t_k}^U\right)\right),\min_k\left(H_2\left(\widetilde{A}_{t_k}^U\right)\right)\end{array}\right),\\ \left(\begin{array}{c}\sum_{k=1}^{p}\eta_{t_k}\left(a_{1t_k}^L\right),\sum_{k=1}^{p}\eta_{t_k}\left(a_{2t_k}^L\right),\sum_{k=1}^{p}\eta_{t_k}\left(a_{3t_k}^L\right),\sum_{k=1}^{p}\eta_{t_k}\left(a_{4t_k}^L\right);\\ \min_k\left(H_1\left(\widetilde{A}_{t_k}^L\right)\right),\min_k\left(H_2\left(\widetilde{A}_{t_k}^L\right)\right)\end{array}\right)\end{array}\right) \tag{12}$$

According to the *Definition 7*, the following properties can be extracted:

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Properties. Let $\widetilde{A}_{t_p},\ldots,\widetilde{A}_{t_3},\widetilde{A}_{t_2},\widetilde{A}_{t_1}$ be a set of TIT2FNs at p periods ($t=t_1,t_2,\ldots,t_p$), with $\eta_t=\left(\eta_{t_1},\eta_{t_2},\ldots,\eta_{t_p}\right)^T$ such that $\sum_{k=1}^{p}\eta_{t_k}=1$, and we have $\eta_{t_k}\geq 0$ ($k=1,2,\ldots,p$) weight vectors, then:

- *Idempotency: If all $\widetilde{A}_{t_k}$s are equal for $k=1,2,\ldots,p$, so that $\widetilde{A}_{t_k}=\widetilde{A}_t$, then we have:*

$$\mathrm{MPTIT2FNWA}_{\eta_t}\left(\widetilde{A}_{t_1},\widetilde{A}_{t_2},\widetilde{A}_{t_3},\ldots,\widetilde{A}_{t_p}\right)=\widetilde{A}_t.$$

- *Boundedness: we have, $\widetilde{A}_t^-\leq MPTIT2FNWA_{\eta_t}\left(\widetilde{A}_{t_1},\widetilde{A}_{t_2},\widetilde{A}_{t_3},\ldots,\widetilde{A}_{t_p}\right)\leq\widetilde{A}_t^+$, where $\widetilde{A}_t^-=\min_k\left(\widetilde{A}_{t_k}\right)$ and $\widetilde{A}_t^+=\max_k\left(\widetilde{A}_{t_k}\right)$.*
- *Monotonicity: Let $\widetilde{A}_{t_k}^*$ be a set of TIT2FNs at p periods ($t=t_1,t_2,\ldots,t_p$), and $\widetilde{A}_{t_k}^*\leq\widetilde{A}_{t_k}$ for all k values, then we have:*

$$\mathrm{MPTIT2FNWA}_{\eta_t}\left(\widetilde{A}_{t_1},\widetilde{A}_{t_2},\widetilde{A}_{t_3},\ldots,\widetilde{A}_{t_p}\right)\leq\mathrm{MPTIT2FNWA}_{\eta_t}\left(\widetilde{A}_{t_1}^*,\widetilde{A}_{t_2}^*,\widetilde{A}_{t_3}^*,\ldots,\widetilde{A}_{t_p}^*\right).$$

In order to apply MPTIT2FNWA operator, determination of the vector of weights (η_t) is an important step. In general, this vector (η_t) can be determined by various methods such as decision-maker's opinion [7], or on the basis of different types of probability distribution functions such as BUM probability distribution function [3] and [11], Gamma distribution function [33], normal distribution function [1], [3], etc. Relying on the applied example provided in [40], application of BUM probability distribution function for determining the vector of weights of time periods (η_t) is explained in the following [1] and [3]:

BUM function: the function $Q:[0,1]\longrightarrow[0,1]$ of the following properties:

- $Q(0)=0.$
- $Q(1)=1.$
- $Q(x)\leq Q(y)$ *if* $x>y$.

The function Q is referred to as general monotonic unit distance function. Using this function, one can calculate vector of weights (η_t) as follows:

$$\eta_{t_k}=Q\left(\frac{k}{p}\right)-Q\left(\frac{k-1}{p}\right);\ k=1,2,\ldots,p. \tag{13}$$

Now, letting $Q(x)=(x)^r$ and $r>0$ gives:

$$\eta_{t_k}=Q\left(\frac{k}{p}\right)-Q\left(\frac{k-1}{p}\right)=\left(\frac{k}{p}\right)^r-\left(\frac{k-1}{p}\right)^r=\left(\frac{k}{p}\right)^r-\left(\frac{k}{p}-\frac{1}{p}\right)^r;\ k=1,2,\ldots,p. \tag{14}$$

Then

$$\frac{\partial\left(\eta_{t_k}\right)}{\partial\left(\frac{k}{p}\right)}=r\left(\frac{k}{p}\right)^{r-1}-r\left(\frac{k}{p}-\frac{1}{p}\right)^{r-1}=r\left(\left(\frac{k}{p}\right)^{r-1}-\left(\frac{k}{p}-\frac{1}{p}\right)^{r-1}\right). \tag{15}$$

Therefore:

- *If* $r>1$, *then* $\frac{\partial(\eta_{t_k})}{\partial(\frac{k}{p})}>0$, *so* η_{t_k} *is a monotonic increasing function.*
- *If* $r=1$, *then* $\frac{\partial(\eta_{t_k})}{\partial(\frac{k}{p})}=0$, *so* η_{t_k} *is a constant function.*
- *If* $r<1$, *then* $\frac{\partial(\eta_{t_k})}{\partial(\frac{k}{p})}<0$, *so* η_{t_k} *is a monotonic decreasing function.*

4 | MPTIT2FNWA based MPMAGDM Method

Assumed in this section is a MPMAGDM problem wherein values of attributes are expressed in MPTIT2FNWA in multiple periods of time. Consider a decision-making problem with n attributes $C_j;j=1, 2, 3, \ldots, n$, m alternatives $A_i;i=1, 2, 3, \ldots, m$, q decision-makers $DM_l;l=1, 2, 3, \ldots, q$, in p periods $t_k;k=1, 2, 3, \ldots, p$. Suppose that $x_{ijl}^{(t_k)}$ is the corresponding TIT2FN to a linguistic term expressing the value of jth attribute of the ith alternative determined by lth decision-maker in kth period of time, such that:

$$\tilde{x}_{ijq}^{(t_k)}=\left(\tilde{x}_{ijq}^{(t_k)U},\tilde{x}_{ijq}^{(t_k)L}\right)=\begin{pmatrix}\left(x_{ijl1}^{(t_k)U},x_{ijl2}^{(t_k)U},x_{jil3}^{(t_k)U},x_{ijl4}^{(t_k)U};H_1\left(\tilde{x}_{ijl}^{(t_k)U}\right),H_2\left(\tilde{x}_{ijl}^{(t_k)U}\right)\right),\\ \left(x_{ijl1}^{(t_k)L},x_{ijl2}^{(t_k)L},x_{jil3}^{(t_k)L},x_{jil4}^{(t_k)L};H_1\left(\tilde{x}_{ijl}^{(t_k)L}\right),H_2\left(\tilde{x}_{ijl}^{(t_k)L}\right)\right)\end{pmatrix}.$$

Now, with this information, we can implement the proposed method by following steps.

Step 1. Calculation of average decision matrix in each time period.

If $\varphi(t_k)=\left(\varphi_1(t_k),\varphi_2(t_k),\ldots,\varphi_q(t_k)\right)^T$ such that $\sum_{l=1}^{q}\varphi_l(t_k)=1$, and $\varphi_l(t_k)\geq 0$ (for $l=1, 2,\ldots,q$) represents vector of the weights of decision-makers in k th period, then, decision matrix for each time period, t_k, is formed as follows:

$$\tilde{D}^{(t_k)}=\begin{array}{c} \\ A_1\\ A_2\\ \vdots\\ A_i\\ \vdots\\ A_m\end{array}\begin{array}{c}\begin{array}{cccc}\tilde{c}_1 & \tilde{c}_2 & \cdots & \tilde{c}_n\end{array}\\ \begin{bmatrix}\tilde{x}_{11}^{(t_k)} & \tilde{x}_{12}^{(t_k)} & \cdots & \tilde{x}_{1n}^{(t_k)}\\ \tilde{x}_{21}^{(t_k)} & \tilde{x}_{22}^{(t_k)} & \cdots & \tilde{x}_{2n}^{(t_k)}\\ \vdots & \vdots & \cdots & \vdots\\ \tilde{x}_{i1}^{(t_k)} & \tilde{x}_{i2}^{(t_k)} & \cdots & \tilde{x}_{in}^{(t_k)}\\ \vdots & \vdots & \cdots & \vdots\\ \tilde{x}_{m1}^{(t_k)} & \tilde{x}_{m2}^{(t_k)} & & \tilde{x}_{mn}^{(t_k)}\end{bmatrix}\end{array};i=1,2,\ldots,m\quad;j=1,2,\ldots,n\quad;k=1,2,\ldots,p.$$

Where,

$$\tilde{x}_{ij}(t_k)=\oplus_{l=1}^{q}(\varphi_l(t_k)\times\tilde{x}_{ijl}(t_k)). \tag{16}$$

Where $\tilde{x}_{ij}^{(t_k)}$ is weighted average of the opinions of the q decision-makers on the value of the j th attribute of the i th alternative in the k th period of time. Now, letting $W(t_k)=(w_1(t_k),w_2(t_k),\ldots,w_n(t_k))^T$ be the vector of weights of attributes in k th period of time and $\eta_t=\left(\eta_{t_1},\eta_{t_2},\ldots,\eta_{t_p}\right)^T$ be the vector of weights of time periods, the proposed method is presented in terms of the following steps.

