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Optimization of a Green Supply Chain Network: A Case Study in a Pharmaceutical Industry

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Abstract

Green Supply Chain Management (GSCM) is defined as the integration of environmental thinking into supply-chain management, including product design, material sourcing and selection, manufacturing processes and procurement. Generally, the sustainable supply chain is built on three dimensions: social, economic, and environmental, while the green supply chain emphasizes on the environmental issue considering an economic result. Although many works of sustainable growth heed attention on the profit margin as a first priority, however, the green supply chain focuses on the environmental issue along with an efficacious process, which is a beneficial to the environment as well. There are over 200 papers about GSCM. But among them, very few papers focus on the Pharmaceutical Industry, specially focusing on the supplier. But this paper focuses on implementing GSCM on Pharmaceutical Industry. Multi-objective Optimization Model has been applied here. Performance of the proposed model is evaluated by the Pycharm Solver of Python.

Keywords: Green supply chain, CO_2 emission, Cost, Pharmaceutical company.

1 | Introduction

Climate change is quite obvious at present, and its' impact is falling adversely on population and world economy. Last twelve years have witnessed the worst scenario [1]. As Supply Chain Management (SCM) is a vital part of a business, it should be sustainable, because a significant amount of pollution occurs through it. Hence, optimization needs to be brought in this sector. Carbon Di-oxide emission and cost need to be reduced [2]. This is how the concept of Green Supply Chain Management (GSCM) has been introduced. Manufacturing industries are GSCM in an efficacious way to increase profit. However, pharmaceutical companies are confronting major challenges due to globalization at present [3]. Pharmaceutical supply chain can be considered as a tool for the effective supply of pharmaceutical products [4]. Here in this paper, we suggest a mathematical model which has been solved by Python. Additionally, we attempted to show all stages of a supply chain network, from supplier to manufacturer [5]. Because we believe, considering each stage is crucial to obtain accurate result. By adding a "green" component in the SCM practices, GSCM practices encompass a set of

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green activities in procurement, manufacturing, distribution and reverse logistics. The objectives of this paper are to minimize cost and CO_2 emission in a pharmaceutical company in Bangladesh.

Environmental collaboration includes cooperating with suppliers to achieve environmental objectives and improve waste reduction initiatives, providing suppliers with design specification that include environmental requirements for purchased items, encouraging suppliers to develop new source reduction strategies, working with suppliers, production and helping suppliers to provide materials, equipment, parts and services that support organizational goals. We tried to accumulate data from a pharmaceutical company in Bangladesh and analyze them. Some relationships between costs and carbon di-oxide emission have also been displayed in this paper.

1.1 | Problem Statement

The rate of increase of environmental CO_2 emission was 3.3 ± 0.1 PgC/year between 1980 and 1989 and 3.2 ± 0.1 PgC/year from 1990 to 1999. The pharmaceutical industry is one of the major industrial emitters of greenhouse gases, particularly CO_2 [6]. Medicine production is an energy-intensive process. Additionally, most of the papers did not start their work from the supplier stage of SCM.

1.2 | Research Goals

The optimization of Green Supply Chain in a pharmaceutical industry is our main goal. The optimization will be done by using Mixed Integer Linear Programming (MILP) model. Our goal is to trade-off between the cost of production and reducing the CO_2 emission by optimizing all the steps of supply chain. And our goal will be fulfilled if we can control all the Green House Gas emission rates and make profit better than before. Two models will be used and those are to be traded off. Thus, we will reach our goal.

2 | Literature Review

2.1 | Economic and Environmental Optimization

Optimization of both economic costs and environmental costs and reduction of the industrial wastage has been shown in multiple papers [14] and [15]. The experimental output obtained by GAMS software illustrates the validity of the model. The main objective is to obtain the capability of resisting uncertain conditions. These procedures were Fuzzy Programming, Bi-objective Supply Chain Network design, Multi Objective Differential Evolutionary algorithm (MODE) [14].

2.2 | MILP Model

Two evaluation indicators, namely TC and TE of CO_2, are chosen to create an MO mixed integer linear model [16]. It proposes a new mathematical model to assess and optimize GSC performance. A chance constrained mixed integer programming model is proposed for a single product supply chain [17]. A MILP problem has been formulated considering demand uncertainty [11].

2.3 | Supply Chain in Miscellaneous Sectors

SCM has been implemented in Green Internet of Things (G-IoT) [12]. Supply chain model has been applied in educational institutions [13]. Blood supply chain has been discussed as well [10]. A manufacturing context has been demonstrated clearly [9].

2.4 | Supply Chain in Pharmaceutical Industry

A model for an agile supply chain in the pharmaceutical industry has been developed [18]. The pharmacy supply chain and current managerial practices in a hospital has been demonstrated [19].

2.5 | Research Gap

Very few papers worked with pharmaceutical companies. Additionally, most of them started working with manufacturer stage. Apart from that, in Bangladesh, researchers hardly work with pharmaceutical industry for supply chain optimization. Hence, we tried to work with a pharmaceutical company in Bangladesh, and start with supplier stage.

3 | Methodology

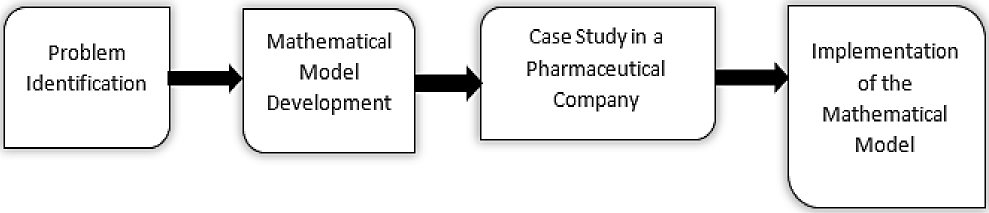

Fig. 1. Steps of research methodology.

The GSCM is a very large field. So, when a planning horizon is tried to be set, we have to consider all the segments of supply chain and apply optimization in each stage. The framework is shown below:

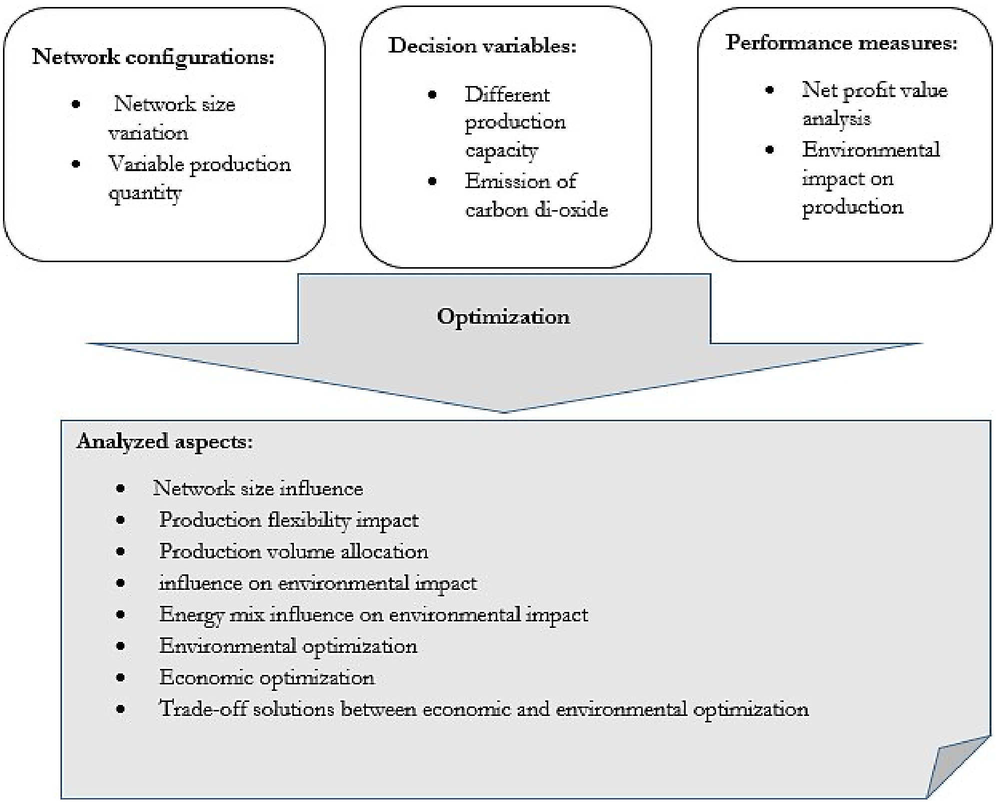

Fig. 2. Conceptual framework for a both economic and ecological GSCM planning horizon.

4 | Model Design

In a supply chain system, there are five stages. Those are supplier – manufacturer – distributor – dealer or, retailer – customer. So, if the pharmaceutical products such as medicine, insulin etc. need to be reached to customer, we should go through the first four stages. Here, we have proposed a multi-objective model that is a MILP. The proposed model is divided into four level of SCM.

Level-1 is indicated for supplier, Level-2 is indicated for manufacturer, Level-3 is indicated for distributor and Level-4 is indicated for dealer.

Now, some limitations and assumptions arise here. In developing the proposed model, the following assumptions and litigations were found out:

I. The model is designed for multiple suppliers, manufacturers, distributors, dealers, products and multi-periods for the whole production process.
II. The amount of demand was assigned to the suppliers at the beginning of the period.
III. The locations of suppliers, plants, distributors, dealers are fixed.
IV. The capacities of the suppliers, manufacturers, the distributors and the dealers are known.
V. The summation of the production time was equal to the duration of each period.
VI. At the beginning or the end of the planning horizon, no inventory is allocated for the distributors.
VII. All dealers are not free all the time. So, sometimes products are temporarily stored at the distributors.
VIII. Transportation mode is controlled by trucks.
IX. The shortage of dealers is allowed and in addition, more limitations arose when the simulation was done.

So, the consideration of limitation added more assumptions. They are:

I. The demands of all stages must be satisfied during the production planning horizon.
II. The production time is limited.
III. Each products have different capacities in storage and they are limited in quantity.
IV. The capacities of the supplier, manufacturer, distributor and dealers are limited.
V. For each perfect product, the capacity in storage is limited.

4.1 | Parameters

The parameters of the proposed model are listed below:

Hdpt = Holding cost of products p at distributor d in period t.

Rsmpt = Receiving cost from supplier s to manufacturer m per item product p in period t.

Kmpt = Production cost per item p by manufacturer m in period t.

Umdpt = Shipping cost of each product p from manufacturer m to distributor d during period t

Vdrpt = Shipping cost of each product p from DC d to dealers r during period t.

Dmpt = Time required to produce product p by manufacturer m in period t.

Tt = Total production time during period t.

Invpdt = Inventory of product p at distributor d during period t.

Demrpt = Demand of dealer r for product p during period t.

Capmpt = Production capacity of manufacturer m for product p in period t.

Sdt = Total storage capacity of distributor d during period t.

Mdpt = Storage capacity of DC d for product p in period t.

Nrpt = Storage capacity of dealer r for product p in period t.

Jrt = Total storage capacity of dealer r during period t.

Srpt = Shortage cost for each product p at dealer r during period t.

Ap = The penalty cost of early/tardy deliveries per unit of product p.

CO_2mdpt = Unit CO_2 emission per product p from manufacturer m to DC d during period t.

CO_2'drpt = Unit CO_2 emission per product p from DC d to dealer r during period t.

4.2 | Decision Variables

The decision variables of the proposed model are listed here:

asmpt = Number of raw materials transported from supplier s to manufacturer m during period t.

bmdpt = Amount of products p transported from manufacturer m to distributor d during period t.

cdrpt = Amount of products p transported from distributor d to dealer r during period t.

dmpt = Total production of products p by manufacturer m during period t.

erpt = Amount of shortage of products p in dealer r during period t.

fpt = Amount of products p which are not deliver on-time in period t in whole network.

4.3 | Set of Parameters

There are six parameters used here. These parameters will help to take the decisions about the optimization of these two functions. The optimized value of the parameters will have to be minimum. The parameters are:

p = set of product types.

P = maximum number of product types, d = set of distributors.

D = maximum number of distributors, t = set of time periods.

T = maximum number of time periods, s = set of suppliers.

S = maximum number of suppliers, m = set of manufacturers.

M = maximum number of manufacturers, r = set of retailers.

R = maximum number of retailers.

4.4 | Model Formulation

Most of the industries implement linear economic optimization models. Other papers divided the proposed model into three levels: level 1 indicates the manufacturers; level 2 represents the distributors and level 3 denotes the dealers [7] and [8]. However, in this paper, we started working with suppliers instead of manufacturers. In the mathematical model we used, the configurations represent different network designs, which, in diverse scenarios and periods, have to satisfy different demands of the markets. The supplier factories produce the pre- products manufactured in the lines and in the nodes. The pre-products yield in the end product, which are produced in the factory, node and line. Investments can be made on the factories, on the nodes or on the lines. The new dimensions integrated to consider the environmental aspects are the monitored and measured emissions categories, the mean of transport used, the different electricity mix compositions and the fuel and heating sources.

Integrating the above environmental parameters to the economic ones and using the decision variables, two model objective functions are formulated in *Eqs. (1)* and *(2)*. The first corresponds to the total supply chain cost minimization and the second to the total carbon di-oxide minimization.

$$\text{Min } Z1=\sum_{m=1}^{M}\sum_{p=1}^{P}\sum_{t=1}^{T}K_{mpt}.d_{mpt}+\sum_{d=1}^{D}\sum_{p=1}^{P}\sum_{t=1}^{T}H_{dpt}.(\sum_{\tau=1}^{t}\sum_{m=1}^{M}b_{mdpt}-\sum_{\tau=1}^{t}\sum_{d=1}^{D}c_{drpt})$$
$$+\sum_{s=1}^{S}\sum_{m=1}^{M}\sum_{p=1}^{P}\sum_{t=1}^{T}R_{smpt}.a_{smpt}+\sum_{m=1}^{M}\sum_{d=1}^{D}\sum_{p=1}^{P}\sum_{t=1}^{T}U_{mdpt}.b_{mdpt} \quad (1)$$
$$+\sum_{d=1}^{D}\sum_{r=1}^{R}\sum_{p=1}^{P}\sum_{t=1}^{T}V_{drpt}.c_{drpt}+\sum_{r=1}^{R}\sum_{p=1}^{P}\sum_{t=1}^{T}S_{rpt}.e_{rpt}+\sum_{p=1}^{P}\sum_{t=1}^{T}f_{pt}.A_{p}.$$

Eq. (1) is the first objective function that abates the total costs of the supply chain, along with the costs of supply, production, holding at the distributor, transportation from the supplier to manufacturers, manufacturers to the distributors, transportation from the distributors to the dealers and dealer shortages due to the shortage of the stock situations. In addition, another cost is included. If the production is hampered for any reason, then the amount of delivery will decrease as per demand. Then a penalty cost is considered for ensuring Just-In- Time deliveries.

$$\text{Min } Z2=\sum_{m=1}^{M}\sum_{d=1}^{D}\sum_{p=1}^{P}\sum_{t=1}^{T}CO2_{mdpt}.b_{mdpt}+\sum_{d=1}^{D}\sum_{r=1}^{R}\sum_{p=1}^{P}\sum_{t=1}^{T}CO2'_{drpt}.c_{drpt}. \quad (2)$$

Eq. (2) denotes the objective function that minimizes total carbon emission in entire supply chain process. Now for calculating the carbon emissions, several methodologies have been applied. Since Greenhouse Gas protocol is the most applied because of its easy application and worldwide scope, we also adopted this methodology in this research. The equivalent carbon emission per product can be calculated as a linear function and it depends on the travelling distance (in kilometers) and the carried vehicle carbon emission (in grams of CO_2 per kilometer). We applied this carbon emission model for a given supplying mode and the carbon emission is proportional to the number of product units that are shipped daily.

4.5 | Constraint Selection

$$d_{mpt}\le Cap_{mpt}, \qquad \forall m,p,t \quad (3)$$

Eq. (3) states that the total products produced by the manufacturer should be equal or less than the production capacity.

$$\sum_{m=1}^{M}\sum_{p=1}^{P} b_{mdpt} \leq S_{dt}, \qquad \forall d,t \tag{4}$$

$$\sum_{m=1}^{M} b_{mdpt} \leq M_{dpt}, \qquad \forall d,p,t \tag{5}$$

Eq. (4) denotes the capacity restrictions of the delivery of distribution centers for each type of product. *Eq. (5)* denotes the capacity restrictions of the delivery of distribution centers for all types of products.

$$\sum_{d=1}^{D}\sum_{p=1}^{P} c_{drpt} \leq J_{rt}, \qquad \forall r,t \tag{6}$$

$$\sum_{d=1}^{D} c_{drpt} \leq N_{rpt}, \qquad \forall r,t \tag{7}$$

Eq. (6) denotes the capacity restrictions of the delivery for the dealers for each type of product. *Eq. (7)* denotes the capacity restrictions of the delivery for the dealers for all types of products.

$$\sum_{r=1}^{R} Dem_{rpt} \leq d_{mpt}, \qquad \forall p,t \tag{8}$$

Eq. (8) considers that total production is equal to the total demand required.

$$\sum_{r=1}^{R} e_{rpt} = \sum_{r=1}^{R} Dem_{rpt} - \sum_{r=1}^{R} d_{mpt} \leq d_{mpt}, \qquad \forall p,t \tag{9}$$

Eq. (9) states the amounts of shortage rather than demand at the dealers.

$$\sum_{d=1}^{D} c_{drpt} = Dem_{rpt}, \qquad \forall r,p,t \tag{10}$$

Eq. (10) shows how the total demands in the supply chain are supplied and fulfilled.

$$\sum_{r=1}^{R}\sum_{\tau=1}^{t} c_{drp\tau} \leq \sum_{m=1}^{M}\sum_{\tau=1}^{t} b_{mdp\tau}, \qquad \forall d,p,t \neq T \tag{11}$$

$$\sum_{d=1}^{D}\sum_{p=1}^{P} c_{drp\tau} \leq \sum_{m=1}^{M}\sum_{p=1}^{P} b_{mdp\tau}, \qquad \forall d,p \tag{12}$$

Eqs. (11) and *(12)* show the inventory at the distribution centers. The point to be noted that there is no inventory in the beginning and at the end of the planning horizon at each distributor.

$$\sum_{m=1}^{M}\sum_{\tau=1}^{P} b_{mdp\tau} - \sum_{r=1}^{R}\sum_{\tau=1}^{t} c_{drp\tau} = Inv_{pdt}, \qquad \forall d,p,t \neq T \tag{13}$$

Eq. (13) represents the balance between the total inputs and outputs of goods moving to the distributors from the manufacturers and to the dealers from the distributors during the planning horizon.

$$\sum_{m=1}^{M}\sum_{p=1}^{P} D_{mpt}.d_{mpt} \leq T_t, \qquad \forall t \tag{14}$$

Eq. (14) represents the available time limitations of the production facilities for all production processes.

$$b_{mdp\tau},\ c_{drpt},\ d_{mpt},\ e_{rpt},\ a_{smpt} \geq 0, \ \text{integer}. \qquad \forall m,d,r,s,p,t \tag{15}$$

Eq. (15) ensures positive values of the supplies from supplier, production amount, deliveries to warehouses and dealers and dealer shortages. Because, non-negative will affect in the final result.

5 | Result and Discussions

The optimization models contain two functions. Both are minimization functions. One function is about the minimization of total supply chain cost included in all stages of supply chain. And other function is about the minimization of the carbon di–oxide through the product flow from manufacturer to

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distributor and from distributor to retailer. In this model, there are seven kinds of costs, two types of time periods, five types of capacity and two stages of carbon di-oxide emission are assumed. Six types of decision variables are also assumed. The whole model is solved using Python. Here, we used 'Pycharm' software for coding and solving it. After solving the model, total six outputs we have got for minimum Z1 and five outputs for minimum Z2 value.

6 | Case Study

When the real model is solved then we need to implement it to a real-life scenario. For this purpose, we visited Beximco Pharmaceuticals Limited which is situated in Tongi, Gazipur. In this industry, we mainly took a survey on the production time and the cost incurred with some specific times. We also have taken some data of costing with changing the number of distributors, number of retailers and the number of suppliers. Also, we have took some information about the whole production planning and the duration of the production planning. On basis of these datasets, some analysis has been done. There are many kinds of pharmaceutical products in this industry. Among those, we have chosen "Napa 500mg" for our all kinds of analyses.

6.1 | Sequence of the Whole Production Planning

A tablet is a pharmaceutical Oral Solid Dosage form (OSD) or solid unit dosage form. Tablets are solid single units containing one or more active ingredients prepared either by molding or compression. The solid production unit follows five stages of production normally. These are:

Stage 1. Dispensing.

Stage 2. Granulation.

Stage 3. Compression.

Stage 4. Coating.

Stage 5. Packaging.

In each and every stage there are a lot of activities which follow a specific sequence. For this reason, a specific production time is allocated for a specific product. From analysis and survey, we have known that there are 11 main sequential activities are allocated among these five stages of solid medicine (tablet) production. These 11 activities are analyzed in the Gantt chart and the total production time is calculated which is used in Value Stream Mapping (VSM) before. The Gantt chart is shown below with the assumed identifications of the activities.

a = checking batch size and batch no.

b = checking the quantity of materials.

c = checking tare weight and selecting container, d = drying.

e = milling.

f = addition of lubrication, g = crushing.

h = blending.

i = seal coating, j = polishing.

k = strip packaging.

Table 1. Table for activities, predecessors and time durations.

Predecessor	Activity	Duration (hour)
-	a	2
-	b	2
-	c	3
b	d	4
a	e	3
c, d	f	1
d, e	g	2
f	h	2
g, h	i	3
I	j	3
j	k	4

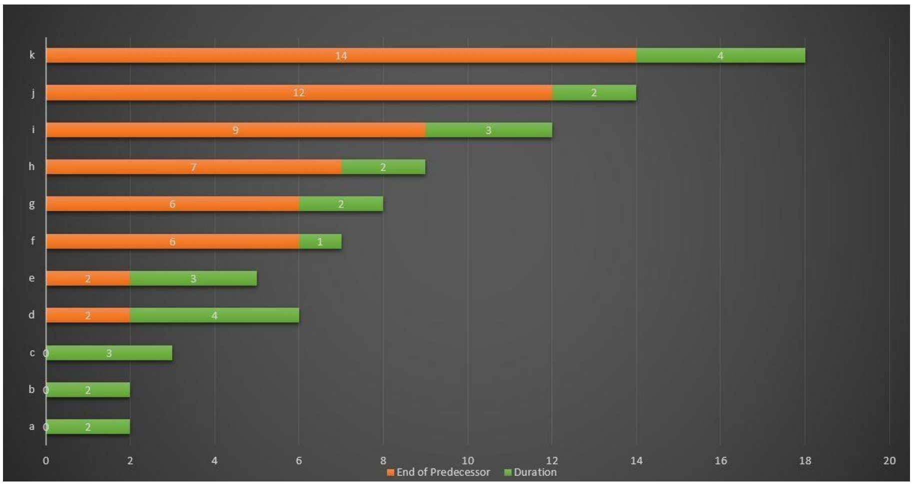

Fig. 3. Gantt chart for total production time.

6.2 | Carbon Di-Oxide Reduction Analysis

Carbon di-oxide emission is a very common issue in any kind of industry. Sometimes, it can be reduced by using different reduction methods. Maximum methods are very costly but highly benefitted. Because our environment is continuously polluted by the emission of this gas.

In our proposed model, two stages of carbon di-oxide emission are incurred. These are:

I. Carbon di-oxide emission with the product flow from manufacturer to distributor.
II. Carbon di-oxide emission with the product flow from distributor to retailer.

In both cases, some specific aspects are responsible for the emission of this gas. For example, transportation, production, packaging is the common source of emission. In this model, we have assumed the total transportation is fixed and the emission during packaging is fixed. Here, we have considered that the carbon di-oxide emission changes over production time. We have set the weightage of emission from 0.1 to 1. The weightage of the lowest emission is 0.1 and the weightage of the highest emission is 1.

Table 2. Carbon di-oxide emission data.

Production Time (Hours)	Cost Incurred	Weightage of CO_2 emission
18	127,50,000 /=	1
17	128,34,000 /=	0.9
16	129,23,000 /=	0.8
15	131,00,000 /=	0.7
14	132,96,000 /=	0.6
13	134,80,000 /=	0.5
12	136,95,000 /=	0.4
11	139,20,000 /=	0.3
10	145,00,000 /=	0.2
9	154,00,000 /=	0.1

These costs have been obtained from the company.

7 | Analysis

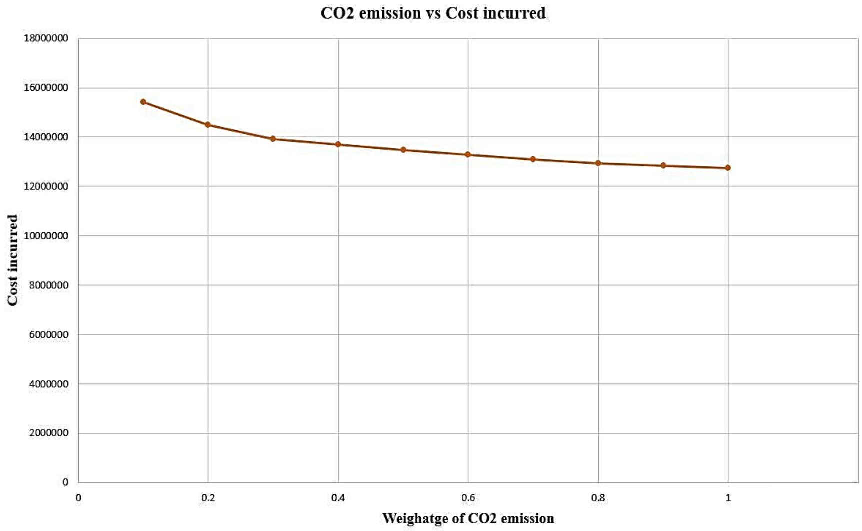

Fig. 4. CO2 emission vs. cost incurred graph.

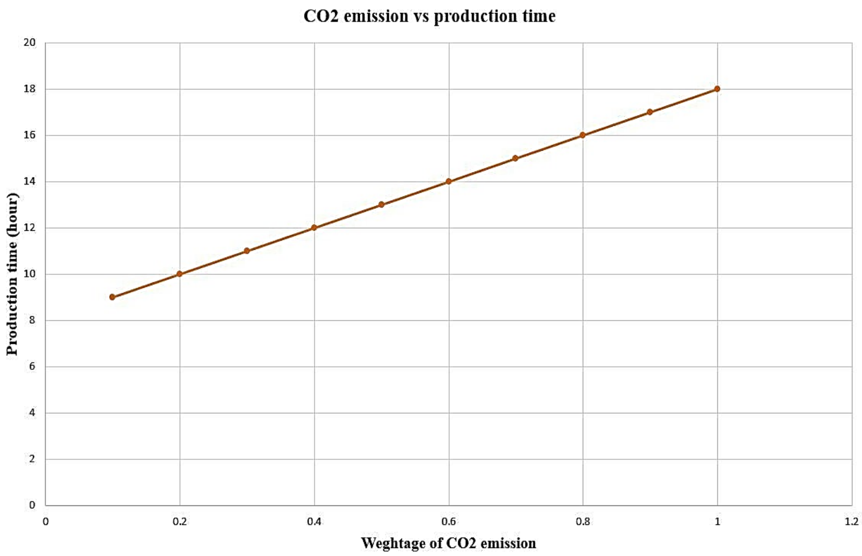

Fig. 5. CO2 emission vs. production time graph.

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7.1 | Tradeoff between Total Cost and Production Time

Above the two analyses, two conflicting situations have been arisen between the total supply chain cost and the total production time. For this reason, a tradeoff solution has been generated for determining the optimum value of the cost, production and the weightage of carbon di-oxide emission.

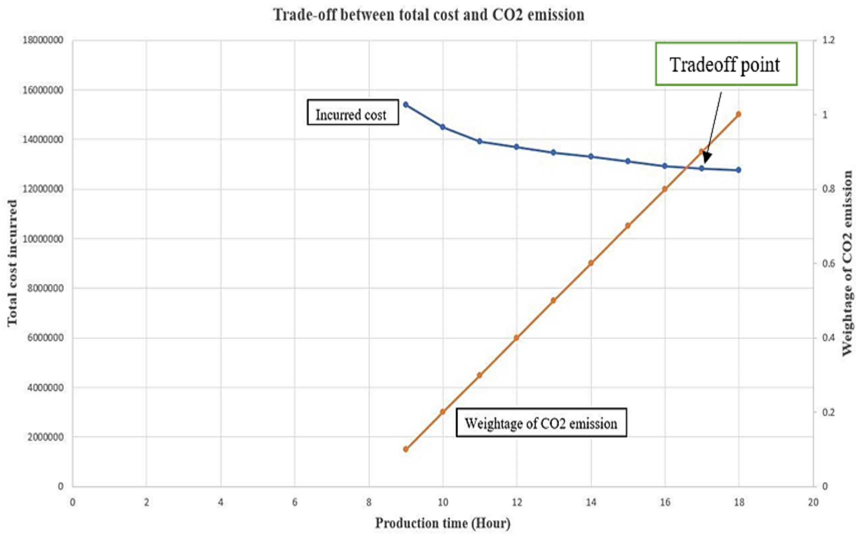

Fig. 6. Tradeoff between total cost and production time.

From this graph, the optimum cost incurred = 128, 00,000, the optimum production time = 16.6 hours. The optimum weightage of carbon di-oxide emission = 0.85.