Step 2. Calculation of weighted decision matrix in each time period.

Decision matrix $\tilde{R}(t_k)=\left[\tilde{r}_{ij}(t_k)\right]_{m\times n}$ is the weighted decision matrix in k the period of time, such that:

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$$\tilde{r}_{ij}(t_k)=w_j(t_k)\times\tilde{x}_{ij}(t_k);\ i=1,2,\ldots,m\ ;j=1,2,\ldots,n;k=1,2,\ldots,p.$$

Step 3. Integration of decision matrices using MPTIT2FNWA operator and formation of general decision matrix.

In this step, the values of the weighted attributes in p periods of time are integrated using the MPTIT2FNWA operator as follows:

$$\tilde{r}_{ij}=\mathrm{MPTIT2FNWA}_{\eta_t}\left(\tilde{r}_{i1}(t_k),\tilde{r}_{i2}(t_k),\tilde{r}_{i3}(t_k),\ldots,\tilde{r}_{in}(t_k)\right)=\left(\sum_{k=1}^{p}\eta_{t_k}\left(\tilde{r}_{t_k}^{U}\right),\sum_{k=1}^{p}\eta_{t_k}\left(\tilde{r}_{t_k}^{L}\right)\right)=$$

$$\left(\begin{array}{l}\left(\sum_{k=1}^{p}\eta_{t_k}\left(r_{1t_k}^{U}\right),\sum_{k=1}^{p}\eta_{t_k}\left(r_{2t_k}^{U}\right),\sum_{k=1}^{p}\eta_{t_k}\left(r_{3t_k}^{U}\right),\sum_{k=1}^{p}\eta_{t_k}\left(r_{4t_k}^{U}\right);\min_k\left(H_1\left(\tilde{r}_{t_k}^{U}\right)\right),\min_k\left(H_2\left(\tilde{r}_{t_k}^{U}\right)\right)\right),\\ \left(\sum_{k=1}^{p}\eta_{t_k}\left(r_{1t_k}^{L}\right),\sum_{k=1}^{p}\eta_{t_k}\left(r_{2t_k}^{L}\right),\sum_{k=1}^{p}\eta_{t_k}\left(r_{3t_k}^{L}\right),\sum_{k=1}^{p}\eta_{t_k}\left(r_{4t_k}^{L}\right);\min_k\left(H_1\left(\tilde{r}_{t_k}^{L}\right)\right),\min_k\left(H_2\left(\tilde{r}_{t_k}^{L}\right)\right)\right)\end{array}\right)$$

$$;\ i=1,2,\ldots,m\ ;\ j=1,2,\ldots,n.$$

In this case, general decision matrix is obtained as $\tilde{R}=[\tilde{r}_{ij}]_{m\times n}$.

Step 4. Determination of Positive Ideal Solution (PIS), and Negative Ideal Solution (NIS).

Letting $\tilde{r}^{+}=(\tilde{r}_1^{+},\tilde{r}_2^{+},\ldots,\tilde{r}_n^{+})$ and $\tilde{r}^{-}=(\tilde{r}_1^{-},\tilde{r}_2^{-},\ldots,\tilde{r}_n^{-})$ denote PIS and NIS, respectively, we will have:

$$\tilde{r}^{+}=\begin{Bmatrix}\max_i\{\mathrm{Rank}(\tilde{r}_{ij})\}\ ,\ \text{for } j=1,2,\ldots,n;\ j\in X_b\\ \min_i\{\mathrm{Rank}(\tilde{r}_{ij})\}\ ,\ \text{for } j=1,2,\ldots,n;\ j\in X_c\end{Bmatrix}, \tag{17}$$

$$\tilde{r}^{-}=\begin{Bmatrix}\max_i\{\mathrm{Rank}(\tilde{r}_{ij})\}\ ,\ \text{for } j=1,2,\ldots,n;\ j\in X_c\\ \min_i\{\mathrm{Rank}(\tilde{r}_{ij})\}\ ,\ \text{for } i=1,2,\ldots,n;\ j\in X_b\end{Bmatrix}. \tag{18}$$

where X_b is the set of all positive attributes (e.g. profit) and X_c is the set of all negative attributes (e.g. cost).

Step 5. Calculation of distances of alternatives from PIS and NIS.

In order to calculate the distance from the ith alternative to PIS and NIS, one could act as follows:

$$d^{+}(A_i)=\sqrt{\sum_j\left(\mathrm{Rank}(\tilde{r}_{ij})-\tilde{r}^{+}\right)^2};\ i=1,\ 2,\ \ldots,\ m, \tag{19}$$

$$d^{-}(A_i)=\sqrt{\sum_j\left(\mathrm{Rank}(\tilde{r}_{ij})-\tilde{r}^{-}\right)^2};\ i=1,2,\ldots,m. \tag{20}$$

where $d^{+}(A_i)$ denotes the distance from ith alternative to PIS and $d^{-}(A_i)$ is the distance from the i th alternative to NIS.

Step 6. Ranking the alternatives using closeness coefficient of the alternatives.

$$CC(A_i)=\frac{d^-(A_i)}{\left(d^-(A_i)+d^+(A_i)\right)};\ i=1,2,\ldots,m. \tag{21}$$

where $CC(A_i)$ is the closeness coefficient of the ith alternative. In order to rank the alternatives, values of $CC(A_i)$ $(i=1,2,\ldots,m)$ should be sorted in decreasing order. It is obvious that the best alternative will be that with highest closeness coefficient. Accordingly, A^* will be chosen as the best alternative if and only if $CC(A^*)=\underset{i}{max}\{CC(A_i)\}$.

5 | Numerical Example

In this example, the numerical example previously presented by Zhu and Hipel [6] is used, with all data and coefficients being used in the same sense as they were used in the reference. In 2008, Chinese government decided to undertake a research and development plan in commercial aviation industry to enhance economic and production capacities of the industry. One of the most important parts of this plan was vendor evaluation which is based on criteria whose performances were considered over multiple periods of time. On this basis, five vendors (A_1, A_2, A_3, A_4, A_5) of an electronic navigation system are studied in this research. Schedule (C_1), quality (C_2), technology (C_3), and level of service (LOS) (C_4) were considered as the problem attributes; these were evaluated along a three years period by three experts (*Table 2*). In this research, the attributes had their weights expressed in terms of the vector $W(t_k)=(0.45,0.40,0.1,0.05)^T$ such that $W(t_1)=W(t_2)=W(t_3)$. Weights of the time periods were defined as the vector $\eta_t=(1/3,1/3,1/3)^T$ and weights of the decision-makers were considered as $\varphi(t_k)=(0.2,0.40,0.40)^T$.Accordingly, the proposed method was undertaken by taking the following steps.

Step 1. Calculation of average decision matrix in each time period.

By using *Eq. (18)*, weighted average of the decision-makers' opinions are considered as the value of each attribute, as can be seen in *Table 3*.

Table 2. Linguistic values of the evaluated attributes by three decision-makers in three periods of time.

DM$_1$																	
t$_1$						t$_2$						t$_3$					
C_1	A_1	A_2	A_3	A_4	A_5		A_1	A_2	A_3	A_4	A_5		A_1	A_2	A_3	A_4	A_5
C_2	F	P	VP	P	MP	C_1	MG	F	P	MG	MG	C_1	VG	MG	P	G	G
C_3	P	F	P	MP	VP	C_2	F	G	MP	P	MP	C_2	P	G	P	VG	MG
C_4	VP	P	F	VP	MP	C_3	P	P	MG	MG	MP	C_3	VP	G	F	P	G
C_5	VP	VP	MP	F	P	C_4	P	MP	P	G	G	C_4	MG	MP	G	G	P
DM$_2$																	
t$_1$						t$_2$						t$_3$					
	A_1	A_2	A_3	A_4	A_5		A_1	A_2	A_3	A_4	A_5		A_1	A_2	A_3	A_4	A_5
C_1	P	P	P	P	MP	C_1	MP	F	F	MP	P	C_1	MG	F	MP	MG	F
C_2	P	MP	P	MP	P	C_2	P	MG	P	F	MG	C_2	VP	VG	VP	G	MG
C_3	VP	P	F	P	MP	C_3	MP	MP	G	F	MP	C_3	MP	P	MG	P	MP
C_4	MP	VP	MP	F	P	C_4	F	MP	MP	MG	P	C_4	P	MP	G	MG	G
DM$_3$																	
t$_1$						t$_2$						t$_3$					
	A_1	A_2	A_3	A_4	A_5		A_1	A_2	A_3	A_4	A_5		A_1	A_2	A_3	A_4	A_5
C_1	P	P	P	MP	P	C_1	P	F	P	MP	MP	C_1	VP	G	MP	F	MG
C_2	P	MP	P	F	VP	C_2	VP	MG	MP	G	P	C_2	VP	VG	P	MG	MP
C_3	VP	P	MP	P	VP	C_3	MP	MP	MG	MP	P	C_3	G	P	F	MP	P
C_4	P	VP	MP	F	P	C_4	F	MP	MP	MG	VP	C_4	P	MP	MG	G	G

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Table 3. Weighted average of the attributes evaluated by decision-makers in three periods of time.

t₁				
	C_1	**C_2**	**C_3**	**C_4**
A_1	((0.06,0.108,0.23,0.38;1,1),(0.12,0.17,0.18,0.28;0.95,0.95))	((0,0.01,0.15,0.3;1,1),(0.05,0.1,0.1,0.2;0.95,0.95))	((0,0,0,0.1;1,1),(0,0,0,0.05;0.95,0.95))	((0.03,0.066,0.16,0.3;1,1),(0.07,0.11,0.12,0.21;0.95,0.95))
A_2	((0,0.01,0.15,0.3;1,1),(0.05,0.1,0.1,0.2;0.95,0.95))	((0.18,0.34,0.39,0.54;1,1),(0.24,0.29,0.34,0.44;0.95,0.95))	((0,0.01,0.15,0.3;1,1),(0.05,0.1,0.1,0.2;0.95,0.95))	((0,0,0,0.1;1,1),(0,0,0,0.05;0.95,0.95))
A_3	((0,0.008,0.12,0.26;1,1),(0.04,0.08,0.08,0.17;0.95,0.95))	((0,0.01,0.15,0.3;1,1),(0.05,0.1,0.1,0.2;0.95,0.95))	((0.21,0.38,0.43,0.58;1,1),(0.28,0.33,0.38,0.48;0.95,0.95))	((0.15,0.3,0.35,0.5;1,1),(0.2,0.25,0.3,0.4;0.95,0.95))
A_4	((0.09,0.184,0.27,0.42;1,1),(0.14,0.19,0.22,0.32;0.95,0.95))	((0.24,0.42,0.47,0.62;1,1),(0.32,0.37,0.42,0.52;0.95,0.95))	((0,0.008,0.12,0.26;1,1),(0.04,0.08,0.08,0.17;0.95,0.95))	((0.3,0.5,0.55,0.7;1,1),(0.4,0.45,0.5,0.6;0.95,0.95))
A_5	((0.06,0.126,0.23,0.38;1,1),(0.11,0.16,0.18,0.28;0.95,0.95))	((0,0.002,0.03,0.14;1,1),(0.01,0.02,0.02,0.08;0.95,0.95))	((0.06,0.12,0.14,0.26;1,1),(0.08,0.1,0.12,0.19;0.95,0.95))	((0,0.01,0.15,0.3;1,1),(0.05,0.1,0.1,0.2;0.95,0.95))
t₂				
	C_1	**C_2**	**C_3**	**C_4**
A_1	((0.13,0.206,0.31,0.46;1,1),(0.19,0.24,0.26,0.36;0.95,0.95))	((0.06,0.102,0.14,0.26;1,1),(0.09,0.11,0.12,0.19;0.95,0.95))	((0.12,0.242,0.31,0.46;1,1),(0.17,0.22,0.26,0.36;0.95,0.95))	((0.24,0.402,0.47,0.62;1,1),(0.33,0.38,0.42,0.52;0.95,0.95))
A_2	((0.3,0.5,0.55,0.7;1,1),(0.4,0.45,0.5,0.6;0.95,0.95))	((0.54,0.74,0.79,0.92;1,1),(0.67,0.72,0.76,0.84;0.95,0.95))	((0.12,0.242,0.31,0.46;1,1),(0.17,0.22,0.26,0.36;0.95,0.95))	((0.15,0.3,0.35,0.5;1,1),(0.2,0.25,0.3,0.4;0.95,0.95))
A_3	((0.06,0.108,0.23,0.38;1,1),(0.12,0.17,0.18,0.28;0.95,0.95))	((0.12,0.242,0.31,0.46;1,1),(0.17,0.22,0.26,0.36;0.95,0.95))	((0.54,0.74,0.79,0.92;1,1),(0.67,0.72,0.76,0.84;0.95,0.95))	((0.12,0.242,0.31,0.46;1,1),(0.17,0.22,0.26,0.36;0.95,0.95))
A_4	((0.22,0.38,0.43,0.58;1,1),(0.28,0.33,0.38,0.48;0.95,0.95))	((0.48,0.642,0.71,0.8;1,1),(0.66,0.71,0.72,0.76;0.95,0.95))	((0.25,0.42,0.47,0.62;1,1),(0.32,0.37,0.42,0.52;0.95,0.95))	((0.54,0.74,0.79,0.92;1,1),(0.67,0.72,0.76,0.84;0.95,0.95))
A_5	((0.19,0.322,0.39,0.54;1,1),(0.25,0.3,0.34,0.44;0.95,0.95))	((0.13,0.206,0.31,0.46;1,1),(0.19,0.24,0.26,0.36;0.95,0.95))	((0.06,0.126,0.23,0.38;1,1),(0.11,0.16,0.18,0.28;0.95,0.95))	((0.14,0.182,0.22,0.32;1,1),(0.2,0.22,0.22,0.27;0.95,0.95))
t₃				
	C_1	C_2	C_3	C_4
A_1	((0.28,0.34,0.35,0.44;1,1),(0.31,0.33,0.34,0.39;0.95,0.95))	((0,0.002,0.03,0.14;1,1),(0.01,0.02,0.02,0.08;0.95,0.95))	((0.45,0.6,0.64,0.72;1,1),(0.61,0.65,0.66,0.69;0.95,0.95))	((0.1,0.148,0.27,0.42;1,1),(0.16,0.21,0.22,0.32;0.95,0.95))
A_2	((0.58,0.78,0.83,0.92;1,1),(0.77,0.82,0.84,0.88;0.95,0.95))	((0.86,0.98,0.99,1;1,1),(0.95,1,1,1;0.95,0.95))	((0.14,0.188,0.31,0.44;1,1),(0.23,0.28,0.28,0.36;0.95,0.95))	((0.15,0.3,0.35,0.5;1,1),(0.2,0.25,0.3,0.4;0.95,0.95))
A_3	((0.12,0.242,0.31,0.46;1,1),(0.17,0.22,0.26,0.36;0.95,0.95))	((0,0.008,0.12,0.26;1,1),(0.04,0.08,0.08,0.17;0.95,0.95))	((0.34,0.54,0.59,0.74;1,1),(0.44,0.49,0.54,0.64;0.95,0.95))	((0.58,0.78,0.83,0.94;1,1),(0.74,0.79,0.82,0.88;0.95,0.95))
A_4	((0.42,0.62,0.67,0.8;1,1),(0.55,0.6,0.64,0.72;0.95,0.95))	((0.62,0.8,0.84,0.94;1,1),(0.74,0.79,0.82,0.88;0.95,0.95))	((0.09,0.184,0.27,0.42;1,1),(0.14,0.19,0.22,0.32;0.95,0.95))	((0.66,0.86,0.91,0.98;1,1),(0.88,0.93,0.94,0.96;0.95,0.95))
A_5	((0.5,0.7,0.75,0.88;1,1),(0.63,0.68,0.72,0.8;0.95,0.95))	((0.29,0.46,0.51,0.66;1,1),(0.36,0.41,0.46,0.56;0.95,0.95))	((0.17,0.246,0.35,0.48;1,1),(0.26,0.31,0.32,0.4;0.95,0.95))	((0.56,0.722,0.79,0.86;1,1),(0.77,0.82,0.82,0.84;0.95,0.95))

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Table 4. Weighted decision matrix in three periods of time.