7 | Discussions

After all the analyses, overall result is generated and decisions can be taken. The fundamental research objective was to minimize the overall supply chain cost with the environmental concerns. Therefore, the cost should be as lower as possible but not sacrificing the environmental issues. Firstly, we have seen the relation of costing with the changing the number of suppliers, retailers and distributors. From that part, we have chosen the minimum value of product types, time periods, suppliers, manufacturers, distributors and retailers. These minimum values will be responsible to minimize the supply chain cost and the carbon di-oxide and other hazardous gas emissions. Finally, we have taken decisions on carbon di-oxide emission purpose. Here, two cases were arisen. Firstly, increase of the total supply chain cost with the reduction of carbon di-oxide. Secondly, increase the carbon di-oxide emission with the increase of the total production time. Then a tradeoff is done between these two aspects and we have got an optimum solution of cost, time and emission.

Besides, Production scheduling using Gantt chart is discussed to introduce all the systems involved with this model. Our whole research calculation is based on the datasets collected from Beximco Pharmaceuticals Limited which is located in Tongi, Gazipur. Here, we have got some ideas from some of their consultants in the firm. We have selected Napa 500 mg for research. Because, it is a Fast-Moving Consumer Good that is called FMCG. So, if we take it as survey product then we can get available data and information.

Again, the emission of carbon di-oxide is measured in two stages. One is the product flow from manufacturer to distributor and other one is the product flow from distributor to retailer. In these two stages, we have considered the total carbon di-oxide emission is high. In these stages, the emission can be done from transportation, production, packaging etc. But we considered here only production because its dataset is available to us. But in the context of our Bangladeshi industries, there is no specialized system to measure carbon di-oxide emission specifically. So, we have used weighted emission value to compare with the costing and production time. We have got an idea from their consultancy that if their production time is normal, then the emission occurs normally. But if their production time can be reduced into half of the normal production time, then the carbon di-oxide emission reduced to one-tenth of normal emission. But the overall supply chain cost increases. Additionally, our model can be implemented in the pharmaceutical companies to minimize the total cost and CO_2 emission.

8 | Conclusion

To conclude, we can summarize our whole research work in a few words. We truly acknowledge our limitations. Hence, we will continue our work to obtain a better result. Our goal of the research was to trade-off between cost and carbon di-oxide emission in a pharmaceutical company. To achieve our goal, we visited some pharmaceutical companies and accumulated data which were implemented in our mathematical model to scrutinize that. However, we used Python to solve the model to be more ascertained. This study proposes a mathematical model for optimizing supply chain costs with respect to environmental impact. The aim of the model is to optimize total costs, including production, holding, shipping, and dealer shortages due to out of stock as well as minimizing carbon di-oxide emission in the whole logistics system.

Following recommendations and suggestions for future work are highlighted in this section in order to improve the implementation process of the mathematical model.

I. ERP Software can be used to get precise result. For using the mathematical model, accurate data would be beneficial.
II. More Pharmaceutical Companies can be visited and more people who are involved with the production process should interviewed.
III. Some modifications in Value Stream Mapping can be brought to make it more efficient.

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Identifying and Ranking the Factors Affecting the Transportation of Products from a Marketing Perspective Using the Fuzzy Analytic Hierarchy Process Approach

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Abstract

Economic researchers consider the development of transportation as blood circulation in the economic body of the country, which accelerating its circulation satisfies the vital and basic economic needs faster, and any kind of disorder and slowness in it causes great damage to the process of growth and development. Therefore, some economists and planners have considered the economic growth to be dependent on the development of the transportation sector and the existence of enough vehicles and related facilities. In this regard, Iran is very important due to its special position near the borders of Central Asia and the Caucasus and because it's a bridge between the two major continents of Asia and Europe and has access to open waters. The main problem of this research is to identify all the dimensions and components that affect the transportation (transit) of goods and products, and because we finally want to present a strategy, the next problem is to rank these factors and consider the most important components and dimensions of Islamic Republic transportation. Analytic hierarchical process was used to determine the importance and weight of the criteria and then Expert Choice software was used for ranking. Given that the subject of transportation is a specific and macro issue, investigating it from a marketing perspective is a novel perspective, the studies conducted by students have remained at the research level, and there have not been many related studies on marketing in Iran, the most important limitation of the research was the ability to explain the reason of conducting this research and controlling the interviews. The results of pairwise comparisons and weights are expressed in the following. Accordingly, among the 20 sub-criteria, the use of information technology is ranked first. After that, sustainable development and smart transportation ranked second and third, respectively.

Keywords: Transportation, Marketing, Fuzzy analytic hierarchy process, Uncertainty.

1 | Introduction

The transportation in a comprehensive sense, including management, infrastructure and superstructure is one of the basic needs of today's societies, so that in any country, the promotion of economic, cultural, security and political power depends on having an extensive and reliable transportation network as one of the basic needs [1] and [2]. Some researchers believe that transportation is one of the primary tools of development. In the past, because transportation was considered a secondary need to meet basic needs such as trade, travel, employment, etc., it was not properly addressed [3] and [4]. But today in most countries, transportation has an important role due to the growth and development of the global economy and in order to make the best use of capabilities and opportunities. Other reasons are the intensified competition in the global market and

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also the direct role of transportation in reducing manufacturing costs and easy access to the market. So, special attention is paid to transportation in management, planning, and investment [5] and [6]. There have also been many studies in the field of transit and transportation. All these issues have caused the previous attitudes towards transportation to change and it is seen as an important economic-service sector. The movement of products, places of living and human beings has been one of the most important human economic activities from the earliest days of human existence on earth until today, and will most likely remain important until the end of human life in this world [7]. In fact, transportation was born at the same time as human beings and has been developed and evolved with the expansion of human civilization and culture, and its development process in the future will follow human progress in various fields [8] and [9]. Nowadays, transportation is considered as one of the important components of the economy, in national and international level and even in the micro sectors, and due to its infrastructure role, it has a great impact on the process of economic growth of countries. Macro environment is very important in strategy development [10] and [11].

Economic researchers consider the development of transportation as blood circulation in the economic body of the country, which accelerating its circulation satisfies the vital and basic economic needs faster, and any kind of disorder and slowness in it causes great damage to the process of growth and development. Therefore, some economists and planners have considered the economic growth to be dependent on the development of the transportation sector and the existence of enough vehicles and related facilities [12]. Transportation is considered as a result and also a reason of progress and development of countries [13]. In other words, it acts as an effective factor in the progress of countries and, when countries have progressed, the transportation sector grows as an indicator of progress. Therefore, the transportation indicator can be considered as a factor indicating developed or undeveloped of societies and countries. Special attention is paid to transportation (in the general sense) in regional development planning [14].

Transportation is one of the necessities of any human community that leads to the dynamism of economic and social development. Sustainable development in general and sustainable transportation in particular seeks to find a balance between present and future environmental, social and economic qualities in the field of transportation facilities. In fact, sustainable transportation planning and design seeks to achieve solutions to reduce tolls in various sectors.

Transportation networks provide more opportunities for regional development. Major transportation networks between countries promote economic and cultural relations and increase regional security [15] and [16]. The transportation sector is directly and indirectly effective in job creation. In international trade and based on demand, products are transported from countries of origin to destinations that are sometimes located on other continents. Since the transportation sector is responsible for delivering these products to the desired destinations through road - rail - sea - air, so it must necessarily pass through the territory of other countries along this route [17]. In this regard, Iran is very important due to its special position near the borders of Central Asia and the Caucasus and because it's a bridge between the two major continents of Asia and Europe and has access to open waters. Iran has access to the open waters of the region such as the Persian Gulf and Oman Sea. Also, the growth of economic processes in East and South Asia and their interest in trade with the West along with the insecurity of rival routes in Russia and Dagestan, Afghanistan and Pakistan have made this region to be considered by the business world. Special geographical location, actual and potential economic power, security and political stability, having long miles of beach along the international waters and proper equipment are factors that can play an important role in the transit of products from Central Asia and the Caucasus. The main research question is as follows: Which criteria are selected for prioritization of marketing strategies for the transportation development in the Islamic Republic of Iran?

The objective of this study is to design a model for transportation in the Islamic Republic of Iran and according to experts, the model should be designed hierarchically to control costs. To achieve this objective, first existing and influential factors in the transportation development model are identified through the grounded theory of Glaser and by interviewing experts and specialists, and then the Analytic Hierarchy Process (AHP) method is used to prioritize the identified factors. The main problem of this research is to identify all the dimensions and components that affect the transportation (transit) of goods and products, and because we finally want to present a strategy, the next problem is to rank these factors and consider the most important components and dimensions of Islamic Republic transportation. In the first section of this research, an introduction was presented and a review of the literature will be presented in the second section. In the third section, the analysis method and in the fourth section, the criteria and sub-criteria will be defined. The results will be prioritized in the fifth section and the conclusion will be presented in the sixth section.

2 | Literature Review

Goodarzian et al. [18], presented a fuzzy bi-objective multi-period, three-echelon, multi-product, and multi-modal transportation model. The main aim of their research is to consider the environmental impacts related to the establishment of pharmacies and hospitals. Finally the presented model was solved using Firefly Algorithm and Simulated Annealing algorithm. Majumdar et al. [19], used fuzzy AHP was to incorporate the vagueness of perception of experts regarding the impact of various supply chain risks. They developed vulnerability matrix where each specific risk was mapped based on their respective impact and probability. The outcome of their paper would be very helpful for developing strategies for resilient green clothing supply chains. Koçak and Yercan [20], used fuzzy AHP in cost-effectiveness analyzes. Eleven container ships' voyage cost, revenue and profit are analyzed and compared. The results indicate the proper performance of the proposed model. Arahish [21], identified and prioritized the factors affecting the increase of productivity in ports in his thesis.

In this research, the technique of experts and AHP were used. Based on the results, political factors between countries, equipping and physical development of the port, location of the port, availability of specialized human resources, increasing speed and reducing loading and unloading time, access to railways, roads and air, providing the necessary infrastructure, marketing, using modern technologies and reduction of transportation time were respectively the first to tenth priorities in increasing productivity in Chabahar port. In his thesis, KarimiNasab [22], identified and developed entrepreneurial opportunities in the field of Chabahar-Milk transit road transportation.

This research was applied in terms of purpose and combined (survey, descriptive and analytical) in terms of nature. The statistical population of this study was the experts and authorities of the Department of Transportation, the Department of Roads and Urban Development, authorities and experts related to starting a business, and heavy truck drivers. Fifty people were selected as a sample using the non-probability purposive sampling method. Mousavi et al. [23], analyzed discourse of the Supreme Leader of the Revolution and presented strategic policies for the management of maritime transport. They showed the six components of the development of maritime transport support industries, purposive scientific, research and applied development in maritime transport activities, data collection and dissemination and the fight against abuse and corruption, economic development with emphasis on maritime and port activities, completing management elements of maritime transport, and the development and improvement of maritime transport management system as the main criteria and dimensions of maritime transport strategic policies.

SohrabiFakher [24], developed a model for prioritizing privatization strategies in the framework of public interest in Iran's rail transportation industry. This research aimed to challenge the ways to attract and involve the private sector in the most effective economic leverage, namely the rail transportation industry, considering the public interest, achieving financial goals and changing the role of government from management to regulation. Therefore, the data was obtained from the return of 160 questionnaires sent to 220 experts in this industry and professors and policy makers. The validity and reliability of the questionnaires were confirmed through statistical tests such as non-parametric binomial test that works the same as the parametric *t* test. SeyedAmini [25], identified and prioritized the factors affecting the capabilities of the border terminals of West Azerbaijan Province in the transit of products to/from the ports of the Mediterranean Sea. In order to achieve this objective, the border terminals of West Azerbaijan province were selected and due to the multiplicity of these terminals, Tamrchin border terminal was selected as a case study. Also, due to the importance of transit to Mediterranean ports, the two ports of Mersin and Latakia along with the countries that are on the route between this terminal and ports were also examined. In this study, 6 criteria and 17 sub-criteria affecting the capability of border terminals in the transit of products were identified as research findings. Among these factors, equipment and facilities had the most weight and importance, and among the related sub-criteria, the Rail and Road transportation network was more important than other sub-criteria. Liu et al. [26], used a Global Malmquist-Luenberger Index approach to evaluate the green productivity growth of this industry at the provincial level based on the Data Envelopment Analysis and Directional Distance Function. Further, it decomposed green productivity growth into changes in various types of efficiency and technological progress. Finally, this study structured a novel quadrant matrix analysis framework based on the green productivity growth rate and stability, using the matrix to analyze the performance of provincial road transportation industries. This analysis results showed that a fluctuating and slowly upward trend of green productivity over time exists. Solaymani and Kari [27], examined the effects of energy subsidy reform on the transportation sector of the Malaysian economy. The transportation sector in this research was divided into four parts: land, sea, air and other services (port, airport, highway, bridge and tunnel).

The results of simulating the energy subsidy reform policy in Malaysia using the CGE model showed that the implementation of this policy was beneficial for the country's economy and increased trade and nominal and real Gross Domestic Product, reduced demand for a variety of energy carriers and also reduced pollution. However, changes in household welfare especially for the native household were negative. Mukundan [28], conducted a study in China and stated that government policies played a significant role in achieving an average growth of 9.3% in the last 30 years. These policies included the promotion of shipping, ocean trade, shipbuilding and container building by giving low-interest loans from state-owned banks, partnerships with the Korean and Japanese shipping development industries, technology and knowledge transfer, and the training of Chinese engineers by Korean and Japanese counterparts.

The main research gap of this research is the lack of attention of other researches to the identification of all dimensions and components that are on the transportation (transit) of cargo and goods, especially in Iran. In addition, other studies have not considered the ranking of these factors and considering the most important components and dimensions of transportation.

3 | Method of Analysis

This section analyzes the research data. The objective of this study is to design a model for transportation in the Islamic Republic of Iran and according to experts, the model should be designed hierarchically to control costs. To achieve this objective, first existing and influential factors in the transportation development model are identified through the grounded theory of Glaser and by interviewing experts and specialists, and then the AHP method is used to prioritize the identified factors. In AHP method, first the effective factors are identified and then they are weighed and determined by AHP method. All calculations are performed in Expert Choice software. The advantages and contributions of presented FAHP are as follows:

- *The presented FAHP can take into consideration the relative priorities of factors or alternatives and represents the best alternative.*
- *The presented FAHP provides an easy applicable decision making methodology that assist the decision maker to precisely decide the judgments.*

- *Any level of details about the main focus can be listed or structured in this method. By this way the overview of the main focus or the problem can be represented very easily.*
- *Decision maker can analyze the elasticity of the final decision by applying the sensitivity analyzes.*
- *It is possible to measure the consistency of decision maker's judgments.*

The fuzzy AHP framework is suggested as follows [29]:

STEP 1: Draw the hierarchical chart.

STEP 2: Define fuzzy numbers for performing the pair-wise comparisons.

STEP 3: Create the pair-wise comparison matrix $\tilde{A}$ using fuzzy numbers.

The pair-wise comparison matrix can be expressed as follows:

$$\tilde{A}=\left[\begin{bmatrix} 1 & \tilde{a}_{12} & \tilde{a}_{1n} \\ \tilde{a}_{12} & 1 & \tilde{a}_{12} \\ \dots & \dots & \dots \\ \tilde{a}_{n1} & \tilde{a}_{n2} & 1 \end{bmatrix}\right]. \quad (1)$$

STEP 4: Calculate s_i for each row of the pair-wise comparison matrix

$$s_i=\sum_{j=1}^{n} M_{ij}\times\left[\sum_{i=1}^{m}\sum_{j=1}^{n} M_{kj}\right]^{-1}. \quad (2)$$

Where i represents the row number and j denotes the column number and M_{ij} is triangular fuzzy numbers of pairwise comparison matrices.

$$\begin{aligned} M_1^{-1} &= \left(\frac{1}{u_1},\frac{1}{m_1},\frac{1}{L_1}\right). \\ M_2^{-1} &= \left(\frac{1}{u_2},\frac{1}{m_2},\frac{1}{L_2}\right). \end{aligned} \quad (3)$$

In the above formulas l_i, m_i, and u_i are the first, second, and third components of the fuzzy numbers, respectively.

STEP 5: Compute the magnitude of s_i with respect to each other

$$V(M_2\geq M_1)=hgt(M_2\cap M_1)=\begin{cases} 1 \text{if} m_2\geq m_1 \\ 0 \text{if} l_1\geq l_2 \\ \dfrac{l_1-u_2}{(m_2-u_2)-(m_1-l_1)} \text{o.w} \end{cases}. \quad (4)$$

STEP 6: Compute the weight of the criteria and alternatives in the pair-wise comparison matrix

$$d'(A_i)=\text{Min}V(S_i\geq S_K)K=1,2,\dots,n. \quad (5)$$

STEP 7: Calculate the final weight vector

$$w = (d(A_1), d(A_2), \ldots, d(A_n))^T. \quad (6)$$

3.1 | Descriptive Statistics of Respondents

In this section, in order to know the respondents to the pairwise comparison questionnaire in terms of: gender, age, work experience and education, the demographic characteristics of the experts who participated in this study are described. *Table 1* lists the demographic characteristics of the experts (See apendix A).

3.2 | Gender of Respondents

Fig. 1 shows the gender of the respondent experts. As can be seen from *Fig. 1*, there are far more male experts in this industry than women, or at least in this study, the purposive sample includes more men. This can indicate that men most of the activists and decision-makers in this industry are men.

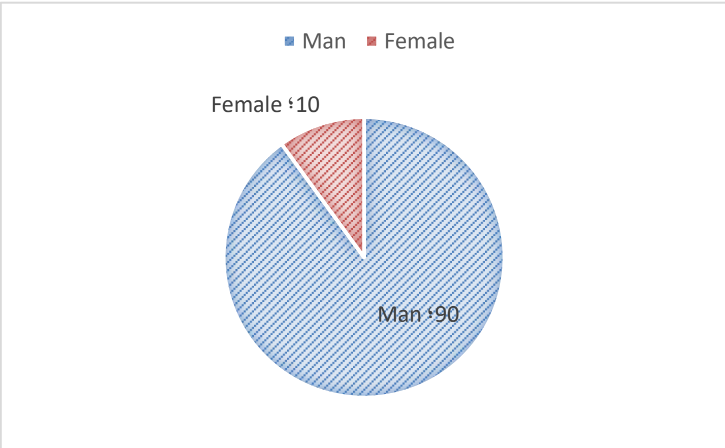

Fig. 1. Gender of respondent experts.

3.3 | Age of Respondents

Fig. 2 shows the age of the respondent experts. As can be seen from *Fig. 2*, 10% of the experts are in the age range of 30 to 40 years and 30% of them are in the range of 51 to 60 years old. The age range with the largest number of experts is between 41 and 50 years which is 60% and indicates that most respondents are middle-aged. Since the development of transportation requires both experience and expertise, these age ranges are acceptable and normal in the whole expert community.

3.4 | Education of Respondents

Fig. 3 shows the education of the respondent experts. As shown in *Fig. 3*, 50% of the respondent experts have a master's degree and 20% of them have a bachelor's degree, and experts with a doctorate or higher degree make up 30% of the respondent population. This shows that most of the respondent experts have experienced postgraduate education, and also, these people need this education in order to be productive in this new field of work.

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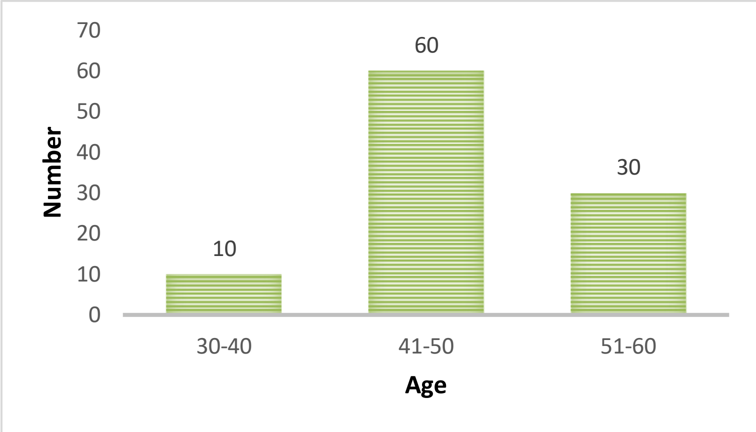

Fig. 2. Age of respondent experts.

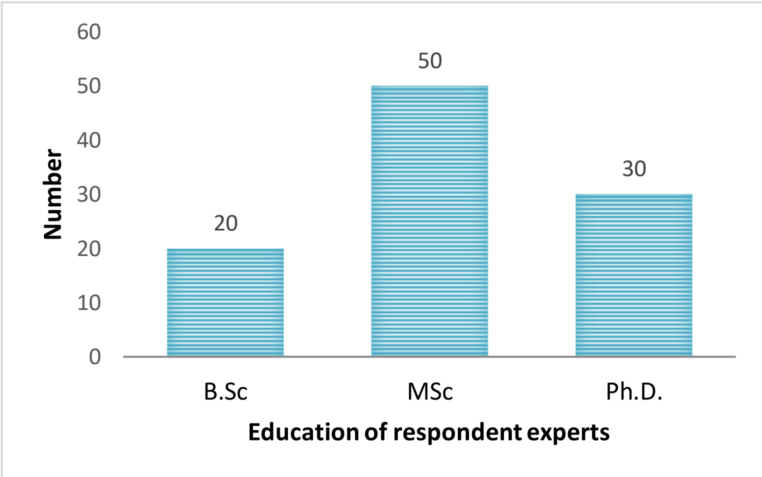

Fig. 3. Education of respondent experts.

3.5 | Work Experience of Respondent Experts

Fig. 4 shows the work experience of respondent experts in the field of transportation. As can be seen from *Fig. 4*, 20% of the respondent experts have had less than 10 years of experience and 20% of them have had more than 20 years of experience. Also, 10% have been working in the field of transportation for 10 to 15 years and 50% for 15 to 10 years. These percentages show that these experts are at a very good level in terms of experience to answer research questions because Iran's transportation has faced fundamental changes in previous decades and it is a growing industry.

3.6 | Specialized Field of Transportation

Fig. 5 shows the specialized field of transportation regarding the respondent experts. As can be seen from *Table 1* and *Fig. 5*, experts in the specialized fields of maritime transport, land transport, air transport, strategic transport headquarters, or researcher and university professor, are respectively 20, 30, 20, 20 and 10 percent. This shows that the respondent experts have the necessary expertise in all fields. This issue is also important because the diversity of expertise in different fields is effective in improving the transportation development model.

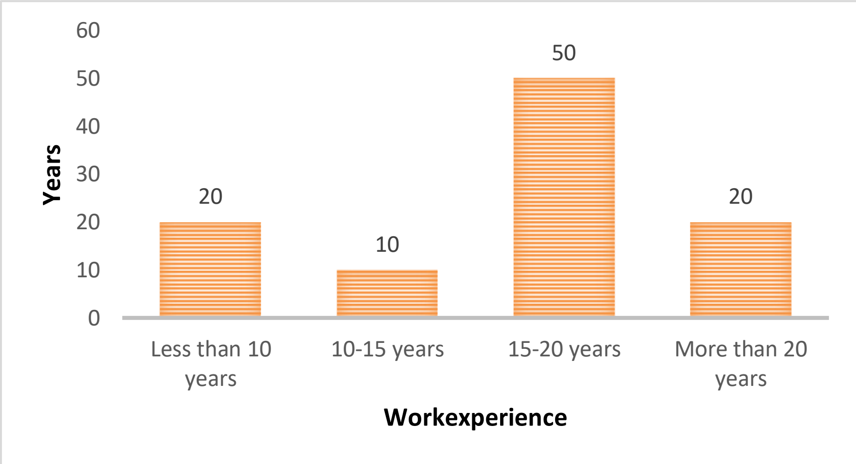

Fig. 4. Work experience of respondent experts.

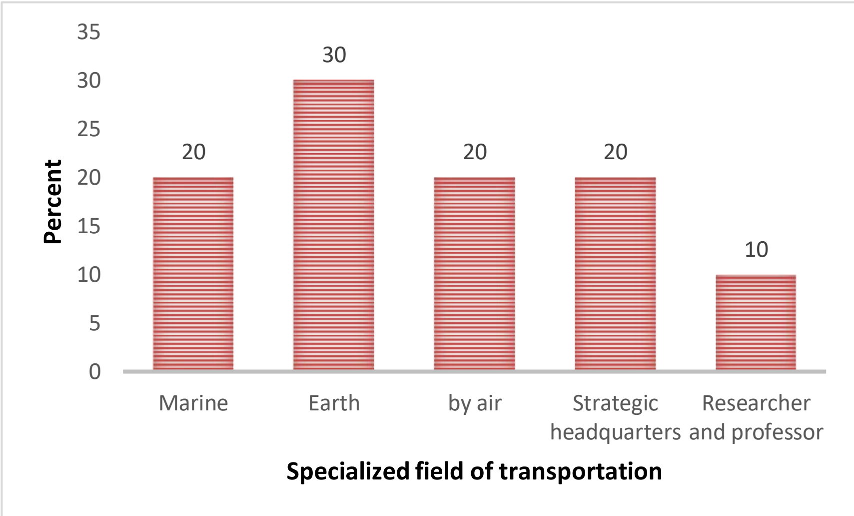

Fig. 5. Specialized field of transportation regarding the respondent experts.

According to interviews with 10 experts in the field of transportation in Iran, it has been shown that strategies such as export marketing, sustainable marketing, smart marketing, political marketing and domestic marketing are effective in developing the transportation model of the Islamic Republic of Iran; each of which consists of some sub-criteria. Because the ranking of activities is considered for each factor in terms of importance and having a long-term plan for strengthening and implementing strategies, these strategies and sub-criteria of each will be ranked in the next section. According to these identified strategies, the model designed by the qualitative part of the research is shown in *Fig. 1*.

4 | Introducing the Code of Criteria and Sub-Criteria

In this section, based on the literature review and previous studies, as well as interviews with experts, 20 factors affecting the transportation development model of the Islamic Republic of Iran were identified and extracted in 5 dimensions, which are shown in *Table 1*. The hierarchical model of the research is shown in *Fig. 2*.

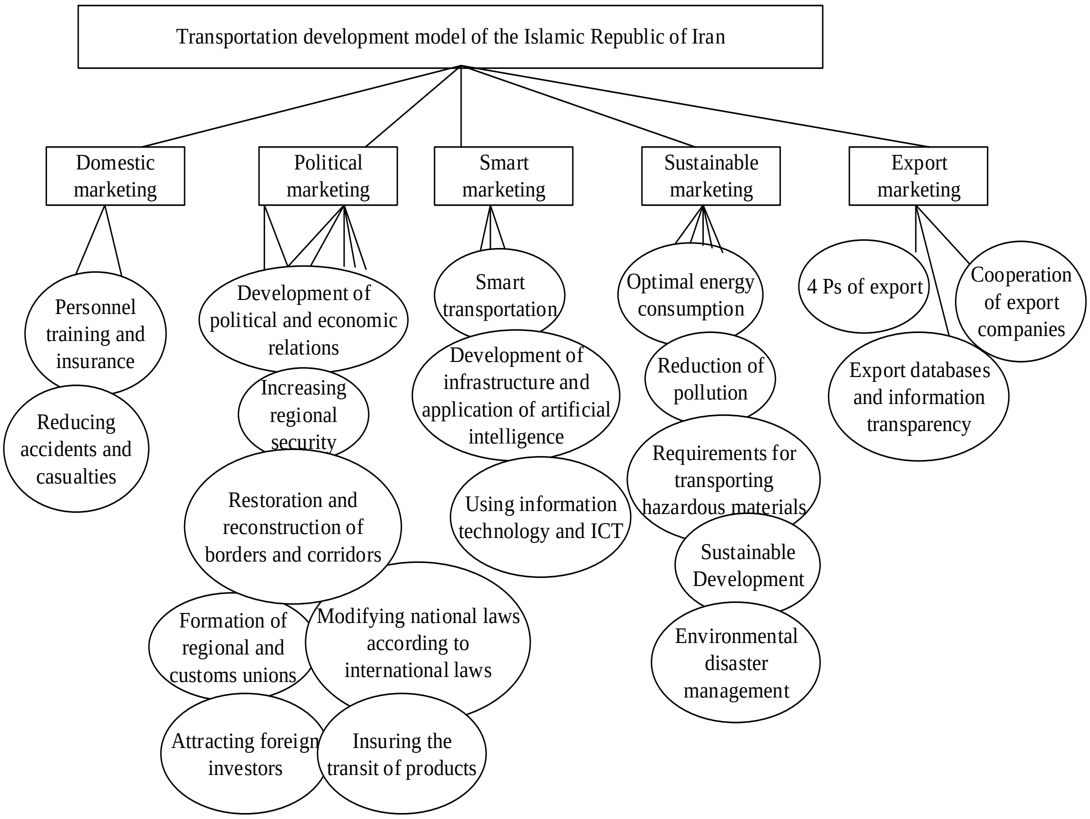

Fig. 1. Transportation development model of the Islamic Republic of Iran.

Table 1. Introduction of research factors.