t_1				
	C_1	**C_2**	**C_3**	**C_4**
A_1	((0.027,0.049,0.104,0.171;1,1),(0.054,0.077,0.081,0.126;0.95,0.95))	((0,0.004,0.06,0.12;1,1),(0.02,0.04,0.04,0.08;0.95,0.95))	((0,0,0,0.01;1,1),(0,0,0,0.005;0.95,0.95))	((0.002,0.003,0.008,0.015;1,1),(0.004,0.006,0.006,0.011;0.95,0.95))
A_2	((0,0.005,0.068,0.135;1,1),(0.023,0.045,0.045,0.09;0.95,0.95))	((0.072,0.136,0.156,0.216;1,1),(0.096,0.116,0.136,0.176;0.95,0.95))	((0,0.001,0.015,0.03;1,1),(0.005,0.01,0.01,0.02;0.95,0.95))	((0,0,0,0.005;1,1),(0,0,0,0.003;0.95,0.95))
A_3	((0,0.004,0.054,0.117;1,1),(0.018,0.036,0.036,0.077;0.95,0.95))	((0,0.004,0.06,0.12;1,1),(0.02,0.04,0.04,0.08;0.95,0.95))	((0.021,0.038,0.043,0.058;1,1),(0.028,0.033,0.038,0.048;0.95,0.95))	((0.008,0.015,0.018,0.025;1,1),(0.01,0.013,0.015,0.02;0.95,0.95))
A_4	((0.041,0.083,0.122,0.189;1,1),(0.063,0.086,0.099,0.144;0.95,0.95))	((0.096,0.168,0.188,0.248;1,1),(0.128,0.148,0.168,0.208;0.95,0.95))	((0,0.001,0.012,0.026;1,1),(0.004,0.008,0.008,0.017;0.95,0.95))	((0.015,0.025,0.028,0.035;1,1),(0.02,0.023,0.025,0.03;0.95,0.95))
A_5	((0.027,0.057,0.104,0.171;1,1),(0.05,0.072,0.081,0.126;0.95,0.95))	((0,0.001,0.012,0.056;1,1),(0.004,0.008,0.008,0.032;0.95,0.95))	((0.006,0.012,0.014,0.026;1,1),(0.008,0.01,0.012,0.019;0.95,0.95))	((0,0.001,0.008,0.015;1,1),(0.003,0.005,0.005,0.01;0.95,0.95))
t_2				
	C_1	**C_2**	**C_3**	**C_4**
A_1	((0.059,0.093,0.14,0.207;1,1),(0.086,0.108,0.117,0.162;0.95,0.95))	((0.024,0.041,0.056,0.104;1,1),(0.036,0.044,0.048,0.076;0.95,0.95))	((0.012,0.024,0.031,0.046;1,1),(0.017,0.022,0.026,0.036;0.95,0.95))	((0.012,0.02,0.024,0.031;1,1),(0.017,0.019,0.021,0.026;0.95,0.95))
A_2	((0.135,0.225,0.248,0.315;1,1),(0.18,0.203,0.225,0.27;0.95,0.95))	((0.216,0.296,0.316,0.368;1,1),(0.268,0.288,0.304,0.336;0.95,0.95))	((0.012,0.024,0.031,0.046;1,1),(0.017,0.022,0.026,0.036;0.95,0.95))	((0.008,0.015,0.018,0.025;1,1),(0.01,0.013,0.015,0.02;0.95,0.95))
A_3	((0.027,0.049,0.104,0.171;1,1),(0.054,0.077,0.081,0.126;0.95,0.95))	((0.048,0.097,0.124,0.184;1,1),(0.068,0.088,0.104,0.144;0.95,0.95))	((0.054,0.074,0.079,0.092;1,1),(0.067,0.072,0.076,0.084;0.95,0.95))	((0.006,0.012,0.016,0.023;1,1),(0.009,0.011,0.013,0.018;0.95,0.95))
A_4	((0.099,0.171,0.194,0.261;1,1),(0.126,0.149,0.171,0.216;0.95,0.95))	((0.192,0.257,0.284,0.32;1,1),(0.264,0.284,0.288,0.304;0.95,0.95))	((0.025,0.042,0.047,0.062;1,1),(0.032,0.037,0.042,0.052;0.95,0.95))	((0.027,0.037,0.04,0.046;1,1),(0.034,0.036,0.038,0.042;0.95,0.95))
A_5	((0.086,0.145,0.176,0.243;1,1),(0.113,0.135,0.153,0.198;0.95,0.95))	((0.052,0.082,0.124,0.184;1,1),(0.076,0.096,0.104,0.144;0.95,0.95))	((0.006,0.013,0.023,0.038;1,1),(0.011,0.016,0.018,0.028;0.95,0.95))	((0.007,0.009,0.011,0.016;1,1),(0.01,0.011,0.011,0.014;0.95,0.95))
t_3				
	C_1	C_2	C_3	C_4
A_1	((0.126,0.153,0.158,0.198;1,1),(0.14,0.149,0.153,0.176;0.95,0.95))	((0,0.001,0.012,0.056;1,1),(0.004,0.008,0.008,0.032;0.95,0.95))	((0.045,0.06,0.064,0.072;1,1),(0.061,0.065,0.066,0.069;0.95,0.95))	((0.005,0.007,0.014,0.021;1,1),(0.008,0.011,0.011,0.016;0.95,0.95))
A_2	((0.261,0.351,0.374,0.414;1,1),(0.347,0.369,0.378,0.396;0.95,0.95))	((0.344,0.392,0.396,0.4;1,1),(0.38,0.4,0.4,0.4;0.95,0.95))	((0.014,0.019,0.031,0.044;1,1),(0.023,0.028,0.028,0.036;0.95,0.95))	((0.008,0.015,0.018,0.025;1,1),(0.01,0.013,0.015,0.02;0.95,0.95))
A_3	((0.054,0.109,0.14,0.207;1,1),(0.077,0.099,0.117,0.162;0.95,0.95))	((0,0.003,0.048,0.104;1,1),(0.016,0.032,0.032,0.068;0.95,0.95))	((0.034,0.054,0.059,0.074;1,1),(0.044,0.049,0.054,0.064;0.95,0.95))	((0.029,0.039,0.042,0.047;1,1),(0.037,0.04,0.041,0.044;0.95,0.95))
A_4	((0.189,0.279,0.302,0.36;1,1),(0.248,0.27,0.288,0.324;0.95,0.95))	((0.248,0.32,0.336,0.376;1,1),(0.296,0.316,0.328,0.352;0.95,0.95))	((0.009,0.018,0.027,0.042;1,1),(0.014,0.019,0.022,0.032;0.95,0.95))	((0.066,0.043,0.046,0.049;1,1),(0.044,0.047,0.047,0.048;0.95,0.95))
A_5	((0.225,0.315,0.338,0.396;1,1),(0.284,0.306,0.324,0.36;0.95,0.95))	((0.116,0.184,0.204,0.264;1,1),(0.144,0.164,0.184,0.224;0.95,0.95))	((0.017,0.025,0.035,0.048;1,1),(0.026,0.031,0.032,0.04;0.95,0.95))	((0.028,0.036,0.04,0.043;1,1),(0.039,0.041,0.041,0.042;0.95,0.95))

Table 5. Integrated decision matrix.

	C_1	Rank	C_2	Rank	C_3	Rank	C_4	Rank
A_1	((0.071,0.098,0.134,0.192;1,1),(0.093,0.111,0.117,0.155;0.95,0.95))	4.575	((0.008,0.015,0.043,0.093;1,1),(0.02,0.031,0.032,0.063;0.95,0.95))	4.084	((0.019,0.028,0.032,0.043;1,1),(0.026,0.029,0.031,0.037;0.95,0.95))	4.074	((0.006,0.01,0.015,0.022;1,1),(0.009,0.012,0.013,0.018;0.95,0.95))	3.972
A_2	((0.132,0.194,0.23,0.288;1,1),(0.183,0.206,0.216,0.252;0.95,0.95))	5.123	((0.211,0.275,0.289,0.328;1,1),(0.248,0.268,0.28,0.304;0.95,0.95))	5.52	((0.009,0.015,0.026,0.04;1,1),(0.015,0.02,0.021,0.031;0.95,0.95))	4.019	((0.005,0.01,0.012,0.018;1,1),(0.007,0.008,0.01,0.014;0.95,0.95))	3.958
A_3	((0.027,0.054,0.099,0.165;1,1),(0.05,0.071,0.078,0.122;0.95,0.95))	4.337	((0.016,0.035,0.077,0.136;1,1),(0.035,0.053,0.059,0.097;0.95,0.95))	4.226	((0.036,0.055,0.06,0.075;1,1),(0.046,0.051,0.056,0.065;0.95,0.95))	4.222	((0.014,0.022,0.025,0.032;1,1),(0.019,0.021,0.023,0.027;0.95,0.95))	4.031
A_4	((0.11,0.178,0.206,0.27;1,1),(0.146,0.168,0.186,0.228;0.95,0.95))	4.961	((0.179,0.248,0.269,0.315;1,1),(0.229,0.249,0.261,0.288;0.95,0.95))	5.391	((0.011,0.02,0.029,0.043;1,1),(0.017,0.021,0.024,0.034;0.95,0.95))	4.036	((0.036,0.035,0.038,0.043;1,1),(0.033,0.035,0.037,0.04;0.95,0.95))	4.116
A_5	((0.113,0.172,0.206,0.27;1,1),(0.149,0.171,0.186,0.228;0.95,0.95))	4.963	((0.056,0.089,0.113,0.168;1,1),(0.075,0.089,0.099,0.133;0.95,0.95))	4.469	((0.01,0.016,0.024,0.037;1,1),(0.015,0.019,0.021,0.029;0.95,0.95))	4.016	((0.012,0.015,0.019,0.025;1,1),(0.017,0.019,0.019,0.022;0.95,0.95))	4.006

Step 2. Calculation of weighted decision matrix in each period.

In this step, by using *Eq. (6)*, weighted decision matrix in each period was calculated, which results came in *Table 4*.

Step 3. Integration of decision matrices using MPTIT2FNWA operator and formation of general decision matrix.

In this step, using MPTIT2FNWA operator, the values of the weighted attributes in three periods of time are integrated, then, their rank were calculated, which came in *Table 5*.

Step 4. Determination of PIS and NIS.

Using *Eqs. (19)* and *(20)*, PIS and NIS vectors were calculated as follows:

$$\begin{aligned} &\tilde{r}^{+} = (5.1230,\ 5.5200,\ 4.2220,\ 4.1160), \\ &\tilde{r}^{-} = (4.3370,\ 4.0840,\ 4.0160,\ 3.9580). \end{aligned} \tag{21}$$

Step 5. Calculation of distances of alternatives from PIS ($d^{+}(A_i)$) and NIS ($d^{-}(A_i)$).

Eqs. (21) and *(22)* were used to calculate distances of alternatives from PIS and NIS, with the results given in *Table 6*.

Table 6. Distances of alternatives from PIS and NIS.

Distance of the alternative from PIS		Distance of the alternative from NIS	
$d^+(A_1)$	1.550819	$d^-(A_1)$	0.245365
$d^+(A_2)$	0.257241	$d^-(A_2)$	1.63704
$d+(A_3)$	1.516396	$d^-(A_3)$	0.260632
$d^+(A_4)$	0.278354	$d^-(A_4)$	1.457048
$d^+(A_5)$	1.088456	$d^-(A_5)$	0.736482

Step 6. Calculation of relative closeness coefficient and ranking.

Eq. (23) was used to calculate relative closeness of different alternatives, based on which the alternatives were ranked, with the results reported in *Table 7*.

Table 7. Relative closeness coefficient and ranking of different alternatives.

Relative closeness coefficient		Rank
$CC(A_1)$	0.136603	5
$CC(A_2)$	0.864201	1
$CC(A_3)$	0.146667	4
$CC(A_4)$	0.839603	2
$CC(A_5)$	0.596435	3

Considering the results of implementing the proposed method, it is clear that the alternative A_2, i.e. the fourth vendor, should be selected as the best vendor, with the alternatives ranked as follows: $A_2 \prec A_4 \prec A_5 \prec A_3 \prec A_1$. *Table 8* presents a comparison between the results of the proposed method and those of the study by Xu and Hipel [6].

Table 8. Comparison with other methods.

Research work	Ranking of the alternatives
Method of Xu and Hipel	A2≻ A4≻ A5≻ A3≻ A1
Proposed method	A2≻ A4≻ A5≻ A3≻ A1

6 | Conclusion

In this paper, A MPMADM method was presented where in initial data was expressed in linguistic terms. In order to cover with the ambiguity and uncertainty of this information, TIT2FNs were used. In order to complete the proposed process, MPTIT2FNWA operator was defined on TIT2FNs. We utilized the BUM function in this operator and its properties were investigated. This operator was used to integrate information from different periods of time and form a general decision matrix. This operator was found to provide required flexibly to select any trend of time series for the specified weights of time periods depending on the problem characteristics. On this basis, the proposed method was presented in several steps. In order to demonstrate efficiency and applicability of the proposed method, the numerical example presented by Zhu and Hipel [6] was used and the problem of vendors of an electronic navigation system for developing commercial aviation industry was evaluated. The results showed that the ranking is the same in both methods, which confirms the efficiency of the proposed method.

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Greedy Algorithm for Solving Student Allocation Problem in Internship Program: A Case Study

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Abstract

This paper presented greedy algorithm for solving student allocation problem that has arisen in internship program. In internship program, engineering students stay one semester in industries which are located across the country and teachers visit students once/twice for supervision during the program. As the industries scatter across the country, teachers spend long time on travel. And this results in wastage of teachers working time and money spent for transport. Therefore, allocating students to universities near the internship location extensively reduces the transport time and money spent for transport. For the current study, we consider 4th mechanical engineering students who are currently working in the industry. The proposed approach extensively decrease the distance traveled from 23,210 km to 2,488.8 km and the time spent on the road from 397 hrs. 40 min to 51 hrs. 30 min. and finally, the results obtained from the greedy algorithm is compared with other heuristics (i.e., Genetic algorithm and Particle swarm optimization) and the greedy algorithm outperforms the other methods.

Keywords: Internship program, Students, allocation, Greedy algorithm.

1 | Introduction

Internship is program one full semester course where students spend the whole semester working in the industry. This provides the students the opportunity to link the concepts learned in classroom with the industrial practice. In this program, the students search for industries where they are going stayed for the semester based on their preference and mostly students prefer to be near to their family and the students who are unable to get acceptance from industries due to the limited capacity of the industries will be assigned by the industry linkage of the university. And, teacher visit the students once/twice in the semester for supervision in their hosting company. However, as the students scatter to different places across the country, the teachers spent a lot of time in transport. In addition to cost and time, the teaching and learning program of the university will be affected as many teachers will go for supervision. This paper investigates the benefits gained by reallocating the students to different public governmental universities which are located in near the student's internship location.

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The rest of the paper is organized as follows. The related works on assignment problem and greedy algorithm described in Section 2. The problem formulation and the proposed solution for solving the current problem are provided in Section 3. The results presents in Section 4. Finally, section 5 concludes the paper.

2 | Literature Review

2.1 | Assignment Problem

Problems related to assignment arise in a range of fields, for example, healthcare, transportation, education, and sports. In fact, this is a well-studied topic in combinatorial optimization problems under optimization or operations research branches. Besides, problem regarding assignment is an important subject that has been employed to solve many problems worldwide [1]. Within the education domain the assignment problem can be classified into two groups, time tabling [2] and allocating problem. The problem presented in this paper is allocation problem. The approaches for solving allocation problem are divided as exact method, heuristic, metaheuristic, and hybrid techniques. According to [3], exact methods assure to provide optimal solution, while heuristic methods simply try to produce a good but not certainly optimal solution. Mostly heuristic and metaheuristic are used when the problems are too large for exact methods.

Both methods are discussed in the literature. Anwar and Bahaj [4], used integer programming model for solving student project assignment problem. Pan et al. [5] used goal programming model to maximize the number of assigned projects. The student's preference was successfully increased by giving higher weightage to the higher preference. Further, Calvo-Serrano et al. [6] proposed mix-integerprogramming model by incorporating ranking of lecturers and research areas in the allocation process. This results in increased student satisfaction and decreased computation time for a large dataset.

On the other hand, Harper et al. developed population-based metaheuristic approach by applying GA and compared it with an optimal integer programming approach [7]. Ramli and Bakar [8] discussed how 0-1 IP model Analytical hierarchy process can be used to assign projects to students. The model dismissed student's preference for team members and as a result, the total constraints and variables increased significantly. Zukhri and Omar [9] explored new student's allocation problem by using genetic algorithm to allocate new students to classes to satisfy both students and room's requirements. Furthermore, Kenekayoro et al. [10] uses population-based techniques such as gravitational search algorithm, genetic algorithm, and ant colony optimization to solve student project allocation problem and concludes that the ant colony optimization algorithm outperformed the genetic and gravitational search algorithm for finding optimal solution to student project allocation problem. Manlove et al. [11] presented algorithmic and experimental results for finding maximum size stable matchings in instances of student project allocation problem. From algorithmic perspective, they proved that MAX-SPA-P becomes polynomial-time solvable if there is only one lecturer, whilst the problem remains NP-hard to approximate even if there are two lecturers involved. They also proved that it is NP-hard to find a maximum size stable matching if each preference list is of length at most 3.

2.2 | Greedy Algorithm

Greedy algorithm is an algorithmic paradigm that follows the problem solving approach of making the locally optimal choice at each stage with the hope of finding the global optimum. In many problems, a greedy strategy does not in general produce an optimal solution, but nonetheless a greedy heuristic may yield locally optimal solutions that approximate a global optimal solution in a reasonable time. Greedy algorithm is often used to solve the problems of some decisions, such as change money problem (when a shop clerk wants to change money for customers, he always tends to pay the largest domination coins firstly), knapsack problem (put several items with different weights and values into a knapsack whose loading capacity is limited, and maximize the total value of those items put into the knapsack [12].

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Extensive research has been discovered in the literature on greedy algorithm. Haket et al. [13] developed four variations of construction algorithms such as the sequential, the random, the saving regret, and the randomized adaptive greedy adding algorithm for efficiently allocating customers to different facilities near customer's location in Van Dorps service providing industry in the Netherland. They also develop three version of improvement heuristic such as the "first" improvement, the random "best" improvement, and the random "one-opt" local search algorithm and compared the results with CPLEX solution. And report the random greedy adding heuristic outperform other versions of construction algorithms for most of the reasonable capacity relaxation levels and recommend for problems with Van Dorp's characteristic. Qu et al. [14] propose distributed greedy algorithm for solving multi agent task assignment problem where each agent selects a task from its admissible task set. The objective of the study is to find an assignment profile that maximizes the global utility. In the proposed algorithm, when the communication links between agents are consistent with the admissible task sets, each agent can make its own decision in a distributed and asynchronous fashion. And the efficiency ratio of the algorithm is lower bounded by $1/(1+k)$, where $k\in[0,1]$ is a problem dependent curvature parameter. Ribas et al. [15] proposed an Iterated Greedy Algorithm (IGA) for solving both parallel blocking flow shop problem and distributed blocking flow shop problem by minimizing the total tiredness of jobs. Ying et al. [16] developed Iterated Reference Greedy (IRG) algorithm for solving Distributed no-Idle Permutation Flowshop Scheduling Problem (DNIPFSP). The aim of their study was to simultaneously assign jobs to various factories and to determine their production sequence in each factory to minimize the makespan. The performance of the proposed IRG algorithm is compared with a state-of-the-art Iterated Greedy (IG) algorithm, as well as the Mixed Integer Linear Programming (MILP) and the result show that the proposed IRG outperforms the IG algorithm. A new hybrid Deferential Evolution and Greedy Algorithm (DEGA) has been proposed and applied for solving multi-skill resource-constrained project scheduling problem [17]. In their study design of experiments method has been used to adjust parameters for investigated method to reduce the procedure of experiments. Various initializations, clone elimination, mutation and crossover operators have been applied. And finally, the result of the proposed algorithm has been compared with the result of other reference methods (HantCO, GRASP and multiStart Greedy) using the benchmark iMOPSE dataset.

2.3 | Motivation

Generally, service providers who must visit their customer are increasingly interested in reducing their employees' time and distance spent on the road [18]. In Ethiopian Universities, mostly engineering students has internship program which is one semester course where students spend the whole semester in the industry. And teachers visit the students once/twice in the semester for supervision. However, as students' scatter across the country the teachers who has to supervise students spend a long time on travel. Therefore, Ethiopian Universities can save their employees working time and the distance traveled by simply allocating students to other universities near the students' internship location for supervision. Customer allocation problem has received a plenty of attention in the field of facility location research [19], [20] and [21]. The general class of facility location problems concerns the location of facilities and the allocation of customers to those facilities, and if the facilities locations are fixed, it becomes an assignment problem. Thus, rather than focusing on where to locate facilities, we focus on which customer should be assigned to which facility [13].

The contribution of this paper is threefold. (1) to investigate the gains (i.e, time, cost, distance) achieved by allocating students in internship program into universities near student's location. (2) to develop a greedy algorithm for solving the current problem and compare the results with other heuristic methods.