Criterion	Criterion Code	Sub-Criterion	Sub-Criterion Code
Export marketing	C1	4 Ps of export	C11
		Export databases and information transparency	C12
		Cooperation of export companies	C13
Sustainable marketing	C2	Optimal energy consumption	C21
		Reduction of pollution	C22
		Requirements for transporting hazardous materials	C23
		Sustainable Development	C24
		Environmental disaster management	C25
Smart marketing	C3	Smart transportation	C31
		Development of infrastructure and application of artificial intelligence	C32
		Using information technology and ICT	C33
Political marketing	C4	Development of political and economic relations	C41
		Increasing regional security	C42
		Restoration and reconstruction of borders and corridors	C43
		Formation of regional and customs unions	C44
		Modifying national laws according to international laws	C45
		Attracting foreign investors	C46
		Insuring the transit of products	C47
Domestic marketing	C5	Personnel training and insurance	C51
		Reducing accidents and casualties	C52

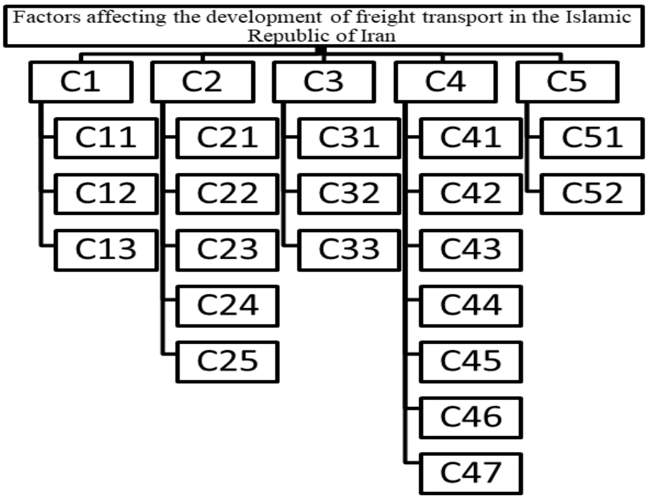

Fig. 2. Hierarchical model of the research (coded).

5 | AHP Results to Prioritize the Factors

In the previous steps, research factors were introduced. In this step, AHP method was used to determine their importance and weight. First, based on *Table 3*, pairwise comparisons of criteria and sub-criteria were created and provided to the experts. The experts in this section are the same 10 people in the previous step. After completing the pairwise comparison matrices, the inconsistency ratios of each were calculated, all of which were less than 0.1, indicating the stability and consistency of the matrices. Then the pairwise comparisons of the experts were integrated by the geometric mean method and then the Expert Choice software was used to determine the weight. The following are the results of pairwise comparisons and weights.

5.1 | Pairwise Comparison of the Main Criteria

Pairwise comparisons of 5 main criteria are given in *Table 2*. The inconsistency ratio of this pairwise comparison is 0.009 which indicates an acceptable consistency because it is less than 0.1.

Table 2. Pairwise comparisons of the main criteria.

	C1	C2	C3	C4	C5
C1		0.461	0.429	0.555	1.382
C2			1.484	2.101	4.072
C3				1.681	4.158
C4					2.698
C5					

The pairwise comparisons of *Table 3* are entered in the Expert Choice software, where the criteria weights are calculated and shown in *Fig. 3*.

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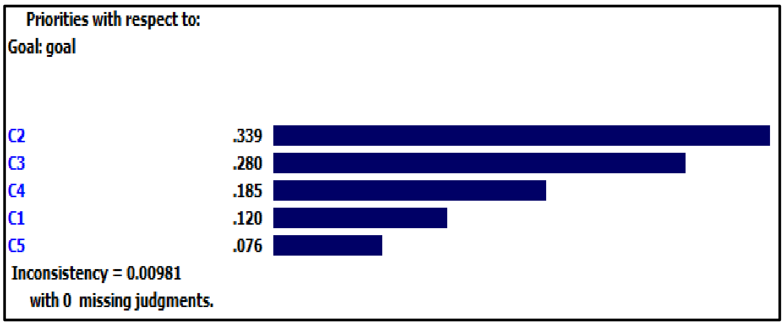

Fig. 3. Weights of the main criteria.

Table 3. Weight and rank of the main criteria.

Rank	Weight	Code	Criteria
1	0.339	C2	Sustainable marketing
2	0.280	C3	Smart marketing
3	0.185	C4	Political marketing
4	0.120	C1	Export marketing
5	0.076	C5	Domestic marketing

According to *Fig. 3*, among the main criteria, sustainable marketing with a weight of 0.339 is ranked first. Smart marketing with a weight of 0.280 is ranked second, political marketing with a weight of 0.185 is ranked third, export marketing with a weight of 0.120 is ranked fourth and domestic marketing with a weight of 0.076 is ranked fifth.

5.2 | Pair Comparison of Export Marketing Sub-Criteria

The export marketing criterion has 3 sub-criteria, the pairwise comparison of which is given in *Table 4*. The inconsistency ratio of this pairwise comparison is 0.05.

Table 4. Pairwise comparisons of export marketing sub-criteria.

	C11	C12	C13
C11		0.498	0.782
C12			0.766
C13			

The pairwise comparisons of *Table 5* are entered in the Expert Choice software, where the criteria weights are calculated and shown in *Fig. 4*.

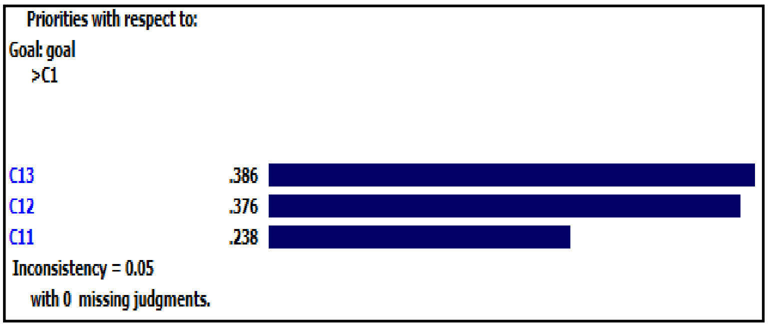

Fig. 4. Weights of export marketing sub-criteria.

Table 5. Weight and rank of export marketing sub-criteria.

Rank	Weight	Code	Sub-Criteria
1	0.386	C13	Cooperation of export companies
2	0.376	C12	Export databases and information transparency
3	0.238	C11	4 Ps of export

Among the export marketing sub-criteria, Cooperation of Transportation Companies with Exporters with a weight of 0.386 has gained the first rank. Export Databases and Information Transparency with a weight of 0.376 and 4 Ps of Marketing with a weight of 0.238 have gained the second and third ranks, respectively.

5.3 | Pairwise Comparison of Sustainable Marketing Sub-Criteria

Sustainable marketing has 5 sub-criteria, the pairwise comparison of which is given in *Table 6*. The inconsistency ratio of this pairwise comparison is 0.02.

Table 6. Pairwise comparisons of sustainable marketing sub-criteria.

	C21	C22	C23	C24	C25
C21		0.384	0.457	0.345	0.466
C22			2.502	1.070	1.069
C23				0.422	0.833
C24					2.092
C25					

The pairwise comparisons of *Table 6* are entered in the Expert Choice software, where the criteria weights are calculated and shown in *Fig. 5*.

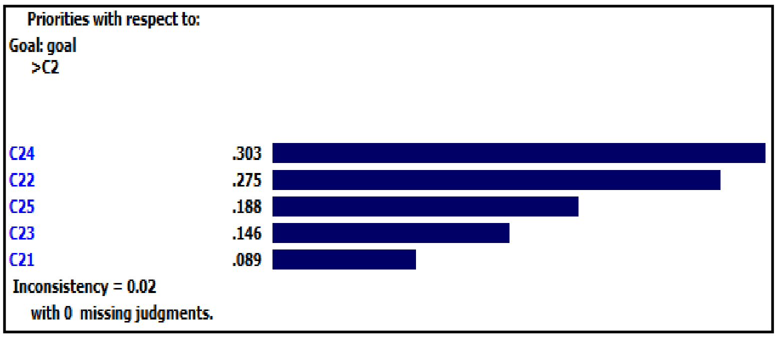

Fig. 5. Weights of sustainable marketing sub-criteria.

Table 7. Weight and rank of sustainable marketing sub-criteria.

Rank	Weight	Code	Sub-Criteria
1	0.303	C24	Sustainable Development
2	0.275	C22	Reduction of pollution
3	0.188	C25	Environmental disaster management
4	0.146	C23	Requirements for transporting hazardous materials
5	0.089	C21	Optimal energy consumption

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Among the sustainable marketing sub-criteria, Sustainable Development with the weight of 0.303 has obtained the first rank. Reduction of Pollution with a weight of 0.275 is ranked second and Environmental Disaster Management with a weight of 0.188 is ranked third. Requirements for Transporting Hazardous Materials and Optimal Energy Consumption with a weight of 0.146 and 0.089 are ranked fourth and fifth, respectively.

5.4 | Pairwise Comparison of Smart Marketing Sub-Criteria

The smart marketing criterion has 3 sub-criteria, the pairwise comparison of which is given in *Table 8*. The inconsistency ratio of this pairwise comparison is 0.0006.

Table 8. Pairwise comparisons of smart marketing sub-criteria.

	C31	**C32**	**C33**
C31		1.523	0.821
C32			0.582
C33			

The pairwise comparisons of *Table 9* are entered in the Expert choice software, where the criteria weights are calculated and shown in *Fig. 6*.

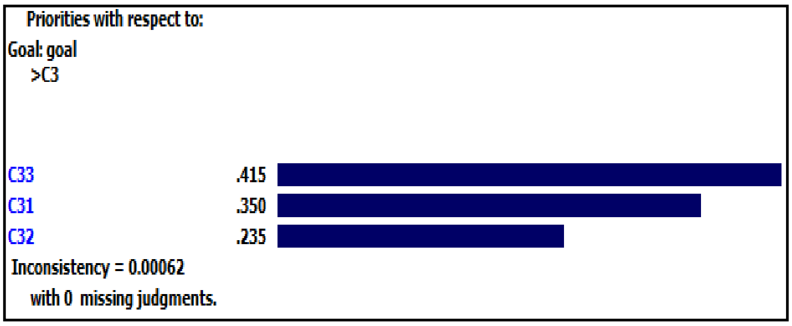

Fig. 6. Weights of smart marketing sub-criteria.

Table 9. Weight and rank of smart marketing sub-criteria.

Rank	Weight	Code	Sub-Criteria
1	0.415	C33	Using information technology and ICT
2	0.350	C31	Smart transportation
3	0.235	C32	Development of infrastructure and application of artificial intelligence

Among the smart marketing sub-criteria, Using Information Technology and ICT with a weight of 0.415 is ranked first. Smart transportation with a weight of 0.350 is ranked second and Development of Infrastructure and Application of Artificial Intelligence with a weight of 0.235 is ranked third.

5.5 | Pair Comparison of Internal Marketing Criteria

The internal marketing criterion has two sub-criteria, the pairwise comparison of which is given in *Table 10*. The incompatibility rate of this pairwise comparison is 0.000.

Table 10. Parallel comparisons of internal marketing criteria.

	C51	C52
C51		0.466
C52		

We enter the pairwise comparisons of *Table 11* in the Expert choice software, where the criteria weights are calculated and shown in *Fig. 7*.

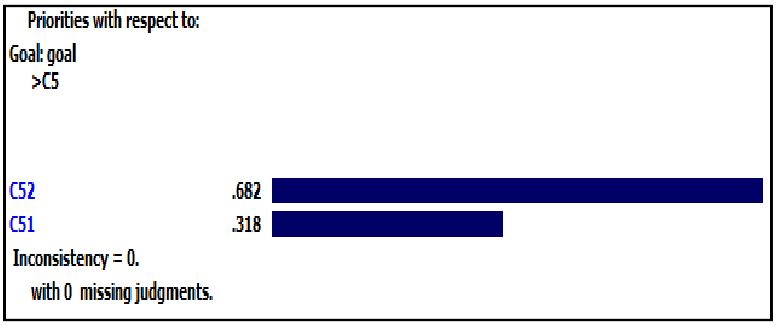

Fig. 7. Weights of internal marketing criteria.

Table 11. Weight and rank of internal marketing criteria.

Criteria	Code	Weight	Rank
Insurance and training of national and international personnel	C52	0.682	1
Reduce accidents and casualties	C51	0.318	2

According to *Fig. 7*, among the domestic marketing sub-criteria, insurance and training of national and international personnel with a weight of 0.682 is ranked first and the reduction of accidents and casualties with a weight of 0.318 is ranked second.

5.6 | Final Weights of Sub-Criteria

The final weight of the sub-criteria is obtained by multiplying the weight of each dimension by the weight of the criterion and then multiplying by the relative weight of the sub-criteria, which is done by Expert Choice software and is shown in *Fig. 8*. So, among the 20 sub-criteria, (Using Information Technology and ICT) (C33) is ranked first. After that, Sustainable Development (C24) and Smart Transportation (C31) are ranked second and third, respectively.

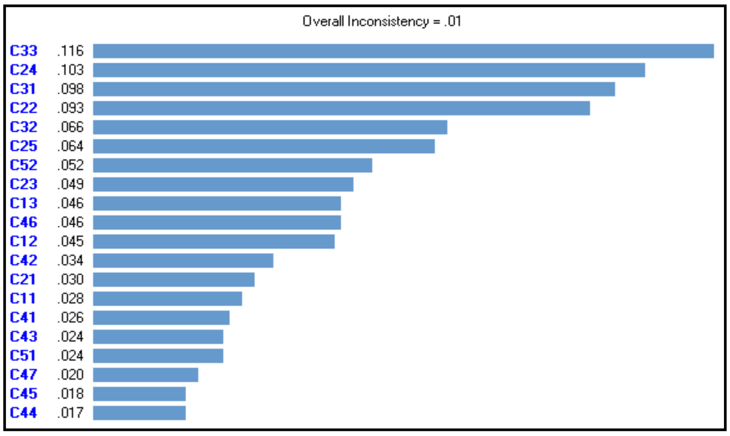

Fig. 8. Final weight and rank of sub-criteria.

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Table 12. Final weight and rank of sub-criteria.

Rank	Final Weight	Sub-Criteria	Sub-Criteria Code
1	0.116	Using information technology and ICT.	C33
2	0.103	Sustainable Development.	C24
3	0.98	Smart transportation.	C31
4	0.93	Reduction of pollution.	C22
5	0.66	Development of infrastructure and application of artificial intelligence.	C32
6	0.64	Environmental disaster management.	C25
7	0.52	Reducing accidents and casualties.	C52
8	0.49	Requirements for transporting hazardous materials.	C23
9	0.46	Cooperation of export companies.	C13
10	0.46	Attracting foreign investors.	C46
11	0.45	Export databases and information transparency.	C12
12	0.34	Increasing regional security.	C42
13	0.30	Optimal energy consumption.	C21
14	0.28	4 Ps of export.	C11
15	0.26	Development of political and economic relations.	C41
16	0.24	Restoration and reconstruction of borders and corridors.	C43
17	0.24	Personnel training and insurance.	C51
18	0.20	Insuring the transit of products.	C47
19	0.18	Modifying national laws according to international laws.	C45
20	0.17	Formation of regional and customs unions.	C44

6 | Conclusion

Regarding the identified factors, due to the special position of product transit in Iran and using interviews with experts, 5 main criteria and 20 sub-criteria that were subsets of the main criteria were identified. There was a doubt about the position of some sub-criteria, which was finalized after the consensus of experts. The main and sub-criteria are as shown in *Table 11*, respectively:

Table 13. The result of the qualitative part of grounded theory.

Sub-Criteria	Main Criterion
4 Ps of export Export databases and information transparency Cooperation of export companies	Export marketing
Optimal energy consumption Reduction of pollution Requirements for transporting hazardous materials Sustainable Development Environmental disaster management	Sustainable marketing
Smart transportation Development of infrastructure and application of artificial intelligence Using information technology and ICT	Smart marketing
Development of political and economic relations Increasing regional security Restoration and reconstruction of borders and corridors Formation of regional and customs unions Modifying national laws according to international laws Attracting foreign investors Insuring the transit of products	Political marketing
Personnel training and insurance Reducing accidents and casualties	Domestic marketing

Given that the subject of transportation is a specific and macro issue, investigating it from a marketing perspective is a novel perspective, the studies conducted by students have remained at the research level, and there have not been many related studies on marketing in Iran, the most important limitation of the research was the ability to explain the reason of conducting this research and controlling the interviews to prevent them from getting out of the main discussion and reaching a model. However in many cases, the words used by experts differed from those found in the literature, which were specific terms for experienced people. In recent interviews, attempts have been made to validate them based on marketing scientific terms. The most important factor influencing the development of transportation and transit of goods is sustainable marketing. Smart marketing and political marketing rank second and third, respectively. Therefore, managers and strategists active in the transit of goods should pay more attention to the issue of sustainability and, in a sense, energy costs and environmental issues. The use of information technology, intelligent systems and automation will certainly be effective because we are in the fourth industrial revolution and the more transparent and accessible information and intelligent systems; the more confident the owners of goods will transit their goods from Iran. This study, in addition to identifying the factors affecting the transportation development in the Islamic Republic of Iran, has prioritized the identified factors. In this section, all identified factors, regardless of the larger categorization, or all sub-criteria have been prioritized separately. This was done for several reasons. First, there are limited resources for any development, so strategy plans, in addition to comprehensively investigation of all the factors and strategies, should allocate most of their resources to the most effective marketing strategy. Also, this study is one of the first studies that have investigated the transportation development in the Islamic Republic from a marketing perspective, therefore, the researcher preferred to provide comprehensive and integrated results to make it more comprehensive and practical.

According to *Table 14*, it can be said that the most important marketing strategies for the development of transportation in the Islamic Republic of Iran are: 1) using information technology and ICT, 2) sustainable development, 3) smart transportation, 4) reduction of pollution, 5) development of infrastructure and application of artificial intelligence, 6) environmental disaster management, 7) reducing accidents and casualties, 8) requirements for transporting hazardous materials, 9) cooperation of transport companies with exporters, 10) attracting foreign investors, 11) export databases and information transparency, 12) increasing regional security, 13) optimal energy consumption, 14) 4 Ps of export, 15) development of political and economic relations, 16) restoration and reconstruction of borders and corridors, 17) insurance and training of national and international personnel, 18) insuring the transit of products, 19) modifying national laws according to international laws, 20) formation of regional and customs unions.

Table 14. Prioritization of marketing strategies for the transportation development in the Islamic Republic of Iran.

Rank	Final Weight	Sub-Criteria	Sub-Criteria Code
1	0.116	Using information technology and ICT	C33
2	0.103	Sustainable Development	C24
3	0.98	Smart transportation	C31
4	0.93	Reduction of pollution	C22
5	0.66	Development of infrastructure and application of artificial intelligence	C32
6	0.64	Environmental disaster management	C25
7	0.52	Reducing accidents and casualties	C52
8	0.49	Requirements for transporting hazardous materials	C23
9	0.46	Cooperation of export companies	C13
10	0.46	Attracting foreign investors	C46
11	0.45	Export databases and information transparency	C12
12	0.34	Increasing regional security	C42
13	0.30	Optimal energy consumption	C21
14	0.28	4 Ps of export	C11
15	0.26	Development of political and economic relations	C41
16	0.24	Restoration and reconstruction of borders and corridors	C43
17	0.24	Personnel training and insurance	C51
18	0.20	Insuring the transit of products	C47
19	0.18	Modifying national laws according to international laws	C45
20	0.17	Formation of regional and customs unions	C44

According to the results of the research, some suggestions are made in this field:

- *Resources and costs must be identified before any strategy can be developed. Although the development of Iran's transportation and product transit is one of the important goals in Iran's development plans. It is suggested that first of all, a suitable model would be used to supply fuel for transportation in a way that does not cause pollution.*
- *It is suggested that, like many other industries and based on the experience of many other countries, the presence of information technology in various forms be welcomed in the transportation industry and the required infrastructure be created for it. Iran is in a good position in the field of information technology in the region, but updating measures need to be taken.*
- *The important issue observed in this study is that the development of political and economic relations in the field of political marketing has an effect on improving transportation conditions. This is important, both regionally and globally. The Middle East is very insecure, and Iran, as a regional power, must try to ensure security. The result will improve transportation development.*
- *Considering that some hazardous materials should be transported from Iran through roads or by ships, special measures should be taken in this regard. Although it is effective in the development of transportation, but regarding the sustainable development, it should not be done in a way that threatens the environment.*

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Appendix A

Table A. Demographic characteristics of experts.

Percentage	Abundance	Concept
Gender		
90%	9	Man
10%	1	Female
Age		
10%	1	30 to 40
60%	6	41 to 50
30%	3	51 to 60
Education		
20%	2	Masters
50%	5	Masters
39%	3	PhD and above
History of activity in the field of transportation		
20%	2	Less than 10 years
10%	1	10 -15 years
50%	5	15-20 years
20%	2	More than 20 years
Specialized field of transportation		
20%	2	Maritime Transportation
30%	3	Ground transportation
20%	2	Air transport
20%	2	Strategic Transportation Headquarters
10%	1	Researcher or professor

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Identifying and Examining the Internal Relationships Between Accounting Management Factors in Imam Khomeini University of Marine Sciences

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Abstract

Accounting action believes that people, like economic traders, seek to make the maximum profit in their behavior in different contexts. These people are very calculating and seek to invest in friendly relationships that have made the target people indebted to them. To be used in times of need of this capital and to provide a platform for legal and illegal requests in the future.The purpose of this study is to identify the factors affecting accounting action of employees in organization, for this purpose, 29 Latin articles and 78 Persian articles that were selected by snowball method were used after theoretical saturation, and textual content analysis was performed then, the obtained data were provided to 14 experts by a questionnaire and after three rounds of qualitative Delphi, 27 main effective criteria were identified and then, using network analysis technique and Dematel, internal relations and prioritization were examined. The results based on network analysis method showed that utilitarianism was the highest factor and competitive advantage factor was the last factor affecting accounting action. Also, based on Dematel's method, we came to the conclusion that the factor of opportunistic behavior was the most effective and the factor of selfish individualism was the most effective among the factors.

Keywords: Accounting actions, Employees-Organization.

1 | Introduction

Understanding organizational behavior has never been more important to managers than it is today. Nowadays, organizational behaviors are a systematic study of personal and intra-personal perceptions and behaviors of organizational employees and describe the relationships between the factors that shape behavior to find cause-and-effect relationships and solve organizational behavioral problems. And its main goals are to predict, explain and control the behaviors that occur in the organization [13]. The Career provides an opportunity for people to take part in their work activities and bring perspectives to do diverse life goals, so people put a lot of energy into their jobs and careers and save their time and energy. They spend pursuing their work ideals [24]. In other words; organizations are made up of people with personal plans that act as if to gain power and influence over others. This agenda or game is called the world of organizational politics. People's actions to achieve their goals

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also cause political behaviors in the organization. Political behavior, like other aspects of organizational dynamics, is not a simple process and varies from organization to organization and even from unit to unit. On the other hand, because power plays a major role in the interactions and relationships of managers during organizational life, its impact on organizational actions cannot be easily ignored. Power in its most naked form is the use of violence, wealth and wisdom to force people to do something special.

Government agencies as governmental and political institutions with the change of governments, see extensive changes in their management body and some of these elected managers sometimes lack the necessary expertise. Existence of these managers who mostly do not have the necessary qualifications to hold managerial positions, causes political behaviors such as organizational hypocrisy, underwater, flattery, gossip, confectionery and the like this leads to unrest in the organization this evolution is progressing to the point that the ethics of these managers has become a reference value system and others must follow this pattern of behavior. As a result, managers; they move government agencies to troubled organizations; it causes job instability, low motivation (prevalence of depression and depression), poor participation, high differences, low professionalism, double organizational culture, lack of respect for high human values such as self-esteem and honesty, high corruption and lawlessness [11].

Moral evils in any set show the unhealthy relationships and norms that govern that organization. Such an atmosphere is formed in the organization when the statuses and roles and in fact meritocracy, do not exist in the true sense. If each person is in a position worthy of them and enjoys a proper social role in accordance with his or her true status, relationships with social relationships will also be sufficiently healthy [12]. With these interpretations, we encounter the concept of action and accountability in organizations by individuals. It is a fundamental concept in philosophy and social sciences such as sociology, psychology and economics [6]. And in its most basic form, it is the behavior that an actor performs and is based on intention [34]. On the other hand, the term accounting, which is rooted in the principles of economics, believes that individuals, like economic traders, seek to make the most profit in their behavior in different contexts. The crystallization of this theory in the realm of criminal law gives rise to the idea that criminal decisions are the end product of the thinking of potential offenders. Hence, criminals turn to criminal phenomena with cost-benefit analysis and greatest profit at the lowest cost. In this approach, criminals as economic actors, by organizing their decisions, seek to do the greatest pleasure of committing a crime [10]. Individuals at different organizational levels work accountable and rationally and are associated with the goals of the organization to the extent that it is accompanied by their individual goals [7]. Most of our activities today are done by connecting with others and using symbols and signs to set up these connections, we convey our intentions. In modern societies, the word communication is very important because it is used to describe errors both in society and in the organization. In addition to being a personal process, communication is a means by which one person can penetrate another and show behavior that can be influenced by these communication exchanges [25]. People in organizations have a strong want to be loved and seen as positive [29]. Enjoying a suitable job provides economic needs, income and a sense of satisfaction, which in turn has many educational, psychological, social and spiritual consequences. In recent decades, employees have shown a great want to use career management strategies for their professional development and success [1].

The purpose of this study is to identify and prioritize the factors affecting accounting practice today's employees think and worry more about their work life than ever before they want a secure, long-term and satisfying job and they want to grow and develop in their job in other words, if employees feel that their job activities are the closest way to achieving their goals and have a satisfying idea of working in this case, they will definitely be more inclined to work, and the opposite is also true. Organizations, as living organisms, take the materials and energy needed for survival from the environment and reciprocally provide services or products to the environment. Therefore, if the organization is sick, it leads to the illness of members and society. Having a healthy society is unthinkable without having a healthy organization .Therefore, we decided to first identify the factors that affect the accounting action of employees in organizations and finally prioritize in order of importance. Organizations are created based on social needs and to meet them, therefore, they play an important role in the survival and evolution of society each

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organization fulfills general or specific goals for society and people by performing multiple or specialized tasks; In fact, organizations play an important role in the growth and development of today's societies and human beings. Therefore, achieving the goals of organizations efficiently and effectively is always a necessity for the well-being of individuals and society as a whole. In this way, employees of organizations take on key responsibilities to achieve organizational goals. In a competitive and rapidly changing world, the success of organizations depends on the performance and attitude of employees. The more coordinated and prepared the employees of the organization are with the behavior of the managers, the better the managers will act in the face of environmental currents. Today, a very important part of any organization is the human resources of that organization. Therefore, knowing all the dimensions related to human capital can be very important. Therefore, in this study, relying on recognizing the behavioral dimension of employees, we addressed a new issue in this field that had not been addressed before. Because we see that in some social interactions, people in different situations start to do completely extreme things to people who are likely to take a position in the organization in the not too distant future; such behaviors can be considered and analyzed. Considering these accounting behaviors, it seems that operating people are looking to invest in friendly relationships in order to influence their target audience. And to accommodate these factors for their own legal or illegal requests in the future [17]. Sometimes people who are not flatterers face serious obstacles in their progress in the organization. People who just want to do their job fail to compete with someone who combines their work with their ability to flatter; therefore, some employees try to behave in a completely calculated way to achieve the goals they have in mind to achieve a better future. This issue leads to the emergence of people who do not have the necessary qualifications to take many organizational positions and control the managers of the organization only with calculating behaviors, which unfortunately cause cultural and moral pollution in the workplace. It becomes and it will be the result of the organization that things fall into the hands of inefficient people, which will lead to a decrease in productivity and dual governance, and eventually the collapse of the organization. Therefore, we decided to find out in this study: what are the factors affecting accounting action? How are the criteria of accounting action prioritized? In this way, by identifying these factors in any organization, we try to prevent such actions from occurring, either by the manager or by the employees, so that we finally have an organization with a strong and growing foundation.

2 | Literature Review

Zahedi and Ghahremani Nahr [35] in an article entitled: modeling of the supply chain of cooperative game between two tiers of retailer and manufacturer under conditions of uncertainty, they concluded that: the results of the implementation of this contract in a numerical example showed that the profit of the whole chain and the amount of economically optimal order in the centralized state increased compared to the decentralized state and the optimal price of the product decreased. Due to the fact that in the decentralized state the retailer determines the values of the optimal variables, the profit of this member decreases in the centralized state and the producer's profit increases.

Amiri and Moghadam [5] in an article entitled: book review democracy; introduction to general selection general selection (court selection) following the application of theories and methods of economic knowledge for analysis of political behavior, this field of economics knowledge developed a lot after that and theoretical and experimental studies numerous mechanisms and political issues, including elections, the constitution, legislators, brokers, rent-seeking, like-minded groups, bureaucracy, dictatorship, size and government takeover, and political-business cycles.

Javan Jafari Bojnourdi and Farhadi Alashti [16], in an article entitled:A reflection on the philosophers' reading of the benefits of preventing the death penalty, they concluded that: the epistemological school of usefulness originality with emphasis on the principle of association of meanings, seeks to increase suffering and reduce benefits possibly due to critical attitude to the usefulness of the death penalty, to the amendment of criminal laws with the death penalty based on the originality of the benefit, the efficiency of the death penalty, increasing its deterrent effect and achieving the main goal of the school originality of benefit, that is, social benefit, will help.

Soltanifar and Heidariyeh [31], in an article entitled: employee performance evaluation using a new preferential voting process, they concluded that twelve performance evaluation indicators extracted from the research background by experts in four categories with the titles of service compensation system in the first category, job security ,perceived organizational support , and physical working conditions in the second category, job characteristics, job clarity and ability of employees in the third category and job feedback, job stress, organizational citizen behavior and motivation in the fourth category were ranked and documented using a linear optimization model based on the goal programming and policy. Data envelopment analysis resulted in a complete ranking for the staff of a university unit with 25 staff members.

Seyed Nezhad Fahim et al. [30], in an article entitled: investigating the effect of investor pressure, on dividend policy, they concluded that: dividend payment decisions are one of the key components in corporate policy and one of the most important issues in the financial literature, on the one hand, it affects the investments of companies. On the other hand, many shareholders with short-term investment horizons want to distribute cash dividends, so managers must always balance between their various interests and profitable investment opportunities in order to maximize wealth.