2.4 | Merits and Limitations of the Paper

The main purpose of this research is to allocate university students who are doing their internship program in different industries across the country into the nearest public university for supervision. And this reduces the teachers valuable working time and distance spent on the road. However, the applicability of this

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research is depending on the willingness of the public universities to collaborate in the proposed solution and the similarity of the courses offered by the universities.

3 | Methodology

3.1 | Problem Formulation

The case study is conducted in Woldia University, Mechanical Engineering Department. One hundred twenty seven 4th year mechanical engineering department students are working in industries located in 23 different towns across the country. And, 18 teachers are assigned by the department to go to the industries where the students are located for supervision. And 13 public governmental universities with similar program are considered in this study, so that the students from these universities can be supervised by other university teachers nears the student's location. Therefore, only for Woldia University Mechanical Engineering Department we have 13x127 matrixes. The distance matrix of the current problem is shown in *Table 1*.

Table 1. Distance matrix.

Universities / Internship Location (no of students)	Shashemene (1)	Addis Abeba (19)	Deredawa (1)	Kombolcha (11)	Dilla (1)
Addis Abeba University	258.5		559.4	379.6	362.6
AASTU	236.7		559.4	384.1	345.1
ASTU	201.3	93.9	366.9	457.5	308.2
Dilla University	108.8	362.6	242.2	910.3	
Bahirdar University	741.4	492.5	938.7	498.2	847.2
Gonder University	907.6	658.8	1105.9	538.5	1014.2
Debretabor University	842.8	594	956.7	389.4	951.2
Debrebirhan University	369.5	130.4	566.9	247.5	477.9
Debremarkos University	554.2	305.4	751.6	483.6	662.6
Wollo University	616.7	398.2	676.2		937
Mizan Tepi University	539.7	568.4	1009.1	966.4	577.5
Woldia University	808.8	705.3	705.6	140	962.6
Hawasa University	24.7	279	584.1	641.1	85.2
Deredawa University	559.4	452.4		655.6	241.6
Arbaminch University	252.4	434.3	811.5	816.7	284.1

Table 1. (Continued).

Bishoftu (6)	Arbaminch (1)	Hawasa (3)	Woldia (1)	Mizan (1)	Dessie (2)	Debremarkos (14)	Debrebirhan (3)	Debretabor (2)	Gonder (8)	Bahirdar (43)
66.2	434.3	279	705.3		398.2	305.4	130.4	594	658.8	492.5
44.4	488.9	260.4	668.5		404.7	321.2	137.1	608.8	673.6	507.5
47.1	453.5	224.9	616.3		476.7	395.2	209.1	681.3	747.6	581.5
314.8	284.1	85.2	960.3		937	662.6	478.2	951.2	1014.2	849.1
546.6	910.3	764.2	359.9		468.2	254	516.3	108.6	171.6	
713.5	1076.4	930.4	400.2		521.1	424.4	681.4	147.5		171.6
654.1	1011.6	865.5	251.1		420	359.5	616.6		153	108.6
175.2	563.7	393.3	386.8		274.5	331.5		616.6	681.4	516.3
363.6	724	578	609.9		475.1		331.5	359.5	424.4	254
451.5	831.3	660.9	119.5			475.1	274.5	420	521.1	468.2
635.1	534.4	539.3	1258.6		987	748.1	719.4	893.7	958.5	788.7
656.4	1054.8	523.7			119.5	609.9	386.8	251.1	398.3	357.6
229.9	245.8		523.7	521.6	660.9	578	393.3	865.5	930.4	764.2
404	811.5	584.1	705.6	1007.1	676.2	751.6	566.9	956.7	1105.5	938.7
451.3		245.8	1054.8	511.5	831.3	724	563.7	1011.6	1076.4	910.3

Table 1. (Continued).

Gaint (1)	Dangla (1)	Burie (1)	Welega (1)	Sekota (1)	Debre Elias (3)	Dejen (2)
643.8	485	411.6	315.2	873.7	346.6	240.7
658.9	500.1	426.8	329.8	851.9	361.7	255.9
732.9	572.6	500.7	401.7	777.1	434.2	329.8
1000.6	841.7	768.4	669.4	1086	703.3	597.5
155.1	80.1	152.2	407.5	434.7	249.7	258.7
197.6	244.9	317	572.2	477.3	414.4	425.6
47.5	185.6	257.7	512.9	327.1	355.1	366.2
616.4	507.8	434.5	436.3	582.9	369.4	263.6
409.6	180.4	107.1	364	689.3	42	75.7
348.6	553.6	580.6	712.6	315.1	515.5	425.4
943.8	714.6	642.9	395.1	1223.4	740.3	821.4
225.9	430.8	503	758.2	195.9	600.4	611.5
915.6	756.8	683.5	584.4	1001.1	618.4	512.6
933	930.9	857.6	758.8	873.7	792.5	686.6
1061.7	902.9	829.5	637.2	1223.2	764.5	658.6

Assumptions.

The following assumptions are made in formulating the problem.

- *The students who are working in industry can be supervised by other university teachers near their work location.*
- *The capacity of the universities to supervised students are known.*
- *The total number of students to be supervised are less than or equal to the university's capacity.*
- *One teacher can supervise a maximum of 7 students.*

Indices.

I: set of students in industries, $i \in I$.

J: set of facilities (universities), $j \in J$.

Decision Variables.

X_{ij}: the number of students i assigned to university j.

C_{ij}: the associated unit cost of assigning student I to university j.

di: capacity of university *j*.

Mathematical Model.

$$\text{Min} \sum_{i\in I} \sum_{j\in J} \frac{1}{7} c_{ij} X_{ij}. \quad (1)$$

Subjected to:

$$\sum_{j\in J} X_{ij} = 1, \qquad \forall i\in I, \quad (2)$$

$$X_{ij} \in \{0,1\}, \qquad \forall i\in I, \forall j\in j, \quad (3)$$

$$\sum_{i\in I} X_{ij} \leq d_j, \qquad \forall j\in J. \quad (4)$$

Where let *I* be the students in different industries $I=\{1\ldots.,i\ldots.,\ldots.,121\}$, *J* the universities in different location that can supervise the students with capacity d_j, $\frac{C_{ij}}{7}$ represent cost or distance between the location of the industry that host the students and the university supervised students (the constant 1/7 is included in cost coefficient to consider the fact that 1 teacher could supervise a maximum of 7 students).

Constraints.

Eq. (1) as an objective function aims to minimize the travel distance of teachers from universities to industries where students are located.

Eq. (2) ensures that each student is assigned to exactly one university.

Eq. (3) simply ensures the integrity of the solution on the decision variables.

Eq. (4) maintains the number of students in industry assigned to the university for supervision are must be less than or equals to the capacity of the university.

3.2 | Proposed Solution

The present work proposed greedy algorithm for solving student assignment problem in internship program. For the current problem we will propose random greedy adding heuristic as it matches with the Van Dorp's case [13], except that in our case reallocation cost is not considered as the facilities or universities serve different students each year. The main steps of the proposed algorithm are described as follow.

Input. The distance matrix (the distance between the student's location and the university) and the capacity of the universities d_j is given.

Step 0. Initialize universities, capacity, cost, and seed.

Step 1. Calculate weighted cost.

Step 2. Generate quasi random distribution of student's *c*.

Step 3. Allocate student c to university *j* from lowest to highest cost of allocation until its full capacity.

Step 4. Update solution and its associated cost.

Step 5. Move to next university.

Output. Assigned students to specific universities and the cost of the assignment.

Table 1. Random greedy algorithm.

1. $S \leftarrow \emptyset$;
2. $f(S) \leftarrow 0$;
3. $F \leftarrow \{i \in E : S \cup \{i\}$ is not infeasible$\}$
4. generate quasi random distribution of students c
5. for c=1 to i do
6. find students with value c
7. for j=1 to k
8. $i^* \leftarrow \text{argmin}\{c_i : i \in F\}$
9. if $i \le d_j$ then
10. allocate student c to university j
11. else $j \leftarrow j+1$ //next university
12. $S \leftarrow S \cup \{i^*\}$;
13. $f(S) \leftarrow f(S) + c_{i^*}$;
14. $F \leftarrow \{i \in F \setminus \{i^*\} : S \cup \{i\}$ is not infeasible$\}$;
13. next c
14. Return S, f(S)

End.

4 | Results

The optimal solution for greedy algorithm is obtained by using matlab programming. The optimal assignment of students to universities is shown in *Table 1*. The comparison of the cost of the greedy algorithm with the optimal solution is presented in *Table 2*.

Table 2. Optimal assignment.

No	Universities	Students Assigned (internship location)
1	Addis Abeba University	10 (Addis Abeba),
2	AASTU	9 (Addis Abeba)
3	ASTU	6 (Bishoftu)
4	Dilla University	1 (Dilla)
5	Bahirdar University	15 (Bahirdar)
6	Gonder University	7 (Bahirdar), (8 Gonder)
7	Debretabor University	8 (Bahirdar), (2 Debretabor), 1 (Dangla), 1 (Gaint), 3 (Debreelias), 1 (Burie), 2 (Dejen)
8	Debrebirhan University	3 (Debrebirhan), 4 (Debremarkos)
9	Debremarkos University	10 (Debremarkos),
10	Wollo University	11 (Kombolcha), 2 (Dessie)
11	Mizan Tepi University	2 (Mizan)
12	Woldia University	1 (Woldia), 13 (Bahirdar), 1 (Sekota)
13	Hawasa University	3 (Hawasa), 1 (Shashemene)
14	Deredawa University	1 (Deredawa)
15	Arbaminch University	1 (Arbaminch)

Table 3. Total cost of the assignment.