Darvishi Selokolayi and Heydari Gorji [9], in an article entitled: a new approach to the economic problem of dumping based on game theory with grey parameters, they concluded that: game theory is one of the advanced scientific phenomena that has provided a solid framework for strategic behaviors. Whereas a lot of data is needed to solve business problems and issues, and given that real-world data usually involves uncertainty; therefore, using definite methods in a real inaccurate environment will not be suitable for optimal decision making. Therefore, this article intends to study the issues of dumping and anti-dumping in the trade relations of countries with the help of game theory in the environment of gray uncertainty.

Ali [8], in an article entitled: implementing Six Sigma DMAIC methodology for increasing the competitiveness of SMEs in Ethiopia, they concluded that: Six Sigma has gained wide acceptance as an improvement methodology to enhance the organization competitiveness in market. For SMEs building a competitive advantage is a difficult task. Changes in the economic environment affect the way such entities perceive factors which could help them not only survive on the market but shape their competitiveness. In the period of significant economic turbulence, the factors that play a major role in shaping the competitive market position are company image (product brand) and lower product price. This study uses Six Sigma DMAIC (define, measure, analyze, improve, and control) approach as a framework to identify, quantify, and eliminate sources of variation in meseret gabi machinery metal works enterprise in dessie (Ethiopia). This helps to improve the competitiveness of the enterprise in the market place by addressing the complaints and requirements of the customer continuously.

Rajab Dorri [23], in an article entitled: the pattern of the relationship between ethical philosophies and information technology with the ethical behavior of auditors: Actors Network Theory (ANT) approach. They concluded that: ethics is a fundamental issue in the accounting and auditing profession and the application of IT-related approaches to the financial sciences is undeniable; therefore, the purpose of this study is to prepare a model and systematize the relationship between ethical philosophies and information technology with the ethical behavior of auditors using the ANT. Findings showed that ethical behavior in auditors is influenced by human and technical actors. Human actors generally include moral theories and technical actors in general include information technology. Ethical theories (human actors) are also divided into two general parts: action-based theories and virtue-based theories (virtuosity). Action-based theories also include duty-based (duty-oriented), outcome-based (utilitarianism and others), divine-based (benevolence), and justice-based (justice-oriented). Information technology (technical actors) also includes the integrated theory of acceptance and use of advanced technology (motivation, hope for effort, and social impact). The overall findings of the study showed that the components of technical and human actors have a positive and significant effect on the development of ethical behavior in auditors.

3 | Method

The main purpose of this study is to prioritize the factors affecting the accounting action. The present study is in the field of applied research in terms of purpose. And on the other hand; considering that in this research, library study methods as well as field methods such as questionnaires and interviews are used, it can be stated that the present research is a descriptive-survey research based on its nature and method. In this research, a mixed (quantitative-qualitative) method has been used to analyze the data.

The choice of research methodology is one of the most important and key steps that the researcher should pay special attention to. The research process in two qualitative and quantitative phases of the present research is as follows:

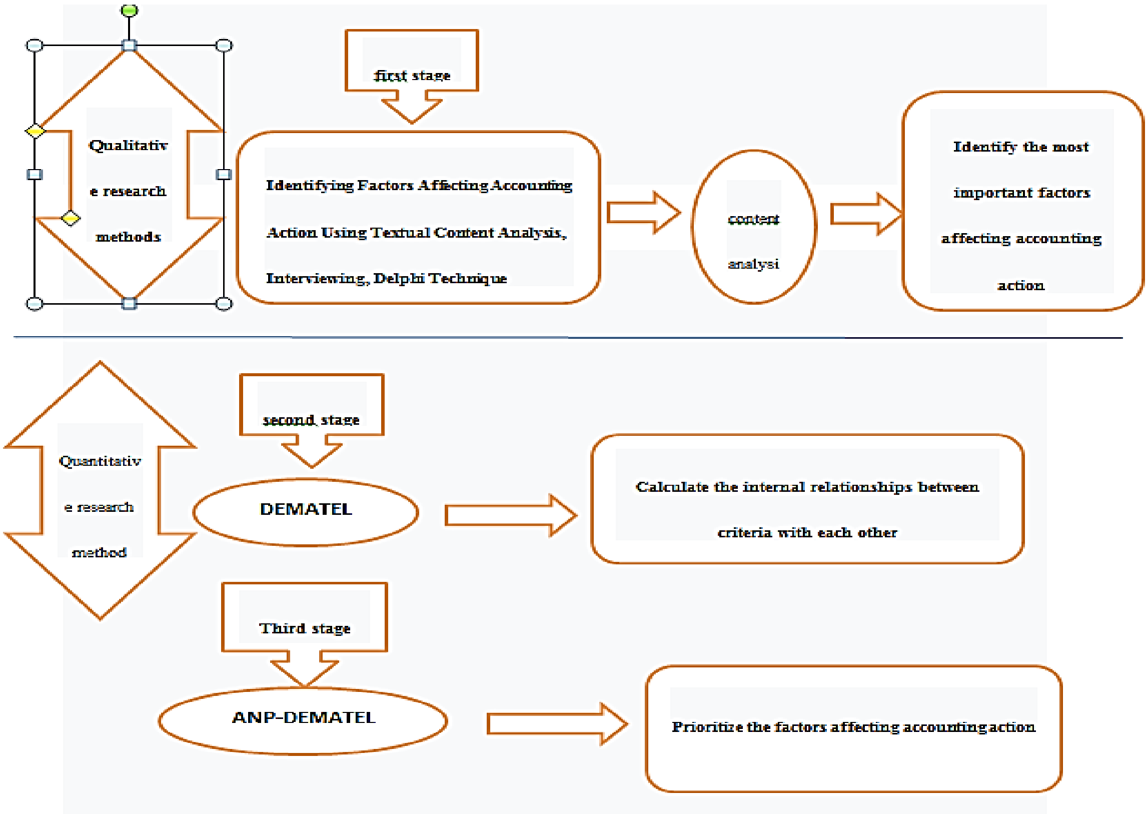

Fig. 1. Research process in two phases, qualitative and quantitative.

The sampling method in the qualitative phase is snowball. The statistical population in this section is upstream documents and records and articles and dissertations related to this field. Finally, after extracting the factors affecting the accounting action from the opinions of experts in order to conduct an interview to identify these factors as the final indicators of the test Qualitative Delphi was used. The statistical population in the quantitative phase is related to the part of multi-criteria decision analysis. In the present study, the snowball technique is used to calculate the number of experts, so that according to the purpose of the research, the snowball sampling or chain sampling method has been used for in-depth interviews. Therefore, after conducting 14 interviews with experts, at the discretion of the researcher and supervisors, the collected data reached saturation point and there was no need to conduct new interviews. Data collection methods in this study are divided into two categories: library and field. Regarding the collection of information related to the literature on the subject and research background, library methods (including upstream documents, articles and dissertations) have been used, and the field method has been used to collect information to review the research objectives. In this study, a questionnaire was used to collect research data.

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In the qualitative part of the present study, the method of textual content analysis and coding has been used to extract the indicators. Then the qualitative Delphi method was used to conclude and extract the ideas and opinions obtained from the collective agreement of the experts after eight steps. Dematel and Analysis Network Process (ANP) techniques were used in a small part the Dematel technique has two main functions. 1- Considering mutual relations; the advantage of this method over network analysis technique is its clarity and transparency in reflecting the interrelationships between wide ranges of components so that experts are able to express their views on the effects (direction and intensity of effects) among the factors with more mastery it should be noted that the matrix obtained from the Dematel technique (internal communication matrix), in fact, forms part of the super matrix in other words, the Dematel technique does not operate independently but as a subsystem of a larger system such as ANP. 2- Structuring complex factors in the form of cause and effect groups this case is one of the most important functions and one of the most important reasons for its widespread use in problem solving processes. By dividing a wide range of complex factors into causal groups, it puts the decision maker in a more appropriate position to understand the relationship. This issue leads to a better understanding of the position of factors and their role in the process of interaction. ANP technique: in decision science where indexing is a priority, multi-criteria or Multi-Criteria Decision Making (MCDM) decision-making methods have been in place for several years. In such decisions, several indicators or goals that are sometimes contradictory are considered if in MCDM it means index criterion, it is known as MCDM. And if multiple criteria are intended, it is called Multiple Objective Decision Making (MODM). After determining the most important criteria of the subject under study and determining their internal relations using the Dimtel technique, each of the identified criteria was prioritized. In order to prioritize the criteria, the ANP technique has been used. In this research, the hourly pairwise comparison model has been used to design an expert questionnaire. Using this model, the relative importance of the criteria is estimated using numbers, which are the principles of ANP. In the algorithm used in the present study, the method of performing steps based on matrix operations is used, which is shown in the figure below.

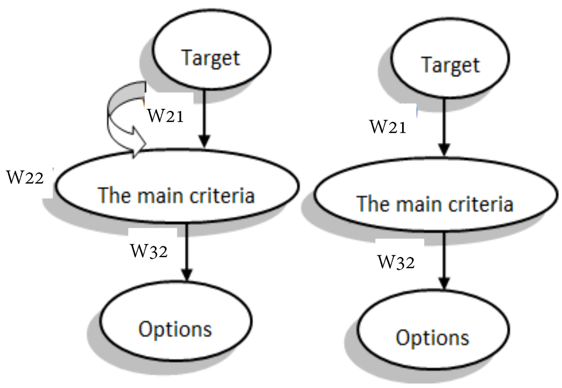

Fig. 2. Form of conceptual research model with AHP and ANP approach, source [36].

In this model, the W21 vector indicates the effect of the research goal on the main criteria. Similarly, the W32 vector indicates the effect of the main criteria on the sub-criteria. Therefore, the research model has a hierarchical relationship. If the internal relationships of the criteria are not considered, the Analytic Hierarchy Process (AHP) model is used. But if there is an interrelationship between the criteria, the model will become a network In this case, the ANP technique will be used the structure of the primary (asymmetric) super matrix of the ANP model will be as follows:

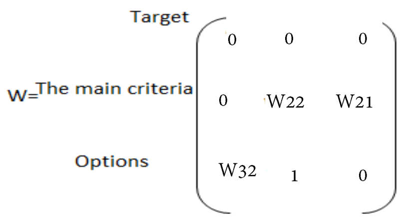

Fig. 3. Primary (unbalanced) structure of the super matrix [42].

3.1 | Introducing Indicators Related to Accounting Practice

In the first part of the present study, the method of textual content analysis and coding from 29 Latin and 78 Persian articles has been used to extract the indicators. Then, Delphi technique was used to "identify" and "sieve" the most important decision-making indicators. In this study, the selected group was university professors. These individuals were selected using a chain sampling approach. There are 14 of them, who were identified and selected in consultation with knowledgeable experts. In selecting experts, it was trying to select people who were frequently recommended by informed people, so the formed group has a high degree of credibility.

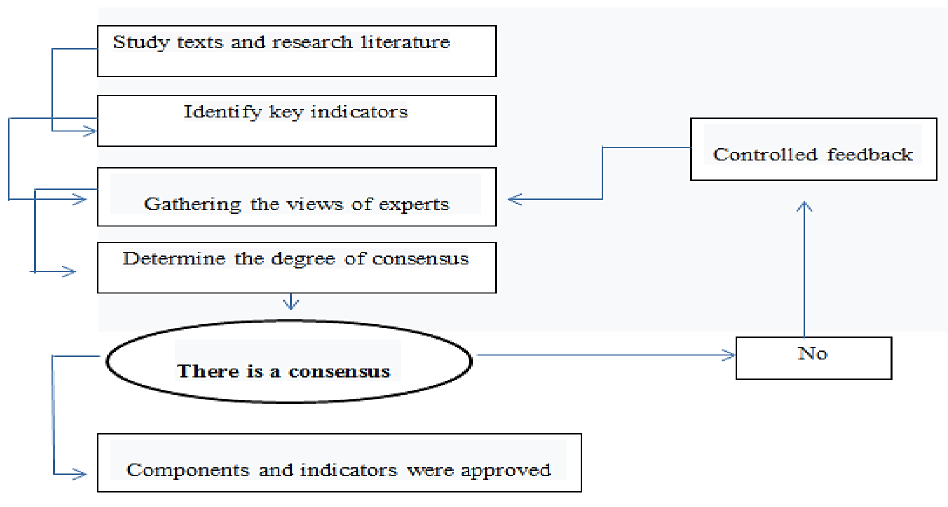

Fig. 4. Confirmation steps for identified components.

Delphi technique has been used to confirm the key indicators and components and the research questionnaire. The Delphi technique can be used to "identify" and "sift" the most important decision-making indicators so although the Delphi technique is not a multi-criteria decision making method. But in many cases, before applying multi-criteria decision-making techniques, this technique is used to screen the indicators or reach an agreement on the importance of decision-making indicators. Delphi technique is one of the methods of group knowledge acquisition that is also used in decision making on quality issues. The Delphi technique is a structured process for gathering information during successive rounds and ultimately group consensus. As mentioned before, first by conducting library studies such as Persian, Latin and Latin books, magazines and texts, the websites of key indicators of accounting practice were identified, then a questionnaire designed based on key research indicators was sent to research experts for a first round survey. After the experts' comments in this regard, the results obtained from all indicators were approved by the experts. Therefore, the questionnaire with the same combination of criteria, sub-criteria and indicators was ready to be sent to experts for the second time this process reached the consensus of research experts on indicators in two rounds of Delphi questionnaire distribution total of 200 indicators were obtained. In this research, the Delphi method has been used to collect, summarize, classify and describe quantitative data, so that by describing the research data, it is possible to use them quickly and better. First, the collected data were evaluated using the components of a summary and classification questionnaire and also according to the questions of a number of components according to the previous steps of identifying the most important indicators and considering three periods of averaging and standard deviation and also community-related components. The questionnaire was statistically analyzed. The Kendall W correlation test (first round Delphi) to calculate the reliability of the first round of Delphi is shown in the table below.

Table 1. Kendall W correlation test (first round Delphi).

Number	W.Kendall Correlation Coefficient	Chi-Square Index	Degrees of Freedom	Significance Index (Sig.)
14	0.309	240.514	199	0.024

As can be seen in the table above, all 196 components have a mean higher than the mean of 3, which indicates their acceptable importance. The value of 0.01 signifies indexes, which is less than the criterion of 0.05, confirms the correlation of the answers. Also a value higher than 0.3 for the Kendall W coefficient shows an acceptable agreement of the opinions on this test. The second rounds of the Delphi method considering that a number of respondents considered the existing variables in the field of the subject to be sufficient therefore, the same approved variables of the first stage are also questioned in the second stage. The second round Delphi questionnaire with all components was delivered to the same 14 people. As can be seen in the table, 196 of the variables have a mean higher than the mean of 3, which indicates their acceptable importance. According to the table below, the value of 0.000 significant indexes, which is less than the criterion of 0.05, confirms the correlation of the answers. Also, a value higher than 0.5 for the Kendall W coefficient shows an acceptable agreement of the opinions in this test.

Table 2. W. Kendall correlation test (Delphi round two).

Number	W.Kendall Correlation Coefficient	Chi-Square Index	Degrees of Freedom	Significance Index (Sig.)
10	0.346	407.393	199	0.000

On the other hand, the comparison of the mean scores of the variables in the first and second stages of Delphi was done. According to the information obtained, 196 variables of the questionnaire remain (the difference is less than 0.1 and has been summarized). Identifies the final approved components with an average greater than 3 Delphi third round. Similar steps were performed as previously announced.

Table 3. Kendall W correlation test (Delphi third round) number of items.

Number	W.Kendall Correlation Coefficient	Chi-Square Index	Degrees of Freedom	Significance Index (Sig.)
15	0.489	268.084	195	0.000

On the other hand, the comparison of the mean scores of the variables in the second and third stages of Delphi was also performed. Based on the results, the remaining 196 components were summarized in the third stage. Therefore, all components have been summarized and the third period is the end period and the final approved components with a difference of less than 0.1 have been summarized.

In the first step, the criteria and sub-criteria of the study were identified and selected. The main criteria of the study are: game theory, public choice, cost of benefit, utilitarianism, organizational behavior, accountants, theory of political behavior, opportunistic behavior, interactive behavior, social behavior, planned behavior, selfish individualism, machiavellianism, power, influence, self-sweetness, flattery, professional ethics, value system / value expectations, job motivation, rewards, progress, career promotion, job duties ,job security, competitive advantage, organizational justice and action. Sub-criteria have been identified for each of the main criteria. A total of 196 sub-criteria have been identified. By default, the internal relations between the main criteria of accounting action and to reflect the interrelationships between the criteria, the Dematel technique has been used. In the first step, we calculated the direct communication matrix (M). Because we had several experts in this research (14 experts), we used the simple arithmetic mean of the comments and formed the direct communication matrix or M. Then, in the second step, the normal direct correlation matrix was calculated: N = K * M: first, the sum of all rows and columns was calculated. The inverse formed the largest number of rows and columns k. Based on the results, the largest number is 84.784 and all values in the table were multiplied by the inverse of this number to obtain a normal matrix.

$$k = \frac{1}{\max \sum_{j=1}^{n} a_{ij}} = \frac{1}{84.784} = 0.012.$$

In the third step, the calculation of the complete correlation matrix was formed. To calculate the complete correlation matrix, first the same matrix (I) is formed, then the same matrix is minus the normalized matrix and the resulting matrix is inverted. Finally, the normal matrix is multiplied by the inverse matrix. Finally, we have a map of the network relationships. Threshold intensity must be calculated to determine the Network Relationship Map (NRM). In this way, partial relationships can be omitted and a network of significant relationships can be drawn only relationships whose values in the T matrix are greater than the threshold value will be displayed in the NRM. To calculate the threshold value of the relations, it is sufficient to calculate the average values of the T matrix. After the threshold intensity is determined, all values of the matrix T that are smaller than the threshold are zero, i.e. that causal relationship is not considered. In this study, the threshold intensity was 0.294.

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Table 4. Pattern of relationships.

C27	C26	C25	C24	C23	C22	C21	C20	C19	C18	C17	C16	C15	C14	C13	C12	C11	C10	C9	C8	C7	C6	C5	C4	C3	C2	C1	
0.333	0.313	0.312	0.318	0.312	0.303	0.328	0.300	0.306	0.307	0.333	0.311	0.314	0.326	0.337	0.357	0.348	0.318	0.319	0.332	0.294	0.315	0.336	0.305	0.294	*	*	**C1**
0.344	0.314	0.325	0.326	0.314	0.297	0.332	0.305	0.302	0.302	0.334	0.311	0.314	0.327	0.337	0.356	0.351	0.321	0.322	0.336	0.302	0.316	0.344	0.311	0.314	*	0.297	**C2**
0.334	0.315	0.313	0.314	0.309	*	0.335	0.294	0.305	0.298	0.335	0.313	0.297	0.327	0.336	0.355	0.351	0.320	0.323	0.336	9	0.309	0.342	0.301	*	*	*	**C3**
0.322	0.308	0.302	0.308	0.303	*	0.318	0.295	0.298	0.300	0.323	0.299	0.303	0.319	0.327	0.338	0.342	0.310	0.315	0.328	*	0.297	0.328	*	0.297	*	*	**C4**
0.333	0.313	0.302	0.312	0.304	0.297	0.327	0.301	0.307	0.305	0.330	0.304	0.302	0.316	0.324	0.347	0.346	0.321	0.321	0.331	0.299	0.309	*	0.300	*	*	*	**C5**
*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	0.269	*	*	*	*	**C6**
0.295	*	*	*	*	*	*	*	*	*	*	*	*	*	*	0.296	0.296	*	*	0.280	*	*	*	*	*	*	*	**C7**
0.355	0.333	0.329	0.323	0.321	0.311	0.348	0.318	0.321	0.319	0.348	0.326	0.325	0.338	0.352	0.371	0.366	0.336	0.338	*	0.315	0.330	0.350	0.318	0.325	*	0.300	**C8**
0.340	0.321	0.317	0.318	0.316	0.305	0.337	0.307	0.312	0.309	0.335	0.314	0.317	0.327	0.340	0.361	0.355	0.320	*	0.331	0.305	0.320	0.342	0.311	0.316	*	0.294	**C9**
0.334	0.312	0.308	0.311	0.305	0.295	0.330	0.300	0.301	0.301	0.328	0.306	0.307	0.319	0.333	0.351	0.345	*	0.314	0.321	0.297	0.308	0.331	0.303	0.308	*	*	**C10**
0.322	0.302	0.302	0.305	0.300	*	0.315	0.298	0.295	*	0.316	0.297	0.295	0.312	0.328	0.341	*	0.301	0.307	0.319	0.298	0.302	0.332	0.295	0.306	*	*	**C11**
0.318	0.294	0.295	0.300	*	*	0.319	*	*	*	0.311	*	*	0.308	0.321	*	0.333	0.299	0.300	0.313	*	0.297	0.324	*	0.301	*	*	**C12**
0.321	0.299	0.297	0.302	0.297	*	0.316	0.295	*	*	0.316	*	0.296	0.307	*	0.334	0.337	0.301	0.304	0.319	*	0.303	0.326	*	0.295	*	*	**C13**
0.323	0.298	*	0.296	*	*	0.309	*	*	*	0.310	0.292	*	*	0.314	0.336	0.329	0.295	0.298	0.314	*	0.304	0.322	0.296	0.297	*	*	**C14**
0.302	0.281	0.278	*	*	*	0.293	*	*	*	*	*	*	0.284	0.296	0.317	0.308	*	*	0.294	*	*	0.303	*	*	*	*	**C15**
0.329	0.308	0.310	0.307	0.307	0.299	0.324	0.304	0.302	0.296	0.325	*	0.302	0.319	0.332	0.346	0.344	0.310	0.313	0.326	0.298	0.310	0.336	0.303	0.304	*	*	**C16**
0.306	*	*	*	*	*	0.295	*	*	*	*	*	*	*	0.303	0.315	0.314	*	*	0.299	*	*	0.308	*	*	*	*	**C17**
0.285	*	*	*	*	*	*	*	*	*	*	*	*	*	*	0.295	0.297	*	*	*	*	*	*	*	*	*	*	**C18**
*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	0.296	*	*	*	*	*	*	*	*	*	*	*	**C19**
*	*	*	*	*	*	0.243	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	**C20**
*	*	*	*	*	*	*	*	*	*	*	*	*	*	0.295	0.309	0.300	*	*	*	*	*	0.295	*	*	*	*	**C21**
0.308	0.290	*	*	*	*	0.302	*	*	*	0.309	*	*	0.306	0.315	0.331	0.328	0.303	0.297	0.308	*	*	0.311	*	*	*	*	**C22**
*	*	0.269	*	*	0.277	0.259	*	*	0.279	*	*	0.275	*	0.305	0.295	*	*	*	*	*	*	*		*	*	*	**C23**
0.301	0.307	*	0.296	*	0.323	*	*	*	0.318	0.299	0.298	0.314	0.327	0.344	0.341	0.305	0.308	0.317	*	0.300	0.326	0.297		0.299	*	*	**C24**
0.305	*	0.310	0.303	*	0.323	0.297	0.295	0.302	0.328	0.300	0.305	0.319	0.335	0.349	0.343	0.305	0.306	0.318	*	0.308	0.332	0.299		0.313	*	*	**C25**
*	0.314	0.317	0.307	0.300	0.326	0.302	0.307	0.301	0.327	0.306	0.312	.323	0.331	0.347	0.354	0.320	0.322	0.335	0.310	0.320	0.332	0.313		0.314	*	*	**C26**
0.301	0.294	0.301	0.301	*	0.311	*	0.293	0.294	.315	0.296	0.303	0.313	0.319	0.334	0.336	0.315	0.313	0.323	0.298	0.302	0.317	0.303		0.298	*	*	**C27**

According to the pattern of relationships, causal relationships were identified as follows:

Table 5. Pattern of causal relationships of the main criteria.

Criterion	Criterion Symbol	D	R	D+R	D-R
Game theory	C1	8.487	7.230	15.717	1.257
General selection	C2	8.589	6.592	15.181	1.997
Originality of benefit / cost of benefit	C3	8.455	7.778	16.233	0.677
Utilitarianism	C4	8.259	7.693	15.953	0.566
organizational behavior	C5	8.391	8.478	16.868	0.087-
Accounting	C6	6.672	7.890	14.562	1.218-
Political Manner	C7	7.249	7.594	14.842	0.345-
Opportunistic behavior	C8	8.889	8.305	17.194	0.584
Interactive behavior	C9	8.614	8.006	16.620	0.608
social behavior	C10	8.391	7.995	16.386	0.396
Planned behavior	C11	8.186	8.777	16.962	0.591-
Selfish individualism	C12	8.059	8.875	16.934	0.815-
Machiavellianism	C13	8.117	8.423	16.540	0.305-
the power	C14	8.011	8.107	16.118	0.096-
infiltrate	C15	7.545	7.763	15.308	0.218-
Selfishness, flattery, flattery	C16	8.379	7.743	16.122	0.637
Ethics	C17	7.662	8.299	15.961	0.638-
Value system / value expectation	C18	7.180	7.597	14.778	0.417-
Job motivation	C19	6.956	7.646	14.602	0.690-
reward	C20	6.369	7.659	14.028	1.290-
Development	C21	7.305	8.274	15.579	0.970-
Promotion	C22	7.913	7.485	15.399	0.428
job responsibilities	C23	7.211	7.761	14.972	0.550-
Job security	C24	8.186	7.911	16.098	0.275
Competitive Advantage	C25	8.326	7.816	16.141	0.510
Organizational Justice	C26	8.500	7.915	16.415	0.585
Action	C27	8.180	8.471	16.651	0.291-

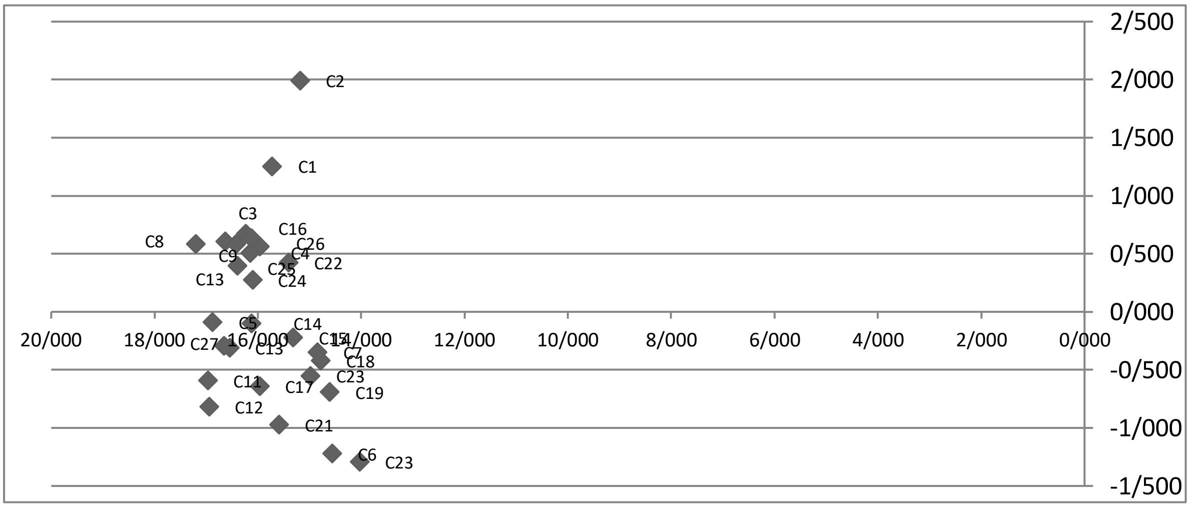

Fig. 4. Cartesian coordinate graph of the output of the Demeter method for criteria.

In *Table 5*; the sum of the elements of each line (D) indicates the extent to which that criterion affects the other criteria of computational action. Accordingly, the criterion of opportunistic behavior is more effective. The sum of the elements of the column (R) for each factor indicates the extent to which that factor is affected by other factors of computational action. Accordingly, the criterion of selfish individualism has a greater impact on accounting action. The horizontal vector (D + R) is the degree of influence and effect of the desired factor on the effectiveness of the accounting action. In other words, the higher the D + R factor, the more it interacts with other system factors. Accordingly, opportunistic behavior is more than other criteria. The vertical vector (D-R) indicates the effect of each factor. In general, if D - R is positive, the variable is a causal variable and if it is negative, it is considered a disability. IN this model, the criteria of game theory, general choice, originality of benefit/cost of benefit, utilitarianism/utilitarianism, opportunistic behavior, interactive behavior, social behavior, sweetness, flattery, flattery, job promotion, job security, competitive advantage, organizational justice causes and criteria of organizational behavior, accounting/accounting, political behavior, planned behavior, selfish individualism, machiavellianism, power, influence, professional ethics, value system/value expectation, motivation, job, reward, progress, job duties, disabled action Is.

3.2 | Network Analysis Process (ANP)

To perform the first analysis, the main criteria of goal-based accounting action were compared in pairs. For this purpose, the opinion of a group of experts has been used and using the technique of geometric mean and normalization of the obtained values, the special vector has been calculated.

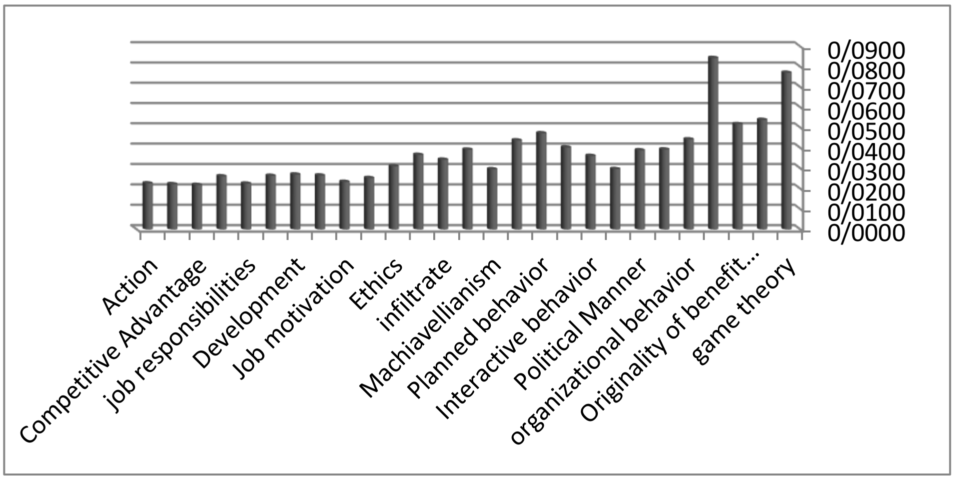

Fig. 5. Superdesizhen software output to prioritize key metrics.

The special priority vector of the main criteria will be W21.

Based on the special vector obtained: the criterion of "utilitarianism / profitability" with a normalized weight of 0.0843 in the first priority, the criterion of "opportunistic behavior" with a normalized weight of 0.0296 in the sixteenth priority and the criterion of "competitive advantage" with a normalized weight of 0.0219 in The last priority is the incompatibility rate of the comparisons performed is 0.097 which is less than 0.1 and therefore the comparisons performed can be trusted.

$$\begin{bmatrix} 0.0771, 0.0538, 0.0517, 0.0843, 0.0442, 0.03924, 0.0388, 0.0296, 0.0361, 0.0402, \\ 0.0472, 0.0437, 0.0795, 0.03915, 0.0342, 0.0366, 0.0310, 0.0252, 0.0233, 0.0265, \\ 0.0270, 0.0263, 0.0225, 0.0261, 0.0219, 0.0222, 0.0226 \end{bmatrix}$$

The comparison between other previous researches can be seen below. Therefore, it can be said that according to previous researches, the model used in the present study, despite the breadth of criteria and considering all appropriate sub-criteria and examining the relationships between criteria, covers the results of previous researches, therefore an efficient model And can be used and confirm the research results.

Previous Research	Criteria Examined	Current Study
Hosseinzadeh et al. [14]	Action	*
Hosseini Suraki [15]	Utilitarianism	*
Ahmadizadeh et al. [4]	Organizational Behavior-Planned Behavior	*
Najafi and Lotfi [21]	job security	*
Nadi and Aghanouri [20]	Social Behavior, Justice	*
Safari et al. [26]	Selfish individualism-Machiavellianism-power	*
Mosayebi et al. [19]	Social Behavior-Action	*
Taghizadeh et al. [32]	Power - Job Tasks - Action	*
Abdullahian and Sheikh Ansari [2]	Political Behavior-Social Behavior-Influence-Interactive Behavior-Value Expectation	*
Molaei Aliabad et al. [18]	Planned Behavior - Expecting Value	*
Abdolahi et al. [3]	Political Behavior - Social Behavior - Job Motivation - Job Duties	*
Sadeghi Fasaei [27]	Accounting-Interactive Behavior	*
Varasteh Far and Mousavi Tazehabadi [33]	Professional Ethics - Competitive Advantage - Job Duties - Organizational Behavior - Accounting	*
Nikobakht and zivar Alam [22]	Interactive behavior	*
Salibi [28]	Organizational Behavior-Social Behavior-Influence	*

4 | Research Findings

According to obtaining the factors affecting the accounting action, the accounting action can be defined as follows: it is a behavior that each person, according to the situation and situation in which he is, using and considering based on utilitarianism/utilitarianism, theory games, general choice, originality benefit/cost benefit, planned behavior, organizational behavior, selfish individualism, social behavior, accounting/accountability, power, political behavior, selfishness, flattery, flattery, interactive behavior, influence, professional ethics, opportunistic behavior, machiavellianism, progress, reward, career promotion, job security, value system/value expectation, job motivation, action, job duties, organizational justice, competitive advantage.

In this regard, all 27 criteria mentioned in the definition of accounting action have been identified as effective factors on accounting action and based on the results obtained from the Dematel technique; the criterion of opportunistic behavior has the most impact and interaction with other criteria. The criterion of selfish individualism is more influential in accounting practice. Criteria of game theory, general choice, originality of benefit/cost of benefit, utilitarianism/utilitarianism, opportunistic behavior, interactive behavior, social behavior, narcissism, flattery, flattery, career advancement, job security, competitive advantage, causal organizational justice and criteria organizational behavior, accounting/accounting, political behavior, planned behavior, selfish individualism, machiavellianism, power, influence, professional ethics, value system/value expectation, motivation, job, reward, progress, job duties, action disability. Based on the results obtained from the ANP technique, the criterion of utilitarianism is the first priority and the criterion of competitive advantage is the last priority, which are presented in the definition of accounting action based on the priority of the criteria.

5 | Discussion and Conclusion

A quick look at human behaviors in different environments, especially in work environments, shows that human beings have an undeniable potential to commit all kinds of positive and negative behaviors independently or simultaneously at a certain point in time. The result of these efforts, all aimed at identifying and then managing different human behaviors, has created a diverse range of positive and negative influential behaviors in the field of scientific texts. Organizational behavior is a field of research in which the effects of group and structure on behaviors within the organization are examined and then used to make the organization more effective. Organizational behavior pays attention to increasing productivity, reducing absenteeism and relocation, and increasing employee job satisfaction. Organizational behavior has caused the correct treatment of issues and problems as well as the use of opportunities for managers, and the manager can use this knowledge to increase the effectiveness of his work and his organization and improve the quality and quantity of employee performance and delegate responsibilities to subordinates. Slowly implement programs to change and reform the structure of the organization. Using this knowledge, it is also possible to improve the human relations of managers and pay attention to the differences between managers and help managers to be able to work with workers in environments. Compromise differently.

The purpose of this study is to identify and prioritize the factors affecting accounting practice. Today's employees think and worry more about their work life than ever before, they want a secure, long-term and satisfying job and they want to grow and develop in their job. Employee action is based on a purposeful approach; thus, it is a tool that a person uses in practice to achieve his goal or to satisfy his needs. In other words, if employees feel that their job activities are the closest way to achieving their goals and have a satisfying image of working, then they will definitely be more inclined to work, and vice versa. Any effort, cunning and service to please others and to achieve personal authority and win in the organization. These behaviors are to gain a competitive advantage for personal growth. What such actors pretend may be different from their performance and outputs. During this study, it has been tried to determine what factors affect the creation of accounting action and how these criteria are arranged.

According to Dematel technique, the criterion of "opportunistic behavior" is the most effective criterion among the criteria and has the most interaction with other criteria. Therefore, it can be said that opportunism is a personal behavior that can be considered a persuasive behavior to achieve personal goals. People with high levels of opportunism may use aggressive, speculative, and deviant behaviors to achieve personal and organizational goals. They do not pay attention to conventional ethics and people who have a higher degree of opportunism are more inclined to lie, play games and pay bribes than people who have a lower degree of opportunism. All of this hurts the growth of organizations. Therefore, employing honest employees who are committed to the welfare of the whole organization instead of their own welfare can be an effective step towards achieving organizational goals. In general, opportunism in organizations means in two ways. First; people who are so poor in personality that even in times of crisis they only think of abusing the situation to their advantage and even at the cost of harming other employees and society. With these interpretations, organizations need employees who prefer the interests of the organization to personal interests and take advantage of the opportunities gained in a positive way. On the other hand, opportunism can also have positive aspects, meaning that the employees in the organizations should be situational and know the current situation in their organization and the surrounding organizations, whether competing organizations or allied organizations, and strengths and recognize the weakness of the organization, its opportunities and threats, and react in the best possible way in the right situation and turn the current situation in favor of the good organization. In this case, it has caused the development of its organization and has done what it has been hired for by the company.

The criterion of "selfish individualism" is the most influential. Therefore, it can be said that selfish individualistic behavior, in that it prefers individual interests to group interests, contradicts the nature of teamwork in organizations because it is considered an obstacle to development and hinders social participation, cooperation and collaboration. Participation and the spirit of collective and teamwork are one of the issues affecting development. Therefore, employing employees with desirable interpersonal and behavioral skills will help synergize the efficiency of organizations. The criterion of "utilitarianism/utilitarianism" is the most important criterion among the criteria. Therefore, it can be said that according to the definition of utilitarianism, the most moral action is the action that is "the most beneficial option for the affected parties", even if it is inherently harmful and evil. Therefore, if utilitarianism and utilitarianism are associated with ethics, it can be useful for organizations; otherwise it leads to organizational degeneration. Utilitarianism generally occurs because of the desire to enjoy and reduce pain, but if the ethical principles are violated in organizations to achieve greater benefits and profits; Organizational justice will be destroyed and will have irreparable consequences. Therefore, the need to observe fairness and ethics in achieving more profit and benefit is felt in organizations, and it is necessary with the growth of organizational culture and the use of efficient and ethical managers, while achieving more profit for the organization, policies should be considered to Ethical rules must be observed.

5.1 | Suggestions for Future Research

In this study, we have tried to achieve the concept of arithmetic action and to identify and prioritize the dimensions and components affecting it and to analyze the relationships between them, but because there are very few field studies in academic forums, especially domestic In a specific field, accounting has been done in a specific organ, so future researchers can increase the range of their studies to improve the designed scale by identifying and modifying items and factors. The following areas are recommended for further exploration and research:

- *The category of accounting action in organizations that have an important role in the future career of individuals and organizations, such as political parties, various ministries, especially the ministry of foreign affairs, the islamic consultative assembly, etc. should be examined and the results compare with each other.*
- *How the accounting actions and important priorities in the accounting actions in other guilds of the country such as the market class, culture including university professors and teachers, politicians, journalists, etc. should be examined and their results should be compared with each other. Be.*
- *It is suggested that the category of accounting action be done with the opinion of political experts and members of parliament or with the consideration of scientific and academic experts and the results of the research should be compared with the results of the present study when the opinions of these people are involved.*
- *Given the current state of the country and the existence of special political relations with some countries, the category of accounting action should be examined at the international level.*

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The Study of Multi-Objective Supplier Selection Problem by a Novel Hybrid Method: COA/ε-Constraint

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Abstract

Today, paying attention to the interests of suppliers in supply chain management strategies is one of the important points in the success of long-term and strategic relationships with suppliers. Not paying enough attention to these points sometimes causes irreparable damage to the overall structure of the organization. In response to this need, researchers have developed and proposed different models according to different approaches. This research has presented a special model with the approach of answering these problems. This approach, which is based on the Cuckoo Optimization Algorithm (COA), can solve the problems in multi-objective methods in addition to single-objective problems. This method based on the COA and the ε-constraint method named COA/ε-Constraint. The general approach of this method is to turn a multi-objective problem into a single-purpose problem, which is associated with increased efficiency. The model studied in this paper, with the aim of creating coordination between buyers and suppliers in the problem of supplier selection, is a three-objective model of cost, quality and delivery time, which is implemented to evaluate the performance of the proposed method. The results show the superiority of the proposed method over similar approaches in terms of creating a Pareto frontier.

Keywords: Cuckoo Optimization Algorithm (COA), Supplier selection problem, Pareto frontier, ε-constraint, Hybrid method.

1 | Introduction

The supplier selection problem is not a new problem, and there is a great of research about conceptual and mathematical modeling of these problems. In fact, before developing supply chain management philosophy, many papers can be found as the vendor selection entitled. The research related to supplier selection problem can be seen in the years before 1950 and when linear programming and numerical computation was at the beginning of their work. The first model of supplier selection is used by the National Bureau of Standards in the United States, and its main purpose was to minimize the cost of purchase contracts in the United States defense industries. It should be noted that often review research in the field of supplier selection, have raised the mid-60s as a period of increased attention to the subject provider [1].

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For separation of the subject matter of supplier selection issues, various indicators have been proposed in papers such as Aissaoui et al. [1] and De Boer et al. [2]. In conclusion, the content of supplier selection problems is defined in six categories.

- *The number and type of objectives/criteria for supplier selection (single-objective or multi-objective).*
- *Intended timespan (single-period or a multi-period).*
- *The number of parts/raw materials for outsourcing/supplying (one or more pieces).*
- *Existence of discount/payment delay strategies.*
- *Certainty or uncertainty of model variables and parameters.*
- *The system of choosing single-source/multi-source (selecting a supplier or several of them).*

The first time, coordination in the supply chain was proposed by Goyal [3]. This problem was examined in Toptal and Çetinkaya [4]. They discussed and examined changeable intervals for improved cost-effectiveness and the issues that may arise in the coordination planning. Also, the model of Goyal was compared to the model of Taptal, considering the transportation problems [5]. They showed that the rate of improvement was better than the purposes of Goyal's paper and interval change is more significant for improvement. In the paper of Ben-Daya et al. [6], a comprehensive study has been done on specific issues of the joint economic lot sizing problem. The initial investigation has been paying more attention to this type of element, but in recent years, investment to reduce the cost of preparing, considering the variable production costs, quality and process control, uncertain demand, transportation costs, and capacity, expanding the number of entities at all levels and considering problems due dates to joint economic lot sizing problem problems were added.

In the study and classification problems in the field of supply chain coordination, Thomas and Griffin [7] considered the coordination problem between buyers and suppliers as a critical argument in coordination with the operational programs. Also, Tan [8] emphasized the need for integrated management in the areas of purchasing, supply, and logistics.

Leung [9] have examined the supply chain coordination in the centralized and decentralized states. Jafarnejad et al. [10] developed a fuzzy decision-making method for selecting preferred suppliers in a single source state. Criteria are presented as fuzzy triangular numbers, and TOPSIS method is used to solve it. Razmi et al. [11] gave a multi-criteria approach to supplier selection and allocation of the purchase by combining hierarchical process and idealistic planning. In another research, Razmi and Rafiei [12] have used a hierarchical process with a combined mixed integer programming method. Moheb-alizadeh and Faez [13] developed a multi-objective model with multiple criteria of data envelopment analysis. First, they provide efficient solutions to obtain a multi-objective problem of supplier selection. Using data envelopment analysis and taking into account economically efficient solutions as incoming entities, provided the appropriate method for selecting suppliers. Jazemi and Ghodsypour [14] by combining the planning and programming interval compromise, proposed a method called adaptive planning interval for selecting preferred suppliers. Their model is a multi-objective problem that minimizes returns of suppliers and costs and maximizes quality, which is considered as objectives of this problem. Sarmah et al. [15] are considered the coordination between a producer and several buyers with an objective function for minimizing the costs. Amid et al. [16] have developed a supplier selection problem concerning discount strategies. A comprehensive overview of using multi-criteria methods in the selection and supplier evaluation was performed by Ho et al. [17]. Moghaddam [18] examined the problem of supplier selection planning for a piece, as a multi-objective and in fuzzy model. Shadkam and Bijari [19] examined the efficiency evaluation by cuckoo optimization algorithms and simulation for the selection of the supplier's problem in a multi-objective model. Arakaw et al. [20] have used the method of combining general data envelopment analysis and Genetic algorithm to produce efcient frontier in multi-objective optimization problems. Georgestani et al. [21] used data envelopment analysis and the cuckoo meta-heuristic algorithm to form the Pareto frontier. Rajabioun [22] provided the cuckoo optimization algorithm to solve optimization problems. Shadkam et al. [23]

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examined the portfolio selection using cuckoo's algorithm. Akbarzadeh and Shadkam [24] examined the problem of production planning using the cuckoo algorithm. Shadkam and Jahani [25] proposed a hybrid method based on COA algorithm and COA/ε-Constraint method. Borhanifar and Shadkam [26] gained the Pareto frontier using the cuckoo algorithm and the simple average weighted methods. From the literature review and research in this area, one finds that the development of models for supplier selection is applicable in three areas:

- *Coordinated Modeling between buyers and suppliers.*
- *The development of multi-objective and multi-product models simultaneously.*
- *Applying uncertain conditions in the supplier selection.*

Concerning this research, in this paper, a model is presented in the supplier selection. The specifications of this model are to take into account the interests between a buyer and a supplier. The problem of coordination in the supply chain is considered, taking into account three objectives between the buyer and supplier in the model. In this case, the final customer raised a certain amount of his needs as demand and Chains of buyers and suppliers to intend to consider three objectives of quality, delivery time, and cost for the entire chain, to meet customer demands. In fact, given the limitations of the problem, they decide which of the providers to meet customer demand and how much to be purchased so that the number of healthy products delivered (indicative target quality) to the final customer with delays of finished products (the true purpose of delivery), and cost of entire chain (indicative target cost) are in optimum balance. Another new issue is raised in this paper, solving using COA/ε-Constraint and obtaining a Pareto frontier using this method. For more information about the COA/ε-Constraint method, refer to Shadkam and Jahani [25].

In the proposed hybrid approach of the simultaneous advantages of both COA and ε-constraint used and leads to an efficient method. On the other hand, it has the speed and accuracy of the COA, and the proposed algorithm can be implemented for large-scale problems. The COA, which was only able to solve single-objective problems, was not used.

In this paper, first the problem is defined and modeled, then the proposed approach is reviewed and validated, and while confirming the necessary efficiency, useful conclusions are presented in this regard.

2 | Presentation of the Model

As mentioned, in the defined model, a buyer is considering to supply the desired products of the final customer, provide a piece of potential suppliers; so that, in this decision-making process, the objectives of cost, quality, and delivery time for both buyers and suppliers must be considered simultaneously at a desirable level of expectations. The specifications of the model will be as follows:

I. The main purpose is planning to ensure the interests of buyers and sellers.
II. Three common objectives are defined as cost, quality, and delivery time between buyers and suppliers.
III. Modeling will be for a period of one year (multi-period).
IV. The problem will be modeled by assuming the absence of discount strategies.
V. All the variables and parameters of the problem are definite.
VI. The annual demand for target customers is constant.
VII. Period order to all suppliers, is the same.
VIII. All suppliers have limited capacity.
IX. The final product is delivered to the customer without delay, and the delay is compensated by spending the extra costs of the buyer.
X. The buyer receives the orders and then determines the failure, corrects, and repairs them at the cost of the supplier.
XI. Orders were received sequentially. This means that after they received the order and entirely using it, the next order is received.

Then, the modeling will be presented, with the definition of variables and factors used.

Parameters.

D: The annual demand of the final product.

n: The number of suppliers ($i = 1, 2, .., n$).

I_i: The percentage of delayed delivery from i_{th} suppliers.

a_i: Percentage of healthy delivery from i_{th} supplier.

c_i: Purchase price from i_{th} supplier.

A_i: Fixed cost of ordering i_{th} supplier.

r: The rates of the annual cost of holding inventory.

z_i: The cost of production (supply) of each original piece by i_{th} suppliers.

s_i: Fixed cost for processing the i_{th} supplier.

G_i: Annual capacity of i_{th} suppliers.

E: Scholarships to suppliers for delay compensation per unit of product.

F: Scholarships to suppliers to compensate for any delay units.

K: The cost of each unit of broken parts which are delivered to the buyer, paid by suppliers.

Variables.

T: Order period (in years).

Q: The total amount of orders to suppliers in each period.

Q_i: Demand predicted by the i_{th} supplier in each period ($\sum_{i=1}^{n} Q_i = Q, Q = DT$).

x_i: The amount of the annual demand of i_{th} which is satisfied by suppliers' ($x_i = \frac{1}{T} Q_i$, $\sum_{i=1}^{n} x_i = D$).

y_i: Binary variable if i_{th} supplier select is equal to 1, otherwise is equal to 0.

According to the definition made, a coordinated multi-objective problem intended by this study is obtained as follows where z_1 shows quality and is defined as "maximizing annually received number of products from suppliers". z_2 is defined to show on delivery time, as "minimizing the number of received goods with a delay from suppliers. z_3 is defined as the total cost of the chain that this cost is equal to the total annual costs of suppliers and annual cost of the buyers, as *Table 1*.

Table 1. The costs of the mentioned model.

$\sum_i c_i x_i$: purchase cost	$\sum_i z_i x_i$: buying cost
$\frac{D}{Q}\sum_i A_i y_i$: delivery cost	$\frac{D}{Q}\sum_i s_i y_i$: preparation cost
$\frac{rQ}{2D^2}\sum_i c_i x_i^2$: inventory cost	$E\sum_i l_i x_i$: overhead costs
$F\sum_i l_i x_i$: delay suppliers'cost	$K\sum_i (1-\alpha_i)x_i$: delivery defective parts cost

According to the above, the objective functions and constraints of the model can be obtained as follows:

$$\text{Max } z_1 = \sum_i \alpha_i x_i , \quad (1)$$

$$\text{Min } z_2 = \sum_i l_i x_i , \quad (2)$$

$$\text{Min } z_3 = \sum_i (c_i + z_i + (E+F)l_i + K(1-\alpha_i))x_i + \frac{rQ}{2D^2}\sum_i c_i x_i^2 \times \frac{D}{Q}\sum_i (A_i + S_i)Y_i, \quad (3)$$

$$\text{s.t: } \sum_i^n x_i = D, \quad (4)$$

$$0 \le x_i \le G_i \quad \forall i = 1,2 \dots n, \quad (5)$$

$$x_i \le DY_i \quad \forall i = 1,2 \dots n, \quad (6)$$

$$x_i \ge \varepsilon Y_i \quad \forall i = 1,2 \dots n, \quad (7)$$

$$Y_i = \{0,1\} \quad \forall i = 1,2 \dots n. \quad (8)$$

Eq. (1) and *Eq. (2)* is clear, but how to achieve the *Eq. (3)* is described below. It is clear that the average inventory level of the buyer in the purchase of i_{th} supplier is equal to $\frac{1}{2}T_i Q_i$. Considering the current fixed rate of inventory by the purchaser and placement of T, the average inventory level of the buyer to purchase from the i_{th} supplier is equal to $\frac{1}{2}\frac{Q_i^2}{D}$. By applying inventory cost and the cost of buying from the i_{th} supplier, the total annual cost of inventory to buyers is equal to $\frac{rQ}{2D^2}\sum_i c_i x_i^2$. *Eq. (4)* are used to satisfy demand, *Eq. (5)* to consider capacity for suppliers, and *Eq. (6)*, *Eq. (7)* and *(8)* to explain the logical relationship between x and y. In recent relations, ε is a sufficiently small amount. The functions, z_1 and z_2, do not have variable Q, and they are linear functions, but objective function z_3 is a non-linear function that comes with the derivation towards Q.

$$\frac{\varrho Z_3}{\varrho Q} = 0 \quad \Rightarrow \quad Q^* = \sqrt{\frac{2D^3\sum_i (A_i + S_i)Y_i}{r\sum_i C_i x_i^2}}.$$

By substituting this value, the model changes as follows:

$$\text{Max } z_1 = \sum_i \alpha_i x_i , \quad (9)$$

$$\text{Min } z_2 = \sum_i l_i x_i , \quad (10)$$

$$\text{Min } z_3 = \sum_i (c_i + z_i + (E+F)L_i + K(1-\alpha_i))x_i + \sqrt{\frac{2r}{D}\sum_i (A_i + S_i)Y_i} \times \sqrt{\sum_i C_i x_i^2}, \quad (11)$$

$$\text{s.t.} \quad \sum_i^n x_i = D, \quad (12)$$

$$0 \le x_i \le G_i \quad \forall i = 1,2 \ldots n, \quad (13)$$

$$x_i \le DY_i \quad \forall i = 1,2 \ldots n, \quad (14)$$

$$x_i \ge \varepsilon Y_i \quad \forall i = 1,2 \ldots n, \quad (15)$$

$$Y_i = \{0,1\} \quad \forall i = 1,2 \ldots n. \quad (16)$$

3 | The Hybrid Proposed Method

The multi-objective optimization problems often are not possible to obtain a solution that simultaneously optimizes all the objectives of the problem, so using the Pareto frontier, an acceptable solution to a multi-objective problem could be obtained. To solve the problem of selecting suppliers, we first give a solution method to the multi-objective problem called ε-Constraint and then using the meta-heuristic cuckoo algorithm, and combination with the above method (COA/ε-Constraint), we have gained the Pareto frontier that went on to explain the procedure described above [25].

3.1 | Introducing the Cuckoo Optimization Algorithm

This algorithm is one of the newest and most powerful evolutionary optimization methods that have been introduced so far. Cuckoo algorithm, inspired by the lifestyle of a bird called the cuckoo in 2009 presented by Shin Ouyang and Deb Savash, in 2011 was developed by Rajabioun [22]. The cuckoo optimization algorithm flowchart is as *Fig. 1*:

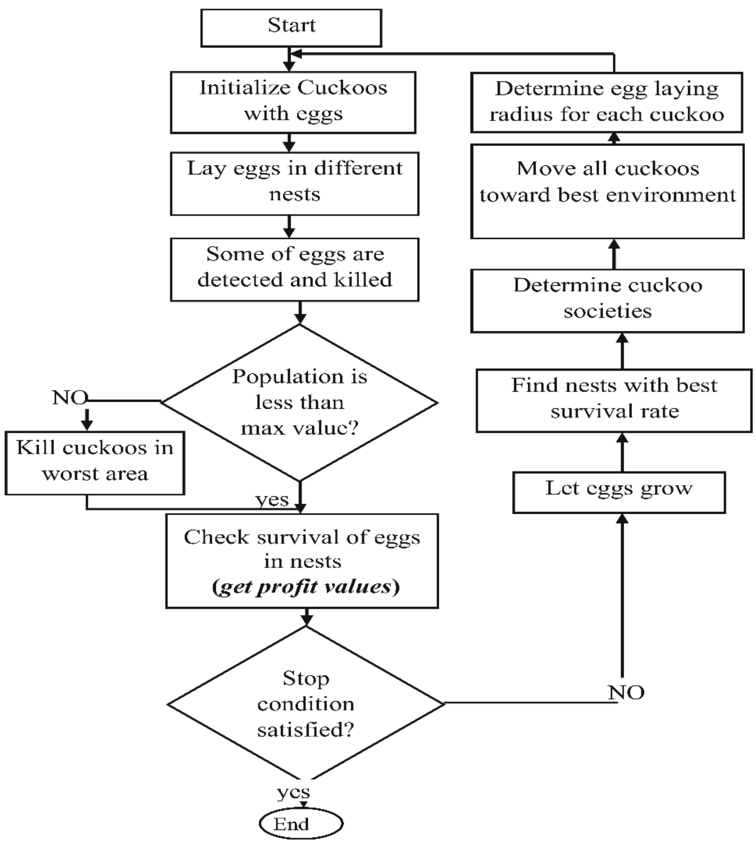


Fig. 1. The flow chart of the cuckoo optimization algorithm [22].

3.2 | The Implementation of the Propose Hybrid COA/E-Constraint Method

Given that the ε-Constraint method is used in multi-objective problems and thus acts as follows: it keeps one of the objective functions as the main objective and puts other objective functions as a part of the restrictions. For this purpose, functions $\sum_{i=1} \alpha_i x_i$ and $\sum_{i=1} l_i x_i$ in the Lingo software programming onetime with min and once with max until upper and lower limit for this objective to be obtained to implement the method of ε-Constraint.

Upper and lower limits of mentioned objective functions, was achieved as follows:

$$20 \le \varepsilon_1 \le 132.2, 1900.2 \le \varepsilon_2 \le 1972.7$$

The conversion mode of a three-objective model to a single model is expressed as follows.

$$\text{Min } z_3 = \sum_i (c_i + z_i + (E+F)L_i + K(1-\alpha_i))x_i + \sqrt{\tfrac{2r}{D}\sum_i (A_i + S_i)Y_i} \times \sqrt{\sum_i C_i x_i^2}, \quad (17)$$

$$\text{S.t.} \quad \sum_i \alpha_i x_i \ge \varepsilon_2, \quad (18)$$

$$\sum_i l_i x_i \le \varepsilon_1, \quad (19)$$

$$\sum_i^n x_i = D, \quad (20)$$

$$0 \le x_i \le G_i \quad \forall i = 1,2 \ldots n, \quad (21)$$

$$x_i \le DY_i \quad \forall i = 1,2 \ldots n, \quad (22)$$

$$x_i \ge \varepsilon Y_i \quad \forall i = 1,2 \ldots n, \quad (23)$$

$$Y_i = \{0,1\} \quad \forall i = 1,2 \ldots n. \quad (24)$$

In this model, the problem is dealt with 10 suppliers (n =10) and the demand equal to 2,000 units (D = 2000) with the assumption that: E=65, F=35, K=40, r=0.25,$\varepsilon = 10^{-14}$, other parameters of this model are as *Table 2*.

Table 2. The parameters of the model.

Supplier Number	C_i	L_i	α_i	G_i	Z_i	A_i	S_i
1	112	0.05	0.98	570	93	7450	9800
2	118	0.08	0.94	670	89	6120	6400
3	114	0.02	0.96	450	90	6590	8600
4	117	0.04	0.97	590	91	6890	9300
5	119	0.01	0.97	610	90	6410	8970
6	120	0.01	0.99	590	89	6700	9100
7	111	0.07	0.95	640	92	6300	9500
8	115	0.05	0.96	470	86	7100	9210
9	108	0.01	0.98	360	88	7800	9700
10	127	0.01	0.99	680	86	6320	9460

As can be seen, the number of initial solution devices will vary depending on the number of suppliers. In the COA, the initial direction is created based on the location of the cuckoo, which is similar to the chromosome in the genetic algorithm and will be as follows.