Greedy Algorithm	Particle Swarm Optimization	Genetic Algorithm	Deviation From The Optimal
2,488.8	3111.2	3235.6	373

5 | Conclusion

In this paper, we have presented greedy algorithm for solving student allocation problem that has arisen in internship program. In internship program, students stay one semester in industries across the country and teachers visit students once/twice for supervision during the program. As industries scatter across the country, teachers spend long time on travel. Therefore, allocating students to universities near the internship location extensively reduces the transport time and money spent for transport. For the current study, we consider 4th mechanical engineering students who are currently working in the industry. The proposed approach extensively decreases the distance traveled from 23,210 km to 2,488.8 km and the time spent on the road from 397 hrs. 40 min to 51 hrs. 30 min. and finally, the results obtained from the greedy algorithm is compared with other evolutionary algorithms (i.e., Genetic algorithm and Particle swarm optimization) and the greedy algorithm outperforms the other methods.

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Using a New Algorithm to Improve the Search Answer in Quadratic Assignment Problem (QAP)

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Abstract

Layout design problem is one of the useful field of study used to increase the efficiency of sources in organizations. In order to achieve an appropriate layout design, it is necessary to define and solve the related nonlinear programming problems. Therefore, using computer in solving the related problems is important in the view of the researchers of this area of study. However, the designs produced by a computer to solve big problems require more time, so, this paper suggests an algorithm that can be useful in better performance of the known algorithms such as Branch and Bound. The proposed study aims to improve the performance of the Branch and Bound (BB) algorithm in solving Quadratic Assignment Problem (QAP) problems. The findings show that the proposed method enables the BB algorithm to produce an optimal solution in the minimum amount of time.

Keywords: Computer algorithms, Exact methods, Branch and bound, Feasible answer.

1 | Introduction

Facility layout design is a position layout of the equipment of good production or service offering. Koopmans and Beckmann [1] were the pioneers defining facility layout design problem as a common industrial problem which aims at configuring facility so that the cost of the transportable materials will be minimized. The facility layout or the Quadratic Assignment Problem (QAP) is a spatial layout of goods production or service provision facilities.

The design of the layout is an optimization problem that tries to make deployment more efficient, taking into account the various interactions between the facilities and materials transportation system [2].

Azadivar and Wang [2] defined layout design problem as a problem determining relative displacement and allocating space to the existed facilities. It is often hypothesized that material

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flow among departments is fixed and the designed layout will be applicable for a long time. But due to competitive atmosphere of the market and change in customers' taste, dynamism is considered as an inevitable element in industry today, that as a result of the production companies, they have to be able to answer it [3]. Due to this point, it can be inferred that facility layout for short time needs can inconsiderably increase the costs resulted from primary facility for a long time. Thus, it seems that considering a dynamic factor is necessary and important [4].

2 | Quadratic Assignment Problem (QAP)

Koopmans and Beckmann [1] defined and formulated QAP to be used in economic activities. Because of its quadratic nature, this problem is known as Quadratic Assignment Problem, which has attracted researchers' attention working in several areas of study. Lots of researchers and scientists used it in areas of mathematics, computer, operations research and economics to model optimization problems. Assignment means that each facility should be conformed into one position and vice versa. In QAP, the number of facilities has to be equal to the number of positions. Mathematic form of this problem is as follows [5]:

$$\text{Min } C=\sum_{i=1}^{n}\sum_{k=1}^{n}\sum_{j=1}^{n}\sum_{s=1}^{n} d_{i,k}w_{j,s}(x_{i,j}\ x_{k,s}),$$

Subject to:

$$\sum_{j=1}^{n} x_{i,j}=1\ ;\ i=1,2,\dots,n, \tag{1}$$

$$\sum_{i=1}^{n} x_{i,j}=1\ ;\ j=1,2,\dots,n,$$

$$x_{i,j}=0 \text{ or } 1\ ;\ i=1,2,\dots,n \text{ and } j=1,2,\dots,n.$$

$d_{i,k}$ refers to the distance between ith and kth cells.

$w_{j,s}$ refers to the transportation flow between the $j$the and sth machines.

$x_{i,j}$ determines whether jth machine is in the ith cell or not.

$x_{k,s}$ determines whether sth machine is in the kth cell or not.

In the event that facility of j is located in cell of i and facility of s is located in cell of k ($x_{i,j}=1$ *and* $x_{k,s}=1$), by calculating the previous condition, the cost of displacement in this route is $d_{i,k}*x_{i,j}*x_{k,s}=d_{i,k}$. In the event that importance of this route is considered, average displacement cost in this route is $d_{i,k}*w_{j,s}*x_{i,j}*x_{k,s}=d_{i,k}w_{j,s}$.

3 | Statement of Problem

QAP is one of the most complex optimization problems of nonlinear integer [6]. In general, (in wide dimension problems) QAP does not include an exact solution because it is located in the group of NP-Hard problems and to solve it, Meta-heuristic algorithm and Invasive weed optimization are often used.

To solve QAP, some exact algorithms such as Dynamic programming, cut page method, Branch and Bound method can be used [7] and [8]. Branch and Bound method proves better function than the previous two methods does [9]. One of the problems of the mentioned three methods is their incapability in solving wide dimension problems. In other words, using the mentioned algorithms are not possible for the problems with size more than 15 [10].

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In real world, all of facilities cannot be settled in all Locations. Thus, by making search space small, we can reach an optimum answer faster in problems having wide dimensions and such limitations.

4 | Literature Review

Stützle [11] offered a new method called Iterated Local Search (ILS) to solve QAP. ILS is a simple random search method. First, some random points are created in search space, then based on the competence of the mentioned points, searching around them is started. One of the biggest challenge in Stützle's method is the radius in local search.

Hicks [12] in a paper developed Genetic algorithm to be used in facility layout in a set of productive cells. The results showed that the approach of redesigning facilities determines intracellular layout, then it localizes the cells among empty departments.

Mak et al. [13] in a paper used Genetic algorithm as a general method to solve layout design problems. They developed a mathematical model to study layout of the devices and material flow pattern for workshop and product manufacturing environment. The suggested Genetic algorithm with the aim of minimizing material displacement cost, extracts an optimum machinery layout.

Pichka et al. [14] solved the Vehicle Routing Problem (VRP) and suggested the use of the Simulated Annealing Algorithm (SAA) to find the optimal routes between the customers and warehouses.

Moradi and Shadrokh [15] investigated the Site Layout Planning (SLP) with equal and unequal surface areas. The SA algorithm was used to find the optimal layout. Comparison of the SA results with those of other algorithms showed the superiority of the SA in finding optimal solutions with high speed in a shorter time.

Jafari et al. [16] investigate the facility layout problem in an industrial workshop. Their problem-solving recommendation was to use a Developed Simulated Annealing Algorithm (DSAA). This new algorithm is an iterative form of the Basic Simulated Annealing Algorithm (BSAA). The results indicate the ability of the proposed algorithm to find better solutions.

Shadkam and Ghavidel [17] investigate the balancing assembly lines problem. The purpose of this paper is to present a multi-objective integer linear mathematical programming model for balancing assembly lines, which is solved using the general criteria method. The three objective functions considered in this model are: (1) Minimizing cycle time (2) Minimize the idle time of each station and (3) increase the efficiency of the assembly line. In order to investigate the model, Iran-Shargh Neishabour Company has been considered as a case study. After implementing the proposed model of the paper, the results show the optimal performance of the proposed model and the studied parameters in line balancing have been significantly improved.

Kane et al. [18] investigate the transportation problem. The aim of this paper is to introduce a formulation of TP involving Triangular fuzzy numbers for the transportation costs and values of supplies and demands. They propose a two-step method for solving fuzzy transportation problem where all of the parameters are represented by non-negative triangular fuzzy numbers i.e., an Interval Transportation Problems and a Classical Transport Problem. To illustrate the proposed approach two application examples are solved. The results show that the proposed method is simpler and computationally more efficient than existing methods in the literature.

Zanjani et al. [19] investigate the Hybrid Flow Shop (HFS) scheduling problem. This study develops a multi-objective Robust Mixed-Integer Linear Programming (RMILP) model to accommodate the problem with the real-world conditions in which due date and processing time are assumed uncertain. The developed model is able to assign a set of jobs to available machines in order to obtain the best trade-off between two objectives including total tardiness and makespan under uncertain parameters. Fuzzy Goal Programming (FGP) is applied to solve this multi objective problem. Finally, to study and validate the efficiency of the developed RMILP model, some instances of different size are generated and solved using CPLEX solver of GAMS software under different uncertainty levels. Experimental results show that the developed model can find a solution to show the least modifications against uncertainty in processing time and due date in an HFS problem.

5 | Branch and Bound Algorithm

Branch and Bound is a public algorithm used to find the optimum solutions of different problems, especially in discrete optimization and combinational optimization. This algorithm counts all the solutions of a problem, meanwhile, there are lots of useless solutions that, by deleting them through estimating upper and lower boundaries, can be optimized. This method was first introduced for discrete programming by Land and Doig [20]. In this algorithm all the states preparing the probability of reaching better answers, will be studied and finally, the best answer will be chosen out of all the studied answers.