The initial solution consists of 5 main parts based on *Fig. 2*. The first part is related to the order period, the second part is related to the total number of orders to suppliers in each period, and the third part is related to the demand estimated by the ith supplier, which consists of one part. Similarly, the fourth and fifth sections are formed by each of the i-classes, the fourth section shows the amount of annual demand supplied by the supplier i and the fifth section represents the variables zero and one, which if the supplier is selected, i is the value of one.

T	Q	Q_1	…	Q_n	x_1	…	x_n	y_1	…	y_n

Fig. 2. Chromosome structure of the cuckoo algorithm.

In order to identify the most effective parameters on the problem, the effect of each parameter on the elapsed time is investigated. The results in *Fig. 3* show that D and n parameters are most effective.

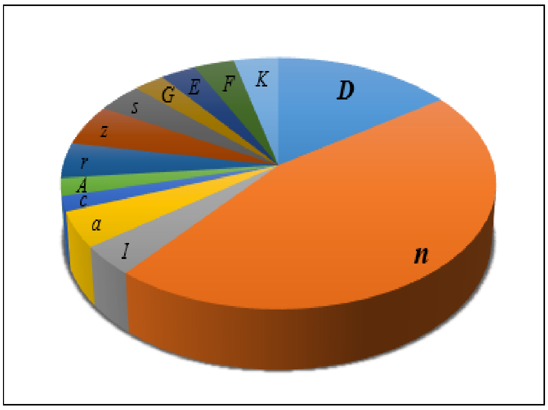
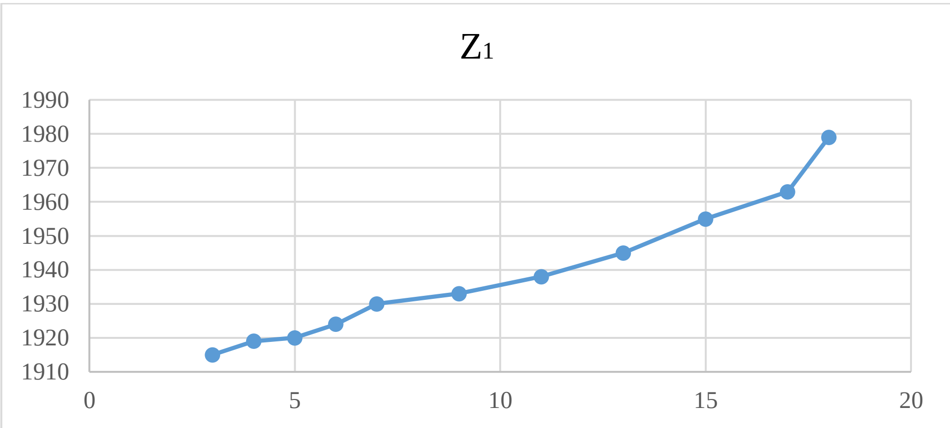


Fig. 3. The effect of each of the problem parameters on the elapsed time.

Considering that D and n are the most effective parameters of the problem, then in *Fig. 4* and *Fig. 5*, the effect of this parameter on each of the objective functions is investigated. As can be seen, by increasing these two parameters, the values of all three objective functions increase and there is a consistent relationship between these parameters and the objective functions.



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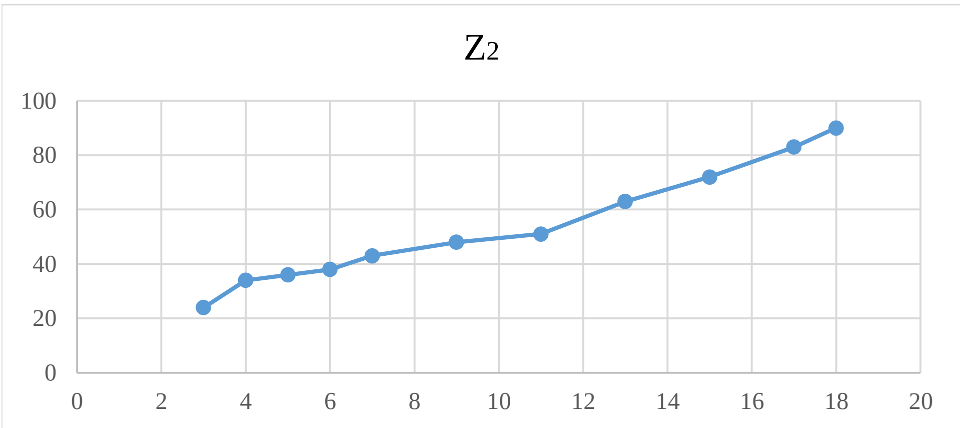

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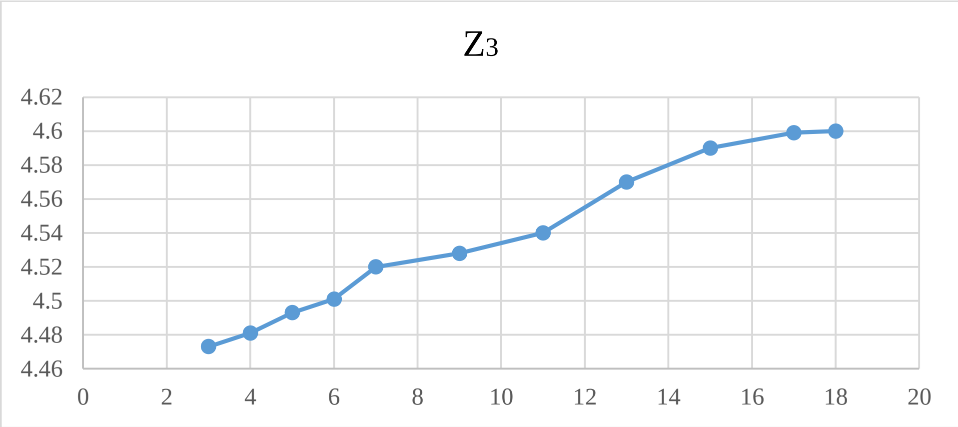

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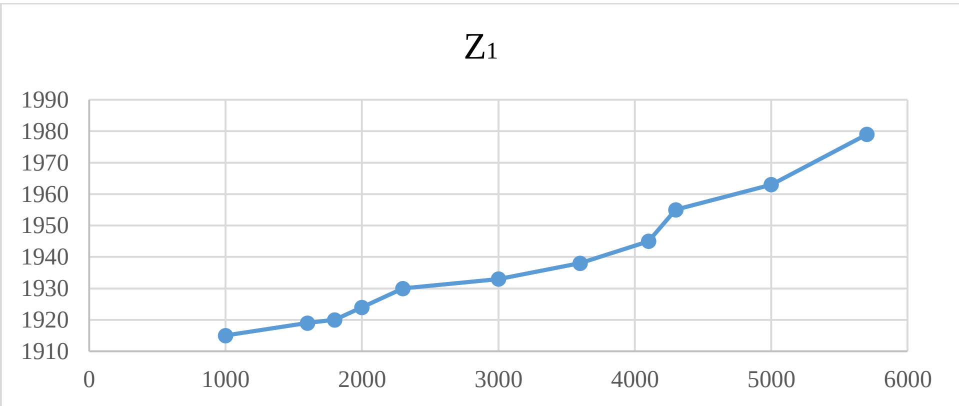
Fig. 4. The effect of n parameter on objective functions.

Also, the effect of these two parameters on the problem decision variables has been investigated and the results are shown in *Table 3*. As can be seen, suppliers 1, 2 and 4 are selected in each case and the suppliers are with the desired performance. In the next category, suppliers 6 and 7 are good and other suppliers do not perform well and are usually not selected.

Table 3. The effect of D and n parameters on decision variables.

n	D	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	y_1	y_2	y_3	y_4	y_5	y_6	y_7	y_8	y_9	y_{10}
3	1000	358	275	367								1	1	1							
4	1600	1050	550	0	0							1	1	0	0						
5	1800	805	589	0	406	0						1	1	0	1	0					
6	2000	987	679	0	265	0	69					1	1	0	1	0	1				
7	2300	1100	599	0	392	0	13079					1	1	0	1	0	1	1			
8	2500	1240	772	39	299	0	59	91	0			1	1	1	1	0	1	1	0		
9	3000	2035	398	21	300	0	79	32	0	135		1	1	1	1	0	1	1	0	1	
10	3300	2290	398	56	397	0	96	39	0	24	0	1	1	1	1	0	1	1	0	1	0



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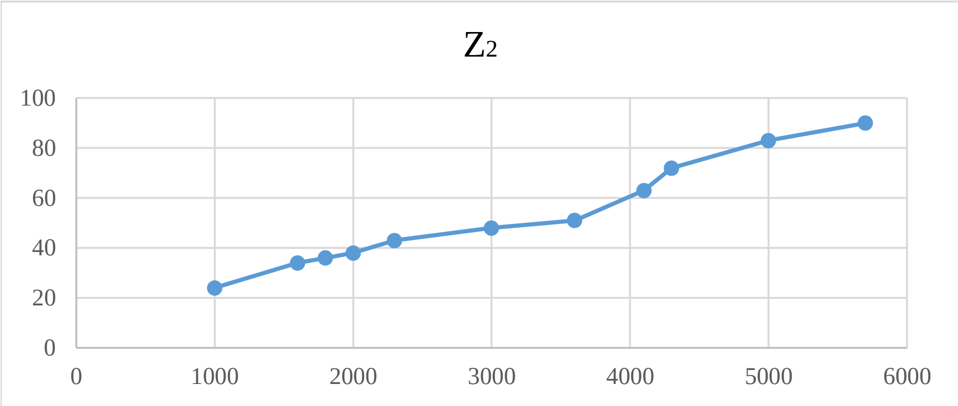

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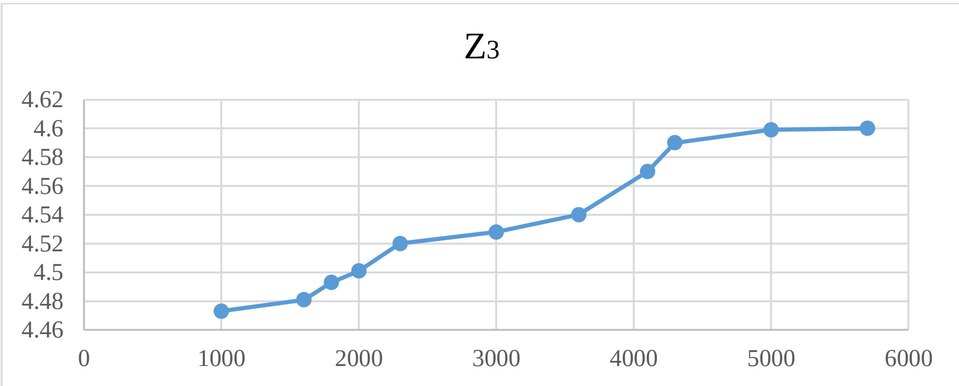

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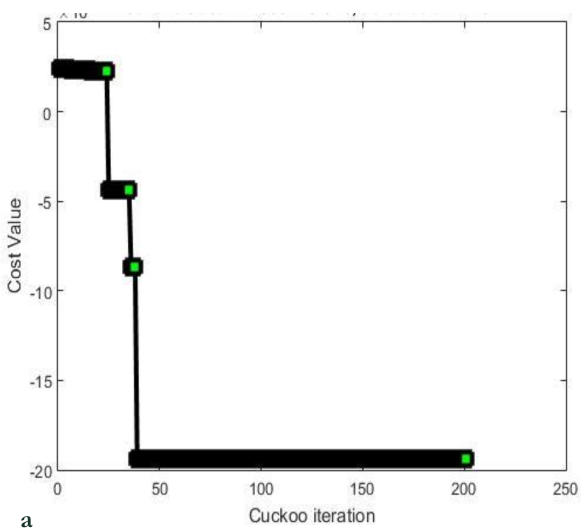
Fig. 5. The effect of D parameter on objective functions.

In the continuation of the sensitivity analysis process, in addition to the effective parameters of the problem, the effective parameters of the cuckoo optimization algorithm are also identified, which are number of clusters, initial number of cuckoos, max number of cuckoos, min number of eggs, and max number of eggs. The number of parameters is very important and effective in the performance of meta-heuristic algorithms, which is usually obtained experimentally. In *Fig. 6*, it can be seen that by examining the different values of the number of clusters parameter on the value of the objective integration function of the problem, the best value for this parameter is identified. This process is performed similarly for the other parameters and the results are shown in *Table 4*.



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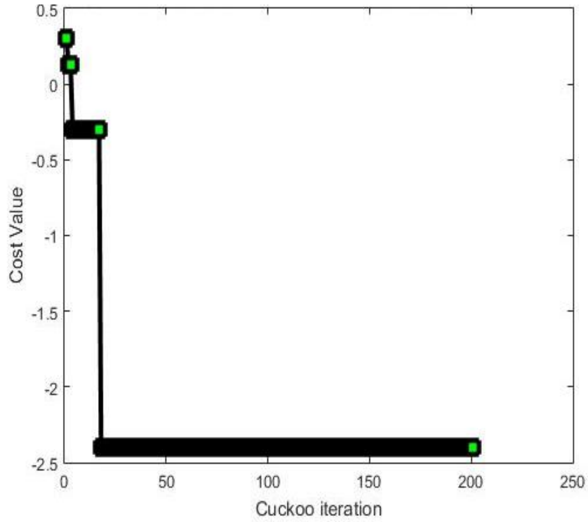

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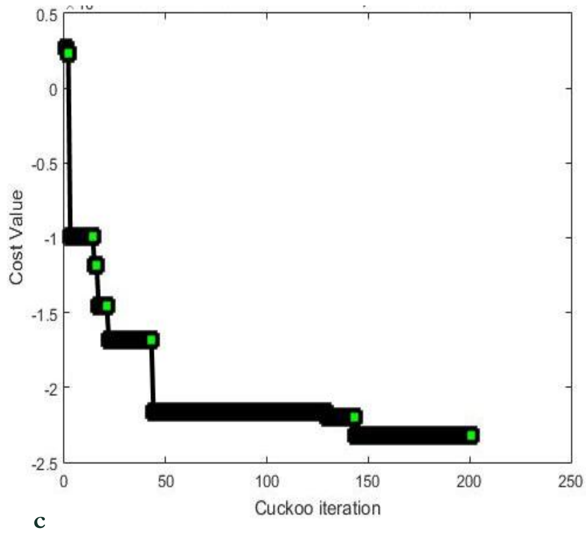

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Fig. 6. The effect of changing the number of clusters on the value of the integration function: (a) The number of clusters=4, (b) the number of clusters=3, the number of clusters=2.

Table 4. Optimal values of the cuckoo meta-heuristic algorithm parameters for the proposed logistics problem.

Parameter	Value
Number of clusters	4
Initial number of cuckoos	4
Max number of cuckoos	20
Min number of eggs	3
Max number of eggs	5

Now, to obtain the Pareto frontier, using the proposed hybrid method by MATLAB software repeated in 1000 iterations with 0.725 strike length for ε_2 and 11.22 for ε_1 parameters, the Pareto frontier can be obtained as *Fig. 7*. Also, the values of the decision variables and the objective functions are shown in *Table 5*.

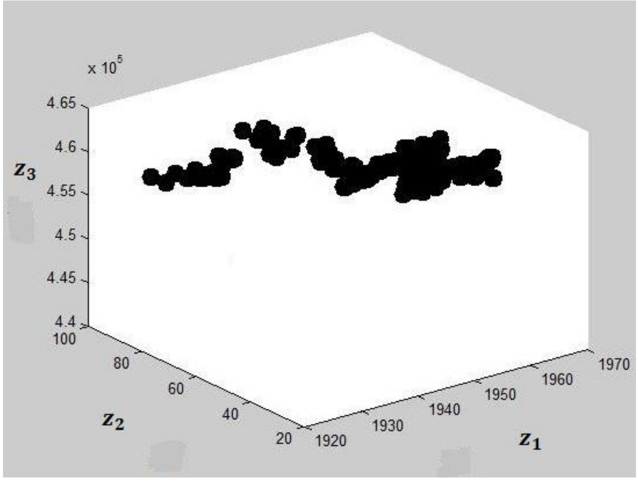

Fig. 7. The Pareto frontier obtained by using the COA/ε-Constraint method for ten suppliers.

Table 5. The Results of implementation of the proposed method.

n	3	4	5	6	7	8	9	10
D	1000	1600	1800	2000	2300	2500	3000	3300
X_1	358	1050	805	987	1100	1240	2035	2290
X_2	275	550	589	679	599	772	398	398
X_3	367	0	0	0	0	39	21	56
X_4		0	406	265	392	299	300	397
X_5			0	0	0	0	0	0
X_6				69	130	59	79	96
X_7					79	91	32	39
X_8						0	0	0
X_9							135	24
X_{10}								0
Y_1	1	1	1	1	1	1	1	1
Y_2	1	1	1	1	1	1	1	1
Y_3	1	0	0	0	0	1	1	1
Y_4		0	1	1	1	1	1	1
Y_5			0	0	0	0	0	0
Y_6				1	1	1	1	1
Y_7					1	1	1	1
Y_8						0	0	0
Y_9							1	1
Y_{10}								0
Z_1	1915	1919	1920	1924	1930	1933	1938	1945
Z_2	24	34	36	38	43	48	51	63
Z_3	4.473	4.481	4.493	4.501	4.52	4.528	4.54	4.57

In this paper, in addition to three-dimensional Pareto frontiers, we have drawn pairwise Pareto frontiers for this reason that, if we do not consider one of the objective functions, we would have the Pareto frontier of the other two functions towards to each other, for example, if the only objective is quality and cost, and we do not consider delivery time, Pareto frontiers of two functions towards each other are like as *Fig. 8-10*. It is worth noting that due to the boundaries related to the values of the objective function, the origin of the coordinates in *Fig. 7* to *Fig. 10* has been changed to have a clearer shape

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In order to validate the method of the article, in addition to the proposed method, the exact method has also been used. The results are presented in the *Table 6* in two ways of exact and meta-heuristic methods. In both methods, the final solutions are obtained for the value of the objective function with approximately 0.1; but the elapsed time of exact method was much longer than proposed method. MATLAB software has been used to code the required programs. It should be noted that for n greater than 18, due to the non-responsiveness of the memory system and the length of time, it is not possible to count the total number of cases with ordinary computers. The same phenomenon that can be seen with the proposed method can be found in less time to a very near solution to the result obtained by solving the exact method.

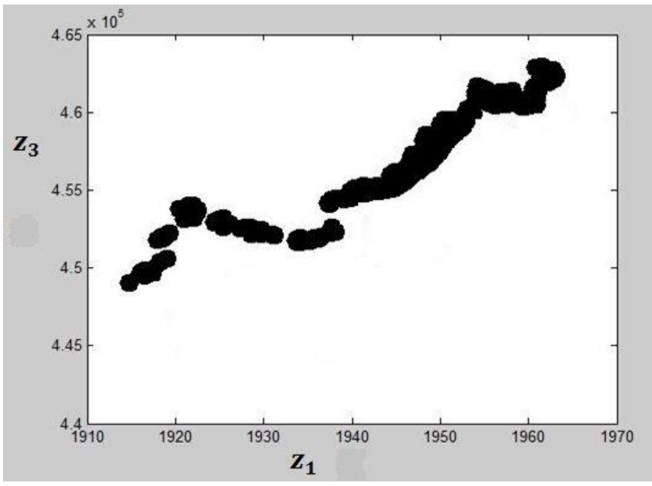

Fig. 8. Quality Paret o frontier, towards the cost.

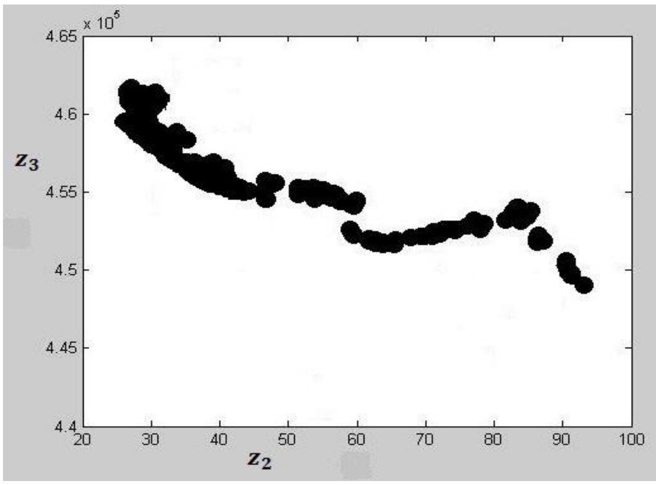

Fig. 9. Pareto frontier for the cost of delivery time.

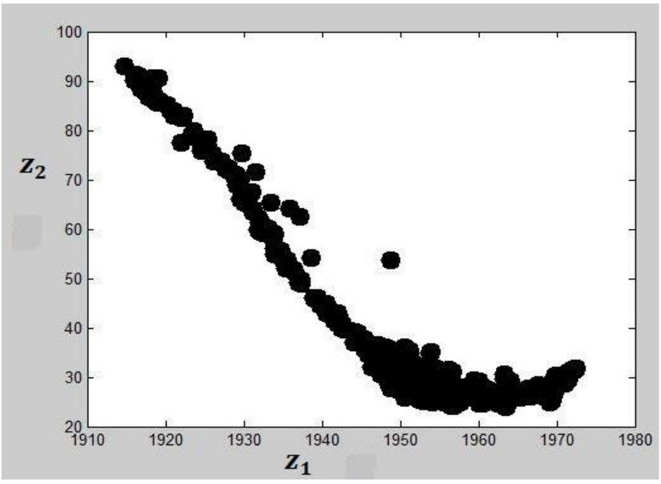

Fig. 10. The Pareto frontiers of quality towards the delivery time.

Table 6. Comparison of exact method and proposed approach.

n	D	Elapsed Time (Sec) Exact	Proposed approach
3	1000	2.79	2.21
4	1600	2.97	2.74
5	1800	2.42	2.03
6	2000	3.12	2.26
7	2300	5.37	2.24
9	3000	13.752	2.27
11	3600	61.45	2.57
13	4100	319.18	2.64
15	4300	1937.4	3.32
17	5000	8114.35	3.7
18	5700	9977.52	3.68

4 | Conclusion

This research; in general, it was done with the aim of optimizing the needs of suppliers and buyers and examining the problems in a multi-objective problem; Due to the general problems in other previous studies, including the existence of some inefficiencies, the problem-solving approach in this study is a solid approach based on problem solving using the cuckoo optimization algorithm; Which eventually led to the creation of an optimization problem model called the COA/ε constraint method.

The main objectives of the problem are to establish a coordination between the three components considered by suppliers, namely quality, cost and delivery time, which we finally achieved an optimal output by implementing the COA/ε-constrain method and creating Pareto frontiers; The proposed method was able to achieve good efficiency in achieving the optimal solution and play a good role in optimizing multi-objective supply chain problems. Therefore, it is suggested to use obtained in general problem of supplier selection and in other similar studies to be studied by researchers. It is also suggested to use other approaches of multi objective decision method such as STEM method and Goal Programing in solving.

Conflict of Interest

The author has no conflicts of interest to declare that are relevant to the content of this article.

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Fabrication and Process Parameter Optimization of a 3D Printer Using Response Surface Methodology

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Abstract

3D printing or additive manufacturing is a technology in which 3D objects are printed by depositing a thin layer of material layer-by-layer until a final product is produced. In this research work, it has been focused on the fabrication of a Portable 3D Printer for the manufacturing of sample parts by using Fused Deposition Modeling (FDM) process. The primary process parameters such as nozzle temperature, extrusion speed and fill density in addition to their interactions are studied. It has been observed that these process parameters influence the dimensional accuracy and extrusion time of the part produced by the process of FDM. The main objective of the research work is to create a reliable and cost efficient 3-D printer and to minimize the dimensional variation that usually occurs to plastic parts produced by 3D printers. Cartesian mechanism has been used where the print bed moves in the Z direction and the extruder moves in both the X and Y directions. The 3D printing filament that has been used is made of Poly Lactic Acid or Poly Lactide (PLA). The process involved 3D solid modeling to design, 3D printing with coated adhesive applied on the printing platform, measurement of dimensional variation of the printed parts and statistical analysis. Response Surface Methodology (RSM) based desirability analysis has been employed for optimization of FDM process parameters namely, nozzle temperature, extrusion speed and fill density. Mathematical models were developed and tested for accuracy and extrusion time using Design Expert 11 software for RSM application.

Keywords: Additive manufacturing, 3D printing, FDM, RSM, Optimization.

1 | Introduction

Additive Manufacturing (AM), contrast to traditional material removal or subtractive manufacturing is the process of manufacturing parts by adding layers in the third dimension. 3D CAD models are used to generate STL (standard triangulation) files containing the deposit layer data. AM is known for reduced supply chain costs, easier manufacturing design and green manufacturing initiatives. In AM, 3D-printing and rapid prototyping are used interchangeably to describe the process [1]. Fused Deposition Modeling (FDM) is a well-known additive manufacturing process for producing strong, robust prototypes [2]. The newer, more advanced manufacturing techniques are better able to deal with smaller, more complex, and custom product. Currently, FDM is used to produce models, visual aids, and prototypes as well as functional parts, such as drill grids in the aerospace industry.

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The application of the FDM process in manufacturing functional parts is still limited due to various drawbacks such as uneven surface, poor mechanical properties, and low accuracy. FDM has a bright future in a variety of industrial and medical fields. Many unsolved problems, such as reproducibility, post-processing, and the low-volume production, persist [3]-[5]. These drawbacks decrease its comparability across traditional manufacturing processes [6]. Reproducibility, ability to produce the replicas of the same part under same conditions with high dimensional accuracy, is one of the major challenges in AM. Several FDM parameters have a big impact on the final pieces. All of these variables affect the bonding between and within layers. Choosing the best process parameters can produce the desired properties. Choosing the best thermoplastic polymer for the part's intended use is also critical [7]. Dimensional precision, mechanical qualities, building time, and surface roughness have all been improved in 3D printers. The right FDM parameter selection can lead to excellent process performance. Research and our desired outcome determine the parameters for this work procedure. Air gap, build orientation, extrusion temperature, infill density, infill pattern, layer thickness, and number of shells are some of the most common process parameters (post-processing parameter). Innumerable studies have looked at the effects of process parameters on dimensional accuracy and mechanical properties. Surface roughness tends to increase with the increament of layer thickness and also with the increamnet of nozzle speed was investigated by Gurminder Singh et al. [8]. In order to get the best surface roughness, various efforts have been made using the traditional optimization approach. To achieve the best surface quality, the optimum process parameters can be found using a variety of optimization techniques, including conventional and non-conventional techniques. In order to optimize the response, the Response Surface Methodology (RSM) uses mathematical and statistical methods to model and analyze a process and to determine the influence of factors (independent variables) and their interactions in order to establish the best circumstances for a dependent variable of interest [9]. RSM studies aim to understand the response surface topography, including local maximum, minimum and ridge lines, and locate the most appropriate response region [10] and [11]. Srinivasan et al. [12] states that RSM is the method that can be used when many input variables affect a process's performance or quality. The input variables are called factors by researchers, and the response quality is called response. The RSM field uses experimental methods to link response and process variables. RSM has many advantages over conventional methods. It takes fewer experiments to study the effects of all variables and find the optimal combination. The interaction (where one factor's behavior is affected by another's level) between factors can be determined [12]. The effects of layer thickness and build orientation on 3D printed part tensile strength were studied by Rai et al. [13]. The number of experiments was determined using the Box-Behnken Design (BBD) of RSM, and the results were analyzed using ANOVA and regression analysis. The results showed that layer thickness reduces tensile strength. Srivastava et al. [14] optimized layout plans for various FDM parameters and spatial orientations. The full factorial central composite design was used. The FDM process parameters contour width, raster width, air gap, raster angle, slice height, and orientation were optimized using RSM.

According to a review of past potential studies, most studies only considered one or two factors at a time, and only a few studies considered three factors at a time. We used a full factorial design of experiment with three factors: nozzle temperature, extrusion speed, and fill density. With both main effects and interaction effects, we want to see if the significant factors remain the same. The key goals of this research are to build a FDM 3D printer, utilize Design Expert software to design an experiment, evaluate the influence of controllable process parameters and their interactions on dimensional accuracy and extrusion time, and apply RSM to optimize the process parameters. The findings of this study will determine the appropriate levels of components that can be employed to generate more precise AM products.

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2 | Development of the 3D Printer

Selecting one of the additive manufacturing processes is the first step in building a 3D printer. The FDM process was chosen because it is clean, easy to use, and environmentally friendly. It is possible to print complex forms and intricate pieces. Because it is primarily utilized by people, FDM is at the very beginning of the market. When compared to other 3D printing technologies, FDM is a more economical option. For X, Y, and Z axis movements, a Cartesian mechanism is chosen after evaluating different factors such as fabrication cost, design simplicity, synchronization, and precision. The bed moves in the Y axis, while the extruder travels in the X and Z axes in this setup. The bed should be minimal in weight with the purpose of maintaining precision. Two stepper motors are used for Z-axis movement, one for X-axis movement, one for Y-axis movement, and one for Extruder filament movement in this system. This mechanism uses a single motor to control lead screws, which are coupled to the Extruder's Z-axis movement. Because the print volume is quite large, using only one motor would produce an interruption in the action. The build volume has been set at 200x200x250 mm3. *Table 1* displays the 3D printer's general parameters, whereas *Fig. 1* depicts the completed built 3D printer.

Table 1. Specification of the fabricated 3D printer including hardware and software.

Elements	Specifications
Frame	Aluminium Channel (1 inch X 2 inch) [53cm, 45cm, 43 cm]
Controller	Ramps 1.4 Shield Arduino Mega 2560
Stepper Motor Drivers	A4988 with heat sink
Stepper Motor	NEMA 17 Stepper Motor 4 Wire Bipolar
Lead Screw	T8 Trapezoidal Lead Screw L8mm Thread 8mm Pitch 300mm
Smooth Rod	M8 500mm
Linear Ball Bearing	LM8UU 8mm Linear Ball Bearing
Timing Belt	2 meter GT2 6mm Open Timing Belt
Pulley	20 teeth Pulley 5mm Bore
Flexible Coupling	5mm*8mm*25mm
Extruder	V6 J-Head Hot End Bowden Extruder
Firmware	Marlin
Slicing Software	Slic3r
Heat Bed	PCB Heat Bed MK2B
Power Supply	12 volt 20 amp Power Supply

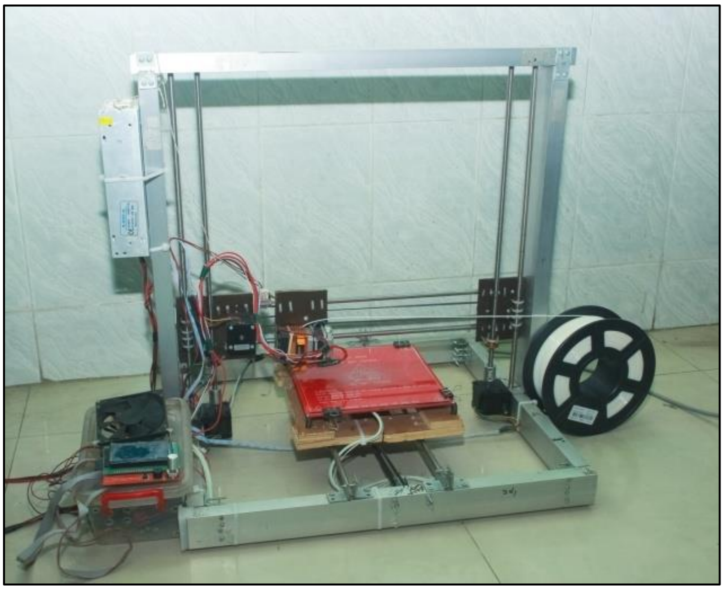

Fig. 1. Final assembly of 3D printer.

3 | Experimental Design

In the experimental design, response surface method was adopted to study the relationship between the process parameters and the output response and the mathematical model that can predict the output response from the actual process. The calculations for the RSM model development was carried out by utilizing the Design-Expert 11 software. The range and level of parameters are shown in *Table 2*. To develop the empirical model for dimensional accuracy and extrusion time, experiment was implemented in accordance with CCD. The CCD has an embedded factorial design which consists of fourteen non-centre points and six centre points for curvature estimation. *Table 3* shows the experimental data for 20 runs with three control factors and two response variables [15].

With the help of SOLIDWORKS software 3D solid model of a Spur Gear is modeled and then converted to STL file which is indicated in *Fig. 2*. STL file is imported to Slic3r software. Control factors listed in *Table 2* are set as per shown experiment plan in *Table 3*. The parts per experiment are fabricated by the use of our 3D printer. PLA is the material used for fabricating the designed part. The average of the three readings of Gear Bore is taken to be the representative value respectively. Digital Slide Calipers was used to measure the dimensions and the response time was taken from the 3D printer display.

Table 2. 3D printer parameters and their levels.

Symbol	Parameter	Unit	Low	High
A	Nozzle Temperature	°C	220	240
B	Extrusion Speed	%	30	40
C	Fill Density	%	20	30

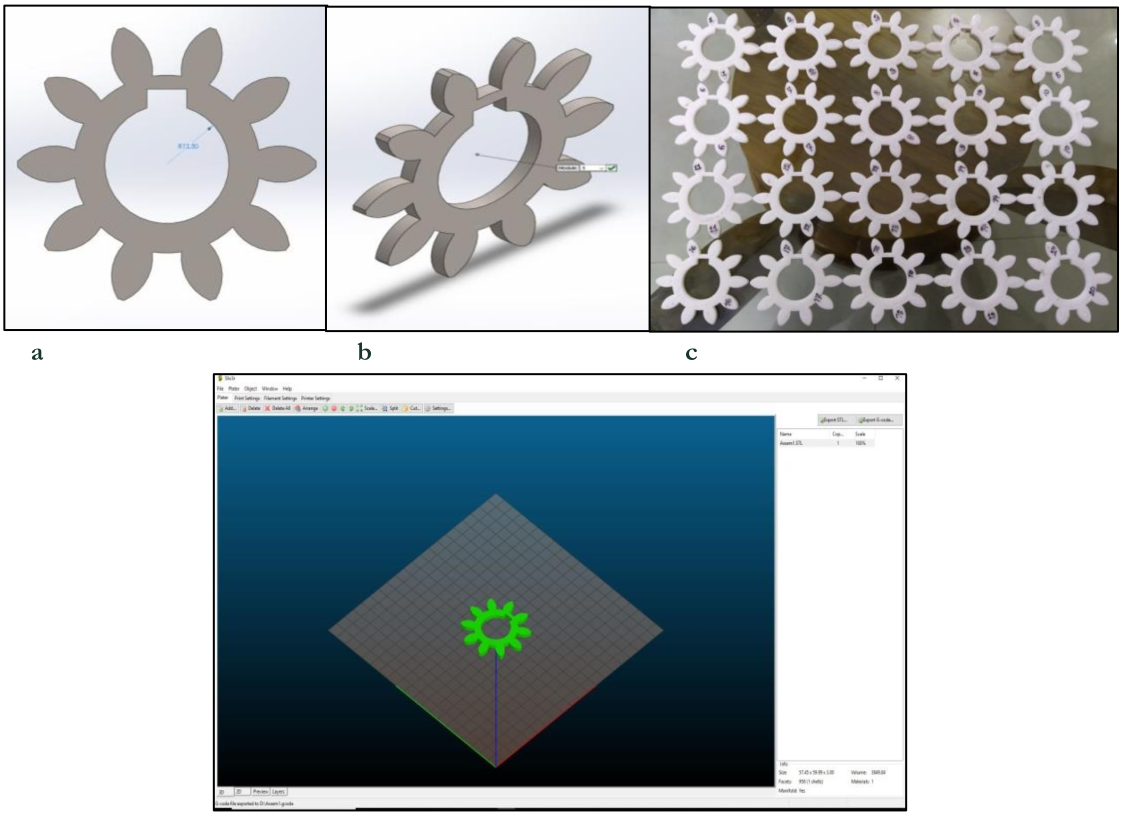

Fig. 2. a. Dimension of the test specimen; b. 20 3D printed spur gears; c. conversion of CAD model into G-code through Slic3r software.

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Table 3. Experimental data for input process parameters and response variable.

Run	Factor 1 A: Nozzle Temperature (°C)	Factor 2 B: Extrusion Speed (%)	Factor 3 C: Fill Density (%)	Response Gear Bore Diameter (mm)	Response Extrusion Time (Min)
1	230.000	35.000	25.000	24.7001	42.06
2	220.000	40.000	20.000	24.6706	36.23
3	230.000	26.591	25.000	24.7214	53.12
4	230.000	35.000	25.000	24.6936	42.06
5	230.000	35.000	25.000	24.6981	41.24
6	230.000	35.000	17.691	24.6916	41.33
7	230.000	35.000	25.000	24.7346	41.32
8	246.818	35.000	25.000	24.699	41.56
9	220.000	30.000	20.000	24.8046	48.15
10	230.000	35.000	34.309	24.6848	41.56
11	240.000	40.000	20.000	24.6991	36.05
12	240.000	40.000	30.000	24.7072	37.33
13	230.000	43.409	25.000	24.7069	34.49
14	213.182	35.000	25.000	24.6379	41.29
15	220.000	40.000	30.000	24.6524	36.28
16	230.000	35.000	25.000	24.7014	41.09
17	240.000	30.000	30.000	24.7259	48.19
18	230.000	35.000	25.000	24.6512	41.57
19	240.000	30.000	20.000	24.8963	47.35
20	220.000	30.000	30.000	24.6357	48.53

4 | Results and Analysis

4.1 | Dimensional Accuracy

ANOVA has been done as shown in *Table 4* to observe the influence of the process parameters which are nozzle temperature (A), extrusion speed (B) and fill density (C) on output response which is gear bore diameter. As per the ANOVA test the calculated "F value" of the second-order model is 3.11 implies the model is significant. There is only a 4.57% chance that an F-value this large could occur due to noise. P-values less than 0.0500 indicate model terms are significant. In this case A, B, C and BC are significant model terms. Values greater than 0.1000 indicate the model terms are not significant. The "Lack of Fit F-value" of 3.82 implies the Lack of Fit is not significant relative to the pure error. There is an 8.37% chance that a "Lack of Fit F-value" this large could occur due to noise. The R2 value is close to 1, which is desirable. The adjusted R2 value is particularly useful when comparing models with different number of terms. Adequate precision compares the range of the predicted values at the design points to the average prediction errors. Ratios greater than 4 indicate adequate model discrimination. In this particular case, the value is 7.478 indicates an adequate signal as it can be seen in *Table 4*. Equation in *Table 5* is valid and can be used to predict the gear bore diameter.

Table 4. ANOVA for main and interaction effects on average gear bore diameter.

Source	Sum of Squares	df	Mean Square	F-value	p-value	
Model	0.0480	9	0.0053	3.11	0.0457	significant
A-Nozzle Temperature	0.0099	1	0.0099	5.79	0.0370	
B-Extrusion Speed	0.0094	1	0.0094	5.47	0.0415	
C-Fill Density	0.0109	1	0.0109	6.37	0.0302	
BC	0.0135	1	0.0135	7.91	0.0184	
Residual	0.0171	10	0.0017			
Lack of Fit	0.0136	5	0.0027	3.82	0.0837	not significant
Pure Error	0.0036	5	0.0007			
Cor Total	0.0651	19				
Std. Dev.	0.0414	R^2		0.7370		
Mean	24.71	Adjusted R^2		0.5002		
C.V. %	0.1675	Predicted R^2		0.5235		
		Adequate Precision		7.4776		

Table 5. Final equation in terms of actual factors.

Gear bore Diameter	=
+25.84549	
+0.002694	Nozzle Temperature
-0.046391	Extrusion Speed
-0.063026	Fill Density
+0.001646	Extrusion Speed * Fill Density

Fig. 3 shows the trace or perturbation plot. The perturbation plot compares the effects of the various factors in the design space. The intersection of the lines is at the reference point (where, X=0.00) and the actual conditions for the factors at the side point are as indicated in the figure. For an instance, in case of factor A, any shift to the right of the reference point (or towards the +1.00 of the deviation from the reference point axis) i.e. as the nozzle temperature (A) increases, the gear bore diameter increases. However, in case of extrusion speed (B) and fill density (C), gear bore diameter tends to decrease with a shift from the reference point to the right. Comparisons of the predicted results and the experimental results of the gear bore diameter were also performed. The experimental and predicted values were compared as shown in *Fig. 4*. For a good fit, the points are located in the vicinity of the fitted line, with narrow confidence bands. Points on the left or right of the plot, furthest from the mean, have the most leverage and effectively try to pull the fitted line towards the point. Points that are vertically distant from the line represent possible outliers. *Fig. 4* shows that the points that have been plot are mostly close to the fitted line so the model that had been generated can be considered as a good prediction in estimating the predicted gear bore diameter values.Fig. 3. Perturbation plot of factors in measuring the dimensional accuracy.

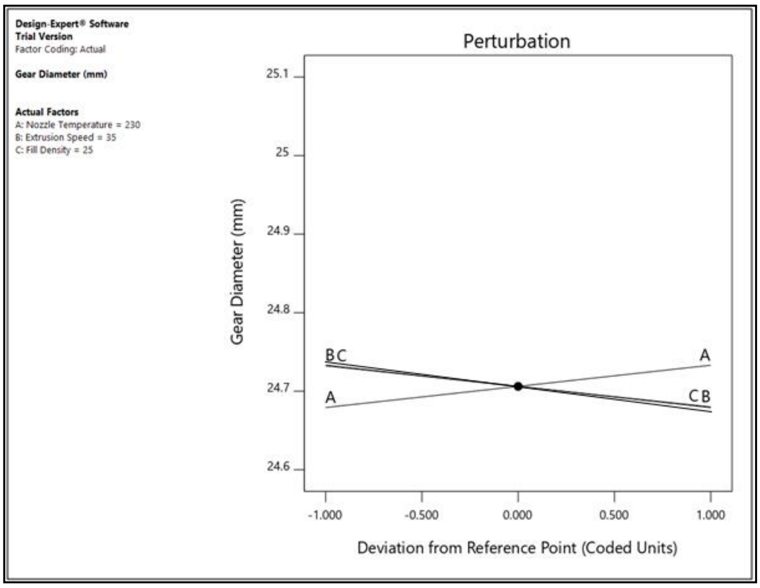

Fig. 3. Perturbation plot of factors in measuring the dimensional accuracy.

The response surface plot is a good tool to estimate the region of optimum response, which is basically similar to the 3-D wire frame plot. *Fig. 5* represents the gear bore diameter as a function of nozzle temperature (A) and extrusion speed (B). In this case, feed (C) was kept at '20' level value. The plot for Figure 5 shows that the gear bore diameter decreases as extrusion speed increases and gear bore diameter decreases with the decrease of nozzle temperature. The response surface plot is a good tool to estimate the region of optimum response, which is basically similar to the 3-D wire frame plot. *Fig. 6* represents the gear bore diameter as a function of extrusion speed (B) and fills density (C). In this case, nozzle temperature (A) was kept at '230' level value. The plot for *Fig. 6* shows that the gear bore diameter decreases as fill density increases and gear bore diameter decreases with the increase of extrusion speed. The interaction between extrusion speeds and fill density also appear to have a dominating effect on gear bore diameter.

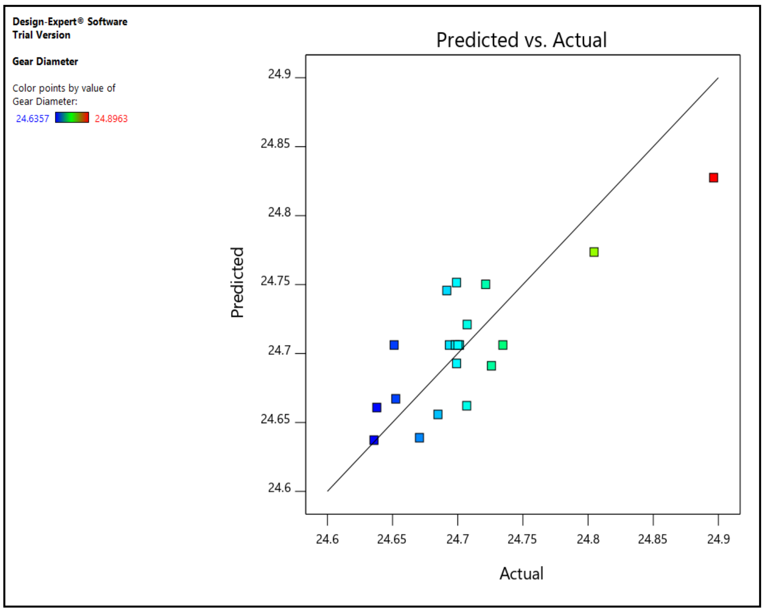

Fig. 4. Comparison of experimental and predicted values (Gear Bore).

The response surface plot is a good tool to estimate the region of optimum response, which is basically similar to the 3-D wire frame plot. *Fig. 5* represents the gear bore diameter as a function of nozzle temperature (A) and extrusion speed (B). In this case, feed (C) was kept at '20' level value. The plot for Figure 5 shows that the gear bore diameter decreases as extrusion speed increases and gear bore diameter decreases with the decrease of nozzle temperature. The response surface plot is a good tool to estimate the region of optimum response, which is basically similar to the 3-D wire frame plot. *Fig. 6* represents the gear bore diameter as a function of extrusion speed (B) and fills density (C). In this case, nozzle temperature (A) was kept at '230' level value. The plot for Fig. 6 shows that the gear bore diameter decreases as fill density increases and gear bore diameter decreases with the increase of extrusion speed. The interaction between extrusion speeds and fill density also appear to have a dominating effect on gear bore diameter.

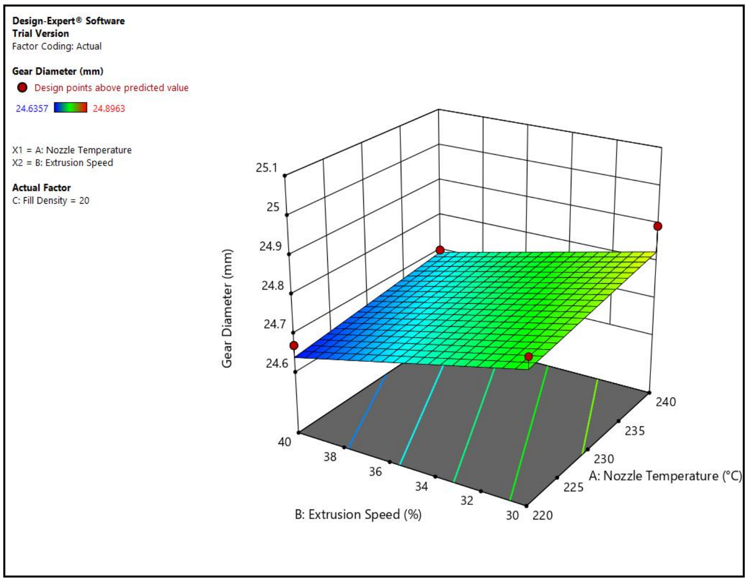

Fig. 5. Interaction effect analysis of factor A and B for the gear bore diameter.

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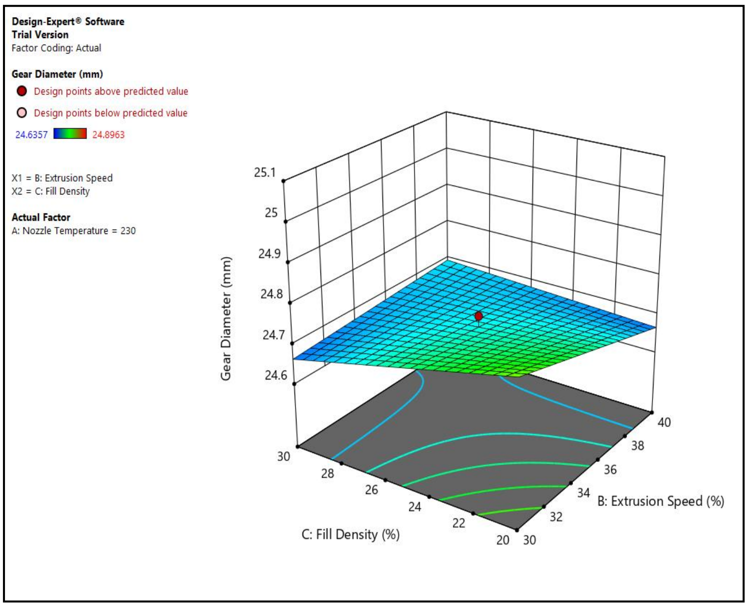

Fig. 6. Interaction effect analysis of factor B and C for the gear bore diameter.

From *Fig. 7*, the first optimum setting that was predicted by the desirability analysis is the nozzle temperature with the maximum value of 240 °C. In addition, the optimum setting for extrusion speed hit the minimum value from the parameter range which is 30%. Lastly, the predicted optimum setting for fill density is also the minimum value from the range that has been set which is 20%. Furthermore, the optimum predicted gear bore diameter by RSM is 24.8275 mm.

Table 6. Values of process parameters for the optimization of gear bore.

Number	A: Nozzle Temperature	B: Extrusion Speed	C: Fill Density	Gear Bore	Desirability	
1	240.000	30.000	20.000	24.827	0.736	Selected
2	240.000	30.000	20.038	24.827	0.734	
3	240.000	30.040	20.000	24.827	0.734	
4	239.718	30.000	20.000	24.827	0.733	
5	240.000	30.000	20.061	24.827	0.733	
6	239.626	30.000	20.000	24.826	0.732	
7	240.000	30.000	20.098	24.826	0.731	
8	240.000	30.100	20.000	24.826	0.731	
9	239.397	30.000	20.000	24.826	0.730	
10	240.000	30.000	20.142	24.826	0.729	

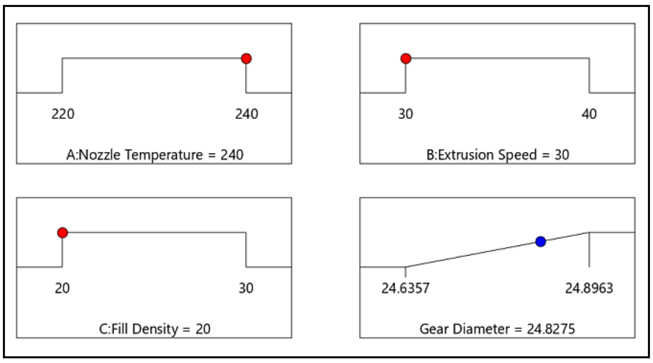

Fig. 7. Optimal parameters for gear bore diameter from RSM optimization.

4.2 | Extrusion Time

ANOVA has been done as shown in *Table 7* to observe the influence of the process parameters which are nozzle temperature (A), extrusion speed (B) & fill density (C) on output response which is extrusion time. As per the ANOVA test the calculated "F value" of the second-order model is 664.64 implies the model is significant. There is only a 0.01% chance that an F-value this large could occur due to noise. P-values less than 0.0500 indicate model terms are significant. In this case B, C & B^2 are significant model terms. Values greater than 0.1000 indicate the model terms are not significant. The "Lack of Fit F-value" of 0.4217 implies the Lack of Fit is not significant relative to the pure error. There is an 86.69% chance that a "Lack of Fit F-value" this large could occur due to noise. The R2 value is very close to 1, which is desirable. The predicted R^2 of 0.9926 is in reasonable agreement with the adjusted R^2 of 0.9953; i.e. the difference is less than 0.2. The adjusted R2 value is particularly useful when comparing models with different number of terms. Adequate precision compares the range of the predicted values at the design points to the average prediction errors. Ratios greater than 4 indicate adequate model discrimination. In this particular case, the value is 95.973 indicates an adequate signal as it can be seen in *Table 7*.

Table 7. ANOVA for main and interaction effects on extrusion time.

Source	Sum of Squares	df	Mean Square	F-value	p-value	
Model	452.57	6	75.43	664.64	< 0.0001	significant
A-Nozzle Temperature	0.0025	1	0.0025	0.0219	0.8847	
B-Extrusion Speed	441.64	1	441.64	3891.47	< 0.0001	
C-Fill Density	0.6246	1	0.6246	5.50	0.0355	
AB	0.5050	1	0.5050	4.45	0.0549	
AC	0.3570	1	0.3570	3.15	0.0995	
B^2	9.54	1	9.54	84.04	< 0.0001	
Residual	1.48	13	0.1135			
Lack of Fit	0.5944	8	0.0743	0.4217	0.8669	not significant
Pure Error	0.8809	5	0.1762			
Cor Total	454.05	19				
Std. Dev.	0.3369	R^2		0.9968		
Mean	42.04	Adjusted R^2		0.9953		
C.V. %	0.8013	Predicted R^2		0.9926		
		Adeq Precision		95.9732		

Table 8. Final equation in terms of actual factors.

Extrusion Time	=
+184.16586	
-0.280152	Nozzle Temperature
-4.55087	Extrusion Speed
-0.928869	Fill Density
+0.005025	Nozzle Temperature * Extrusion Speed
+0.004225	Nozzle Temperature * Fill Density
+0.032254	Extrusion Speed^2

Fig. 8 shows the trace or perturbation plot. The perturbation plot compares the effects of the various factors in the design space. The intersection of the lines is at the reference point (where, X=0.00) and the actual conditions for the factors at the side point are as indicated in the figure. For an instance, in case of factor B, any shift to the right of the reference point (or towards the +1.00 of the deviation from the reference point axis) i.e. as the extrusion speed (B) increases, the extrusion time decreases. However, in case of nozzle temperature (A) and fill density (C), extrusion time tends to increase with a shift from the reference point to the right. Comparisons of the predicted results and the experimental results of the extrusion time were also performed. The experimental and predicted values were compared as shown in *Fig. 9*. For a good fit, the points are located in the vicinity of the fitted line, with narrow confidence bands. Points on the left or right of the plot, furthest from the mean, have the most leverage and effectively try

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to pull the fitted line towards the point. Points that are vertically distant from the line represent possible outliers. *Fig. 9* shows that the points that have been plot are mostly very close to the fitted line so the model that had been generated can be considered as a good prediction in estimating the predicted gear bore diameter values.

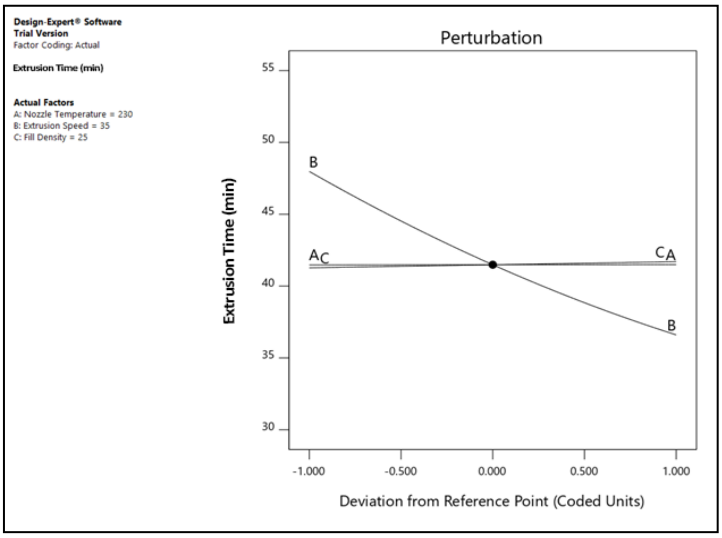

Fig. 8. Perturbation plot in measuring extrusion time.

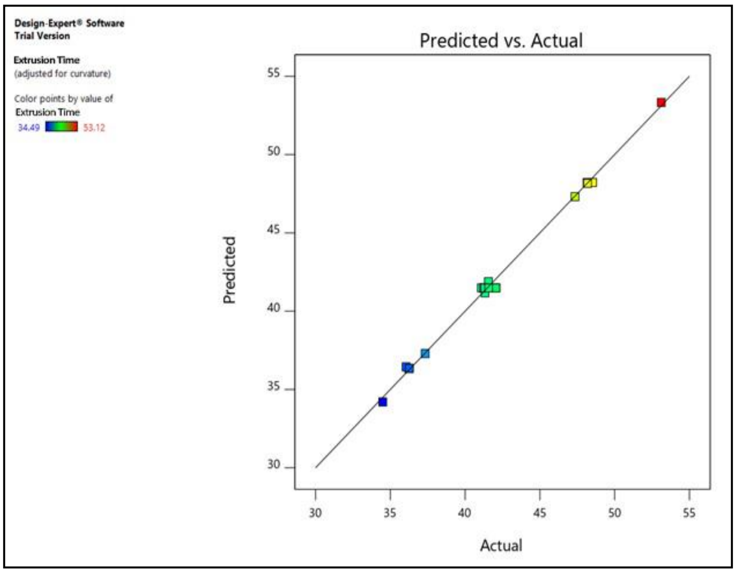

Fig. 9. Comparison of experimental and predicted values (extrusion time).

The response surface plot is a good tool to estimate the region of optimum response, which is basically similar to the 3-D wire frame plot. *Fig. 10* represents the extrusion time as a function of nozzle temperature (A) and extrusion speed (B). In this case, feed (C) was kept at '25' level value. The plot for *Fig. 10* shows that the extrusion time decreases as extrusion speed increases and extrusion time decreases with the decrease of nozzle temperature. The response surface plot is a good tool to estimate the region of optimum response, which is basically similar to the 3-D wire frame plot. *Fig. 11* represents the extrusion time as a function of extrusion speed (B) and fill density (C). In this case, nozzle temperature (A) was kept at '230' level value. The plot for *Fig. 11* shows that extrusion time decreases as fill density decreases and extrusion time decreases with the increase of extrusion speed.

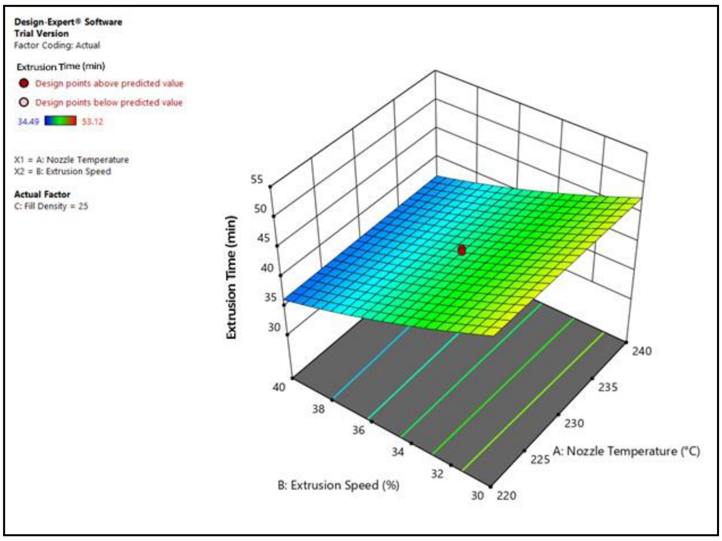

Fig. 10. Interaction effect analysis of factor A and B for the extrusion time.