6 | Introducing Feasible Search Algorithm

The new algorithm is explained in the following order:

1. Start.
2. Put $K=1$.
3. Put $n=N$.
4. Put $MaxCost= +\infty$.
5. Put $NE=0$.
6. Put $i=1$.
7. Put $j=1$.
8. Choose a possible state (feasible) for $X_{i,j}$ from the set $S_{i,j}$ as if solution X is not repetitive.
9. Put $NE=NE+1$.
10. Calculate objective function for the present layout and copy it in variable Cost_{NE}.
11. If $Cost_{NE} < MaxCost$, put Best $\text{Cost}=\text{Cost}_{\text{NE}}$ and Bestsolution=x.
12. If $j \leq n\text{-}1$, add one unit to j and go to step 8, otherwise go to step 10.
13. If $i \leq n\text{-}1$, add one unit to i, and go to step 7, otherwise go to step 14.
14. Print NE.
15. Put Best solution.
16. Print Best cost.
17. The end.

N means the number of facilities, $S_{i,j}$ means all the possible (feasible) states for $x_{i,j}$, also NE depicts the number of evaluations or the measured solutions.

7 | Case Study

The case study in this paper includes an industrial workshop producing different kinds of wooden and metal products. This workshop includes 17 facilities and 17 Locations. The aim of this paper is to reach an optimized settlement of the facilities in the locations based on the distance among the locations and the transportation flow among machines.

8 | Distance of the Locations

The distance among the Locations is shown in *Table 1*.

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Table 1. Distance of locations (meter).

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
1	0	25	38	50	60	77	22	35	49	63	77	28	43	55	67	79	101
2	25	0	16	28	40	55	5.5	17.5	31.5	44.5	58.5	23	38	50	62	74	98
3	38	16	0	17	29	41	17.5	6.5	19.5	33.5	47.5	23	24	36	48	60	68
4	50	28	17	0	16	31	29.5	15.5	7.5	21.5	35.5	35	20	28	32	44	56
5	60	40	29	16	0	19	41.5	27.5	13.5	9.5	23.5	47	32	20	24	36	44
6	77	55	41	31	19	0	56.5	42.5	28.5	14.5	8.5	62	47	35	23	23	25
7	22	5.5	17.5	29.5	41.5	56.5	0	7	31	45	59	8	23	35	47	59	81
8	35	17.5	6.5	15.5	27.5	42.5	7	0	7	31	45	14	9	21	32	44	68
9	49	31.5	19.5	7.5	13.5	28.5	31	7	0	7	31	28	13	7	19	31	55
10	63	44.5	33.5	21.5	9.5	14.5	45	31	7	0	7	44	27	15	5	17	41
11	77	58.5	47.5	35.5	23.5	8.5	59	45	31	7	0	56	41	29	17	5	26
12	28	23	23	35	47	62	8	14	28	44	56	0	20	32	44	56	80
13	43	38	24	20	32	47	23	9	13	27	41	20	0	16	27	39	63
14	55	50	36	28	20	35	35	21	7	15	29	32	16	0	16	28	52
15	67	62	48	32	24	23	47	32	19	5	17	44	27	16	0	16	40
16	79	74	60	44	36	23	59	44	31	17	5	56	39	28	16	0	28
17	101	98	68	56	44	25	81	68	55	41	26	80	63	52	40	28	0

9 | Percentage of Transportation Flows

Percentage of displacements among machines is shown in *Table2*.

Table 2. Matrix of the transportation percentage among facilities.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
1	0	0	0	0	0	0	0	0	16.9	0	0	0	0	0	0	0	0
2	0	0	0	0	0	0	0	0	12.15	0	0	0	0	0	0	0	4.97
3	0	0	0	0	0	0	0	0	1.83	0	10.35	0	0	0	0	0	3.65
4	0	0	0	0	0	12.27	2.79	2.79	0	0	0	0	0	0	0	0	0
5	0	0	0	0	0	0	12.61	0	0	2.23	0	0	0	0	0	0	0
6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
7	0	0	0	0	0	0	0	0	0.01	0	0	0	0	0	0	0	0
8	0	0	0	0	0	0	0	0	0	0	0	0	0.01	0	0	0	0.01
9	0	0	0	0	0	0	0	0	0	0	0.03	0.03	0	0	0	0	0.09
10	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
11	0	0	0.37	0	0	0	0	0	0.41	0	0	0	0	0	0	0	0
12	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
13	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
14	3.67	1.2	5.27	2.39	3.99	0	0	0	0	0	0	0	0	0	0	0	0
15	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
16	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
17	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

In the following discussion, the required model is defined generally and parametrically.

$$\text{Cost}=\sum_{i=1}^{17}\sum_{k=1}^{17}\sum_{j=1}^{17}\sum_{s=1}^{17} d_{i,k}w_{j,s}x_{i,j}\,x_{k,s}.$$

Subject to:

$$\sum_{j=1}^{17} x_{i,j}=1\ ;\ i=1,2,\ldots,17. \tag{2}$$

$\sum_{i=1}^{17} x_{i,j}=1\ ;\ j=1,2,\ldots,17\,.$

$x_{12,j}=0\ ;j\in\{1,2,3,4,5,6,7,8,9,10,11,12,13,15,16\},$

$x_{6,j}=0\ ;j\in\{1,2,3,4,5,6,7,8,9,10,11,12,13,15,16\},$

$x_{1,j}=0\ ;j\in\{1,2,3,4,5,6,7,8,9,10,11,12,13,14,17\},$

$x_{17,j}=0\ ;j\in\{1,2,3,4,5,6,7,8,9,10,11,12,13,14,17\},$

$x_{7,j}=0\ ;j\in\{1,3,4,5,8,10,11,13,14,15,16,17\},$

$x_{8,j}=0\ ;j\in\{1,3,4,5,8,10,11,13,14,15,16,17\},$

$x_{9,j}=0\ ;j\in\{1,3,4,5,8,10,11,13,14,15,16,17\},$

$x_{10,j}=0\ ;j\in\{1,3,4,5,8,10,11,13,14,15,16,17\},$

$x_{11,j}=0\ ;j\in\{1,3,4,5,8,10,11,13,14,15,16,17\},$

$x_{2,j}=0\ ;j\in\{2,6,7,9,12,14,15,16,17\},$

$x_{3,j}=0\ ;j\in\{2,6,7,9,12,14,15,16,17\},$

$x_{4,j}=0\ ;j\in\{2,6,7,9,12,14,15,16,17\},$

$x_{5,j}=0\ ;j\in\{2,6,7,9,12,14,15,16,17\},$

$x_{13,j}=0\ ;j\in\{2,6,7,9,12,14,15,16,17\},$

$x_{14,j}=0\ ;j\in\{2,6,7,9,12,14,15,16,17\},$

$x_{15,j}=0\ ;j\in\{2,6,7,9,12,14,15,16,17\},$

$x_{16,j}=0\ ;j\in\{2,6,7,9,12,14,15,16,17\},$

$x_{12,14}=x_{1,16},$

$x_{12,17}=x_{1,15},$

$x_{6,14}=x_{17,16},$

$x_{6,17}=x_{17,15},$

$x_{i,j}=0$ or 1 ; $i=1,2,\ldots,17$ and $j=1,2,\ldots,17.$

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Due to this point that in the current problem, based on real condition of the studied workshop, some new stipulations are added to that do not exist in base QAP, thus these stipulations are explained briefly in *Table 3*.

Table 3. Explanation of new stipulations.

Mathematical Stipulation	Limitation in Reality
$x_{12,j}=0$;j∊{1,2,3,4,5,6,7,8,9,10,11,12,13,15,16}.	Location 12 can only accept facility 14 and 17.
$x_{7,j}=0$;j∊{1,3,4,5,8,10,11,13,14,15,16,17}.	Location 7 can only accept facilities {6, 7, 9, 12, 2}.
$x_{2,j}=0$;j∊{2,6,7,9,12,14,15,16,17}.	Location 2 can only accept facilities {1, 11, 10, 8, 4, 3, 2, 5, 13}.
$x_{12,14}=x_{1,16}$.	If facility 14 is settled in Location 12, facility 16 shouldbe settled in Location 1.

11 | Comparison of the Results of the Proposed Algorithm and Branch and Bound Algorithm

Table 4 shows the results of performing the two algorithms by a common computer (CPU: 3.2 GHz & RAM: 4096MB).

Table 4. Results of the two algorithms' performance.

Branch and Bound Algorithm			Proposed Algorithm		
Time of performance (seconds)	Number of Evaluations made	Optimum amount	Time of performance (seconds)	Number of evaluations made	Optimum amount
2256.3	19353600	1400.845	1003.1	9676800	1400.845

As it is shown in *Table 4*, the new algorithm could reach the optimum answer by spending less time and making less evaluations. In fact, the proposed algorithm (1003.1 sec) finds the optimal solution in a shorter amount of time than the Branch and Bound (BB) algorithm (2256.2 sec). Moreover, the iteration number of the proposed algorithm (9676800) is lower than the iteration number of the BB algorithm (19353600). The optimal layouts are similar in both layouts. The objective functions of both algorithms are equal to 1400.845. The optimized facility layout is inserted in *Table 5*. For example, facility 16 should be located in the first Location.

Table 5. Optimum facility layout.

0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0
0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0
0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0
0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0
0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0
0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0
1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0

12 | Conclusion

To reach an appropriate layout design, it is necessary to define and solve the related nonlinear programming problems. Thus, using computer to solve the related problems seems to be important to the researchers of this area of study. But usually, the designs produced by computers for solving big problems need more time. In fact, this is a QAP; therefore, if we use a BB algorithm, we should analyze all states. According to the proposed approach, it is impossible to install some pieces of machinery in certain locations (stations) in some QAPs. Therefore, it is better to use an algorithm that ignores unfeasible installations. This minor change in BB algorithm can accelerate it and reduce its runtime.

In this paper, an algorithm is proposed that can be useful for better performance of the known algorithms such as BB and so it can identify the best answers by spending less time.

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