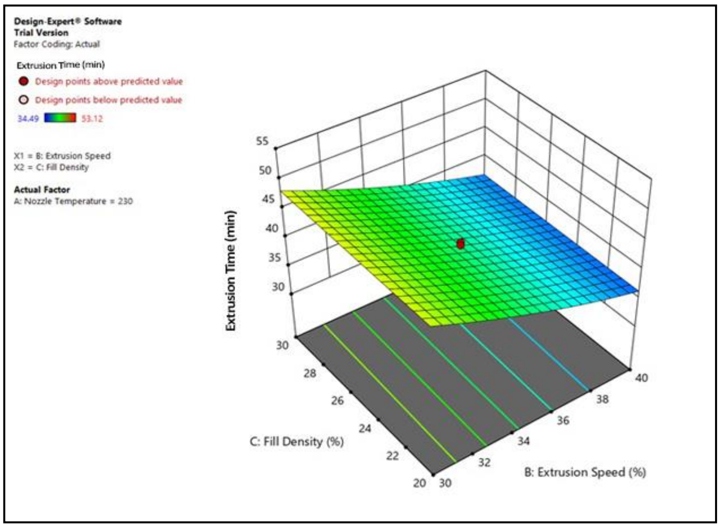

Fig. 11. Interaction effect analysis of factor B and C for the extrusion time.

From *Fig. 12*, the first optimum setting that was predicted by the desirability analysis is the nozzle temperature with the minimum value of 220 °C. In addition, the optimum setting for extrusion speed hit the maximum value from the parameter range which is 40%. Lastly, the predicted optimum setting for fill density is the minimum value from the range that has been set which is 20%. Furthermore, the optimum predicted extrusion time by RSM is 36.3232 min.

Table 9. Values of process parameters for the optimization of extrusion time.

Number	A: Nozzle Temperature	B: Extrusion Speed	C: Fill Density	Extrusion Time	Desirability	
1	220.000	40.000	20.000	36.323	0.901	Selected
2	220.002	40.000	20.072	36.337	0.901	
3	220.002	40.000	20.319	36.337	0.901	
4	220.008	40.000	20.366	36.337	0.901	
5	220.001	40.000	20.578	36.337	0.901	
6	220.048	40.000	20.153	36.337	0.901	
7	220.001	40.000	20.696	36.337	0.901	
8	220.000	40.000	20.997	36.338	0.901	
9	220.002	40.000	21.233	36.338	0.901	
10	220.000	40.000	21.404	36.338	0.901	

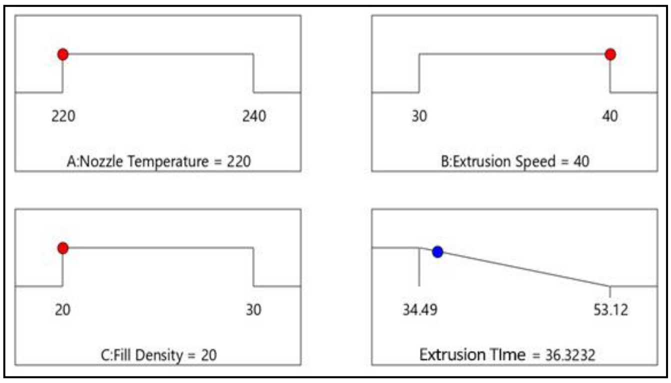

Fig. 12. Optimal parameters for extrusion time from RSM optimization.

5 | Conclusion and Recommendation

The major goal of this research was to build a 3D printer and look into the effects of various factors on the dimensional accuracy and extrusion time of PLA items that were manufactured. The construction of a portable 3D printer has been finished successfully. Aluminum channels are used to make the frame sturdy and compact. The use of a dual motor for vertical movement simplifies bed leveling. Because of the precise orientation of the motors, controlling the mechanism becomes simple, and good synchronization can be obtained with this 3D printing technology. After that, the impact of three process factors, namely nozzle temperature, extrusion speed, and fill density, on the dimensional accuracy of FDM produced components and their extrusion time, is investigated at three distinct levels. The experimental plan is created using RSM. The reduction in diameter of the specimen is observed to be greater than the desired value. RSM is used to identify relevant elements and their interactions. To increase the built part's dimensional accuracy, the parts must be manufactured in such a way that the dimensions are as close to the actual value as possible. As a result, optimum process variables should be determined using a systematic approach. The ANOVA analysis and surface interaction plot demonstrated that nozzle temperature, extrusion speed, and fill density, as well as the interaction between extrusion speeds and fill density, have a significant impact on dimensional accuracy. RSM predicts a gear bore diameter of 24.8275 mm as the best. Extrusion time is also influenced by extrusion speed and fill density. RSM predicts a maximum extrusion time of 36.3232 minutes. As a result, we conclude that a right combination of nozzle temperature, extrusion speed, and fill density can result in higher dimensional accuracy and reduced extrusion time.

The following suggestions for improving our manufactured 3D printer should be considered. For the rapid extrusion of a 3D printed component, multiple extruders could be assembled. To print an exact colored object, a multi-colored filament arrangement could be used. The use of a proximity sensor that can be easily monitored could make bed leveling much easier. The impact of other characteristics such as surface roughness and hardness can be investigated using RSM..

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Solving Semi-Fully Fuzzy Linear Programming Problems

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Abstract

The present paper aims to propose an alternative solution approach in obtaining the fuzzy optimal solution to a semi-fully fuzzy linear programming problem. In this paper, the semi-fully fuzzy linear programming problem is transformed into equivalent semi-fully interval linear programming problems. The solutions to these interval linear programming problems are then obtained with the help of linear programming technique. A set of seven random numerical examples has been solved using the proposed approach.

Keywords: Interval numbers, Fuzzy numbers, Linear programming.

1 | Introduction

Linear programming is a most widely and successfully used decision tool in the quantitative analysis of practical problems where rational decisions have to be made. In order to solve a Linear Programming Problem, the decision parameters of the model must be fixed at crisp values. But to model real-life problems and perform computations we must deal with uncertainty and inexactness. These uncertainty and inexactness are due to measurement inaccuracy, simplification of physical models, variations of the parameters of the system, computational errors etc. Interval and fuzzy analysis are an efficient and reliable tool that allows us to handle such problems effectively.

Several researchers have carried out investigations on the semi-fully fuzzy linear programming problems with interval numbers, triangular fuzzy numbers, trapezoidal fuzzy numbers, pentagonal fuzzy numbers, hexagonal fuzzy numbers, heptagonal fuzzy numbers, octagonal fuzzy numbers, nonagonal fuzzy numbers, decagonal fuzzy numbers, hendecagonal fuzzy numbers and dodecagonal fuzzy numbers.

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Ramesh and Ganesan [1] proposed the solution concepts of primal and dual linear programming problems involving interval numbers without converting them to classical linear programming problems. Nasseri et al. [4] proposed a method to find the fuzzy optimal solution of fully fuzzy linear programming problems with equality constraints having lr flat fuzzy numbers. Siddi [12] proposed a method for solving fuzzy linear programming problem with pentagonal fuzzy number by using a ranking function and compared the solutions with fully fuzzy linear programming problem. Ingle and Ghadle [13] presented fully fuzzy linear programming problem with hexagonal fuzzy number is solved by new ranking function. They converted the fully fuzzy linear programming problem to a crisp valued problem then can be solved using Simplex/Big-M method. Slevakumari and Tamilarasi [14] presented a paper aims at solving linear programming problems in which the parameters are octagonal fuzzy numbers with the help of robust ranking method. Das [16] proposed an approach to optimize the cost of transportation problem based on triangular fuzzy programming problem. Das et al. [17] presented a mathematical model for solving fully fuzzy linear programming problem with trapezoidal fuzzy numbers. Das et al. [17] presented a modified ranking function of linear programming problem directly approach to fuzzy environment.

In this paper, a new improved method for solving the semi-fully fuzzy linear programming problems is proposed. This new method finds the fuzzy optimal solution of semi-fully fuzzy linear programming problems. Moreover, the new method improves the existing methods for solving the interval transportation problems and fully fuzzy transportation problems [6] and [15].

In general, most of the existing techniques provide only crisp solutions for the semi-fully fuzzy linear programming problems. In contrast to most existing approaches, our method of transforming a fuzzy number into interval numbers is the first. So, our method proposed is the first. Also, the fuzzy optimal solution, obtained by using the new method mentioned, will always exactly satisfy the centers of all the constraints and some constraints.

The contributions of the present study are summarized as follows: (a) we introduce new technique for improve the methods for solving the semi-fully interval linear programming problems *Eq. (6)*. (b) We introduce a formulation of semi-fully fuzzy linear programming problems *Eq. (12)*. (c) According to the proposed approach, the *Eq. (12)* is converted into classical linear programming problems and/or interval linear programming problems. The integration of the interval optimal solutions of the sub-problems provides the fuzzy optimal solution of the problem *Eq. (12)*. (d) An algorithm for the new proposed method and is developed to find the fuzzy optimal solution of the problem *Eq. (12)*. (e) The complexity of computation is greatly reduced compared with commonly used existing methods in the literature.

The rest of this paper is organized as follows. In Section 2, some basic definition, arithmetic operations and semi-fully interval linear programming problems are reviewed. Furthermore, we attempt to introduce a formulation of semi-fully fuzzy linear problem. In Section 3, we propose a simple method for solving semi-fully fuzzy linear programming problems and a new fuzzy arithmetic on fuzzy or interval numbers. In Section 4, seven numerical examples are presented to illustrate the proposed method. Advantages of the proposed method over the existing methods are discussed in Section 5. Finally, concluding remarks and future research directions are presented in Section 6.

2 | Materials and Methods

In this section, some basic definitions, arithmetic operations for closed Intervals numbers and of linear programming problems involving interval and fuzzy numbers are presented.

2.1 | A New Interval Arithmetic

In this section, some arithmetic operations for two intervals are presented [1].

Let $\Re = \{\bar{a} = [a^1, a^2] : a^1 \le a^2, a^1, a^2 \in \mathbb{R}\}$ be the set of all proper intervals. We shall use the terms "interval" and "interval number" interchangeably. The mid-point and width (or half-width) of an interval number $\bar{a} = [a^1, a^2]$ are defined as $m(\bar{a}) = \frac{a^2 + a^1}{2}$ and $w(\bar{a}) = \frac{a^2 - a^1}{2}$.

The interval number $\bar{a}$ can also be expressed in terms of its midpoint and width as

$$\bar{a} = [a^1, a^2] = \langle m(\bar{a}), w(\bar{a}) \rangle = \langle \frac{a^2 + a^1}{2}, \frac{a^2 - a^1}{2} \rangle. \tag{1}$$

For any two intervals $\bar{a} = [a^1, a^2] = \langle m(\bar{a}), w(\bar{a}) \rangle$ and $\bar{b} = [b^1, b^2] = \langle m(\bar{b}), w(\bar{b}) \rangle$, the arithmetic operations on $\bar{a}$ and $\bar{b}$ are defined as:

$$\text{Addition: } \bar{a} + \bar{b} = \langle m(\bar{a}) + m(\bar{b}), w(\bar{a}) + w(\bar{b}) \rangle. \tag{2}$$

$$\text{Subtraction: } \bar{a} - \bar{b} = \langle m(\bar{a}) - m(\bar{b}), w(\bar{a}) + w(\bar{b}) \rangle. \tag{3}$$

$$\text{Multiplication: } \alpha\bar{a} = \begin{cases} \langle \alpha m(\bar{a}), \alpha w(\bar{a}) \rangle \text{ if } \alpha \ge 0 \\ \langle \alpha m(\bar{a}), -\alpha w(\bar{a}) \rangle \text{ if } \alpha < 0 \end{cases}. \tag{4}$$

$$\bar{a} \times \bar{b} = \begin{cases} \langle m(\bar{a})m(\bar{b}) + w(\bar{a})w(\bar{b}), m(\bar{a})w(\bar{b}) + m(\bar{b})w(\bar{a}) \rangle \text{ if } a^1 \ge 0, b^1 \ge 0 \\ \langle m(\bar{a})m(\bar{b}) + m(\bar{a})w(\bar{b}), m(\bar{b})w(\bar{a}) + w(\bar{b})w(\bar{a}) \rangle \text{ if } a^1 < 0, b^1 \ge 0 \\ \langle m(\bar{a})m(\bar{b}) - w(\bar{a})w(\bar{b}), m(\bar{b})w(\bar{a}) - m(\bar{a})w(\bar{b}) \rangle \text{ if } a^2 \le 0, b^1 \ge 0 \end{cases} \tag{5}$$

2.2 | Formulation of Semi-Fully Interval Linear Programming Problem

We consider the linear programming problem involving interval numbers as follows [1]:

$$\begin{cases} \text{Max / Min } \bar{Z}^{pq}(\bar{x}^{pq}) \approx \sum_{j=1}^{n} \bar{c}_j^{pq} \bar{x}_j^{pq} \\ \quad \text{Subject to the constraints} \\ \sum_{j=1}^{n} a_{ij} \bar{x}_j^{pq} \left(\approx \right) \bar{b}_i^{pq}, \text{ for } i = 1, 2, \ldots, m. \end{cases} \tag{6}$$

Where:

I. p and q are integers $(\mathbb{N})$ with $q \ge p$.

II. $\bar{c}_j^{pq} = [c_j^p, c_j^q]$ are non-negatives interval numbers.

III. $\bar{x}_j^{pq} = [x_j^p, x_j^q]$ and $\bar{b}_i^{pq} = [b_i^p, b_i^q]$ are unrestricted interval numbers.

IV. a_{ij} are real numbers $(\mathbb{R})$.

Object function transformation.

$$\bar{Z}^{pq}\left(\bar{x}^{pq}\right)\approx\langle m\left(\bar{Z}^{pq}\left(\bar{x}^{pq}\right)\right),w\left(\bar{Z}^{pq}\left(\bar{x}^{pq}\right)\right)\rangle$$

$$Max/Min\,\bar{Z}^{pq}\left(\bar{x}^{pq}\right)\approx\sum_{j=1}^{n}\bar{c}_j^{pq}\bar{x}_j^{pq}=\sum_{j=1}^{n}\langle m\left(\bar{c}_j^{pq}\bar{x}_j^{pq}\right),w\left(\bar{c}_j^{pq}\bar{x}_j^{pq}\right)\rangle \text{ where}$$

$$m\left(\bar{c}_j^{pq}\bar{x}_j^{pq}\right)=\begin{cases} m\left(\bar{c}_j^{pq}\right)m\left(\bar{x}_j^{pq}\right)+w\left(\bar{c}_j^{pq}\right)w\left(\bar{x}_j^{pq}\right) \text{ if } x_j^{p}\geq 0 \\ m\left(\bar{c}_j^{pq}\right)m\left(\bar{x}_j^{pq}\right)+w\left(\bar{c}_j^{pq}\right)m\left(\bar{x}_j^{pq}\right) \text{ if } x_j^{p}\langle 0 \text{ and } x_j^{q}\rangle 0. \\ m\left(\bar{c}_j^{pq}\right)m\left(\bar{x}_j^{pq}\right)-w\left(\bar{c}_j^{pq}\right)w\left(\bar{x}_j^{pq}\right) \text{ if } x_j^{q}<0 \end{cases} \quad (7)$$

And

$$w\left(\bar{c}_j^{pq}\bar{x}_j^{pq}\right)=\begin{cases} m\left(\bar{c}_j^{pq}\right)w\left(\bar{x}_j^{pq}\right)+m\left(\bar{c}_j^{pq}\right)w\left(\bar{x}_j^{pq}\right) \text{ if } x_j^{p}\geq 0 \\ m\left(\bar{c}_j^{pq}\right)w\left(\bar{x}_j^{pq}\right)+w\left(\bar{c}_j^{pq}\right)w\left(\bar{x}_j^{pq}\right) \text{ if } x_j^{p}\langle 0 \text{ and } x_j^{q}\rangle 0. \\ m\left(\bar{c}_j^{pq}\right)w\left(\bar{x}_j^{pq}\right)-m\left(\bar{c}_j^{pq}\right)w\left(\bar{x}_j^{pq}\right) \text{ if } x_j^{q}<0 \end{cases} \quad (8)$$

Transformation of constraints.

$$\sum_{j=1}^{n}a_{ij}\bar{x}_j^{pq}\left(\begin{matrix}\approx\end{matrix}\right)\bar{b}_i^{pq}\Leftrightarrow\sum_{j=1}^{n}\langle a_{ij}m\left(\bar{x}_j^{pq}\right),w\left(a_{ij}\bar{x}_j^{pq}\right)\rangle\left(\begin{matrix}\approx\end{matrix}\right)\langle m\left(\bar{b}_i^{pq}\right),w\left(\bar{b}_i^{pq}\right)\rangle \text{ then}$$

$$\langle\sum_{j=1}^{n}a_{ij}m\left(\bar{x}_j^{pq}\right),\sum_{j=1}^{n}w\left(a_{ij}\bar{x}_j^{pq}\right)\rangle\left(\begin{matrix}\approx\end{matrix}\right)\langle m\left(\bar{b}_i^{pq}\right),w\left(\bar{b}_i^{pq}\right)\rangle.$$

We can write the following remark

Remark 1. For $k\in\left[1,m\right]$, we have

- $\sum_{j=1}^{n}a_{kj}\bar{x}_j^{pq}=\bar{b}_k^{pq}$ *if and only if* $\sum_{j=1}^{n}a_{kj}m\left(\bar{x}_j^{pq}\right)=m\left(\bar{b}_k^{pq}\right)$ *and* $\sum_{j=1}^{n}w\left(a_{kj}\bar{x}_j^{pq}\right)=w\left(\bar{b}_k^{pq}\right)$.
- $\sum_{j=1}^{n}a_{kj}\bar{x}_j^{pq}\neq\bar{b}_k^{pq}$ *if and only if* $\sum_{j=1}^{n}a_{kj}m\left(\bar{x}_j^{pq}\right)=m\left(\bar{b}_k^{pq}\right)$ *and* $\sum_{j=1}^{n}w\left(a_{kj}\bar{x}_j^{pq}\right)\neq w\left(\bar{b}_k^{pq}\right)$.

Remark 2. For $k\in\left[1,m\right]$, we have

- $\sum_{j=1}^{n}a_{kj}\bar{x}_j^{pq}=\bar{b}_k^{pq}$ *if and only if the slack variable* $x_{n+k}^{pq}=0$.
- $\sum_{j=1}^{n}a_{kj}\bar{x}_j^{pq}\neq\bar{b}_k^{pq}$ *if and only if the slack variable* $x_{n+k}^{pq}\neq 0$.

From *Remark 1* and *2*, we can say that

$$\begin{cases} Max/Min\ \bar{Z}^{pq}\left(\bar{x}^{pq}\right) \approx \sum_{j=1}^{n} \bar{c}_j^{pq} \bar{x}_j^{pq} = \sum_{j=1}^{n} \langle m\left(\bar{c}_j^{pq} \bar{x}_j^{pq}\right), w\left(\bar{c}_j^{pq} \bar{x}_j^{pq}\right) \rangle \\ \text{Subject to the constraints} \\ \langle \sum_{j=1}^{n} m\left(a_{ij} \bar{x}_j^{pq}\right), \sum_{j=1}^{n} w\left(a_{ij} \bar{x}_j^{pq}\right) \rangle \left(\approx \right) \langle m\left(\bar{b}_i^{pq}\right), w\left(\bar{b}_i^{pq}\right) \rangle, \text{ for } i = 1,2,\ldots,m. \end{cases} \tag{9}$$

From *Eqs. (7), (8)* and *(9)*, we can get:

$$\begin{cases} Max/Min\ m\left(\bar{Z}^{pq}\left(\bar{x}^{pq}\right)\right) = \sum_{j=1}^{n} m\left(\bar{c}_j^{pq} \bar{x}_j^{pq}\right) = \sum_{j=1}^{n} m\left(\bar{c}_j^{pq}\right) m\left(\bar{x}_j^{pq}\right) \\ \text{Subject to the constraints} \\ \sum_{j=1}^{n} a_{ij} m\left(\bar{x}_j^{pq}\right) \begin{pmatrix} = \\ \geq \\ \leq \end{pmatrix} m\left(\bar{b}_i^{pq}\right), \text{ for } i = 1,2,\ldots,m. \end{cases} \tag{10}$$

Where $\sum_{j=1}^{n} w\left(a_{kj} \bar{x}_j^{pq}\right) = w\left(\bar{b}_k^{pq}\right)$ or $\sum_{j=1}^{n} w\left(a_{kj} \bar{x}_j^{pq}\right) \neq w\left(\bar{b}_k^{pq}\right)$ for $k \in [1, m]$.

And *Eq. (10)* is equivalent to

$$\begin{cases} Max/Min\ m\left(\bar{Z}^{pq}\left(\bar{x}^{pq}\right)\right) = \sum_{j=1}^{n} \left(\frac{c_j^p + c_j^q}{2}\right) x_j^{pq} \\ \text{Subject to the constraints} \\ \sum_{j=1}^{n} a_{ij} x_j^{pq} \begin{pmatrix} = \\ \geq \\ \leq \end{pmatrix} \frac{b_i^p + b_i^q}{2}, \text{ for } i = 1,2,\ldots,m. \end{cases} \tag{11}$$

Optimal solution according to the choice of the decision maker:

$Max/Min\ \bar{Z}^{pq}\left(\bar{x}^{pq}\right) \approx \sum_{j=1}^{n} \bar{c}_j^{pq} \bar{x}_j^{pq}$ with $\bar{x}_j^{pq} = \left[x_j^{pq} - w\left(\bar{x}_j^{pq}\right), x_j^{pq} + w\left(\bar{x}_j^{pq}\right)\right]$ and

$$w\left(\bar{x}_j^{pq}\right) = \frac{x_j^q - x_j^p}{2}.$$

For $x_{n+k}^{pq} = 0$, we have $\sum_{j=1}^{n} a_{kj} \bar{x}_j^{pq} = \bar{b}_k^{pq}$ and $\sum_{j=1}^{n} w\left(a_{kj} \bar{x}_j^{pq}\right) = w\left(\bar{b}_k^{pq}\right)$.

2.3 | Formulation of Semi-Fully Fuzzy Linear Programming Problems

Since the fuzziness may appear in many ways for the parameters of linear programming models, hence the definition of fuzzy linear programming is not unique. One of these models is Semi-Fully Fuzzy Linear Programming problem where the coefficients in the objective function, the right hand side vector and the decision variables are a kind of fuzzy numbers, simultaneity. This paper is assigned to these type of problems. Consider the Semi-fully Fuzzy Linear Programming Problems as follows [3] and [4]:

$$\begin{cases} \text{Max / Min } \tilde{Z}(\tilde{x}) \approx \sum_{j=1}^{n} \tilde{c}_j \tilde{x}_j \\ \text{Subject to the constraints} \\ \sum_{j=1}^{n} a_{ij}\tilde{x}_j \begin{pmatrix} \approx \\ \\ \end{pmatrix} \tilde{b}_i, \text{ for } i = 1,2,..,m. \end{cases} \quad (12)$$

Where non-negatives fuzzy numbers, and are unrestricted fuzzy numbers and are real numbers.

3 | Results

In this section, a solution procedure for solving the problem *Eq. (6)* via *Eq. (11)* is developed in the following steps:

Step 1. Construct the fuzzy linear programming problem *Eq. (12)*, and then convert it into an interval linear programming problem *Eq. (6)* based on the new arithmetic of fuzzy or interval numbers.

Step 2. Convert the problem *Eq. (6)* into the corresponding classical linear programming problems *Eq. (11)* based on the new arithmetic of fuzzy or interval numbers, and then solving *Eq. (11)*:

$$Max / Min\, \left(Z^{pq}\left(x^{pq}\right)\right) = \sum_{j=1}^{n} c_j^{pq} x_j^{pq} \text{ subject to the constraints } \sum_{j=1}^{n} a_{ij} x_j^{pq} \begin{pmatrix} = \\ \le \\ \ge \end{pmatrix} \frac{b_i^q + b_i^p}{2}, x_j^{pq} \ge 0.$$

Step 3. Determine $w\left(\bar{x}_j^{pq}\right)$ with $\bar{x}_j^{pq} = \left[x_j^{pq} - w\left(\bar{x}_j^{pq}\right), x_j^{pq} + w\left(\bar{x}_j^{pq}\right)\right] = \left[x_j^p, x_j^q\right]$ for $j = 1,\ldots,n$ by applying the following conditions: if and only if the slack variable $x_{n+k}^{pq} = 0$ for $k \in \left[1, m\right]$. Considering the $\sum_{j=1}^{n} a_{kj} w\left(\bar{x}_j^{pq}\right) = w\left(\bar{b}_k^{pq}\right)$ following cases:

Case 1. t is odd or even:

I. If t is odd, then $p = q = \frac{t+1}{2}$ and $w\left(\bar{x}_j^{pq}\right) = 0$ with $w\left(\bar{x}_j^{pq}\right) = 0$ if and only if $x_j^{pq} = 0$, and go to Case 2.

II. If t is even, then $p = \frac{t}{2}$ and $q = \frac{t+2}{2}$ do:

a) If $x_j^{pq} = 0$, then $w\left(\bar{x}_j^{pq}\right) = 0$. Else, choose between (b) or (c) or (d):

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b) **Very important decision:** if $\sum_{j=1}^{n} a_{kj} w\left(\bar{x}_j^{pq}\right) = w\left(\bar{b}_k^{pq}\right)$ for all $k \in \left[1, m\right]$, then the current solution is optimal and go to Case 2.

c) **Very important decision:** if $\sum_{j=1}^{n} a_{kj} w\left(\bar{x}_j^{pq}\right) = w\left(\bar{b}_k^{pq}\right)$ for some $k \in \left[1, m\right]$, then the current solution is optimal and go to Case 2.

d) **Important decision:** choose an index k such that $\sum_{j=1}^{n} a_{kj} w\left(\bar{x}_j^{pq}\right) = w\left(\bar{b}_k^{pq}\right)$, then go to Case 2.

Case 2. For $p = q \neq \frac{t+1}{2}$, $p \neq \frac{t}{2}$ and $q \neq \frac{t+2}{2}$, then choose between (a) or (b) or (c):

a) **Very important decision:** if $\sum_{j=1}^{n} a_{kj} w\left(\bar{x}_j^{pq}\right) = w\left(\bar{b}_k^{pq}\right)$ for all $k \in \left[1, m\right]$ with $\left|x_j^{pq} - x_j^{(p+1)(q-1)}\right| + w\left(\bar{x}_j^{(p+1)(q-1)}\right) \leq w\left(\bar{x}_j^{pq}\right)$, then the current solution is optimal.

b) **Very important decision:** if $\sum_{j=1}^{n} a_{kj} w\left(\bar{x}_j^{pq}\right) = w\left(\bar{b}_k^{pq}\right)$ for some $k \in \left[1, m\right]$ with $\left|x_j^{pq} - x_j^{(p+1)(q-1)}\right| + w\left(\bar{x}_j^{(p+1)(q-1)}\right) \leq w\left(\bar{x}_j^{pq}\right)$, then the current solution is optimal.

c) **Important decision:** choose an index k such that $\sum_{j=1}^{n} a_{kj} w\left(\bar{x}_j^{pq}\right) = w\left(\bar{b}_k^{pq}\right)$ with $x_{n+k}^{pq} = 0$ and $\left|x_j^{pq} - x_j^{(p+1)(q-1)}\right| + w\left(\bar{x}_j^{(p+1)(q-1)}\right) \leq w\left(\bar{x}_j^{pq}\right)$ otherwise $w\left(\bar{x}_j^{pq}\right) = \left|x_j^{pq} - x_j^{(p+1)(q-1)}\right| + w\left(\bar{x}_j^{(p+1)(q-1)}\right)$.

3.1 | Solution Procedure for Semi-Fully Interval Linear Programming Problem (t= 2)

For all the rest of this paper, we will consider the following semi-fully interval linear programming problem as follows *Eq. (6)* and $\left(t = 2\right)$ [4] where $\bar{x}_j^{12} = \left[x_j^1, x_j^2\right]$ and $\bar{b}_1^{12} = \left[b_i^1, b_i^2\right]$.

The steps of our method for solving the semi-fully interval linear programming problem as follows *Eqs. (6) and (11)*:

Step 1. Solving *Eq. (6)* via *Eq. (11)*. We have $p = \frac{t}{2} = 1$ and $q = \frac{t+2}{2} = 2$. We get $\bar{x}_j^{12} = \left[x_j^{12} - w\left(\bar{x}_j^{12}\right), x_j^{12} + w\left(\bar{x}_j^{12}\right)\right] = \left[x_j^1, x_j^2\right]$ for $j = 1, \ldots, n$ and $Max / Min\left(\bar{Z}^{12}\left(\bar{x}^{12}\right)\right) = \sum_{j=1}^{n} c_j^{12} x_j^{12}$

subject to the constraints $\sum_{j=1}^{n} a_{ij} x_j^{12} \begin{pmatrix} = \\ \leq \\ \geq \end{pmatrix} \frac{b_i^2 + b_i^1}{2}, x_j^{12} \geq 0$.

Step 2. The optimal solution according to the choice of the decision maker is $Max / Min \bar{Z}^{12}\left(\bar{x}^{12}\right) \approx \sum_{j=1}^{n} \bar{c}_j^{12} \bar{x}_j^{12}$ with $\bar{x}_j^{12} = \left[x_j^1, x_j^2\right]$.

3.2 | Solution Procedure for Semi-Fully Fuzzy Linear Programming Problem with Triangular Fuzzy Numbers (t= 3)

For all the rest of this paper, we will consider the following semi-fully fuzzy linear programming problem with Triangular fuzzy numbers as follows *Eq. (12)* and $\left(t = 3\right)$ [3] and [4] where $\tilde{x}_j = \left(x_j^1, x_j^2, x_j^3\right) = \left(x_j^2; \bar{x}_j^{13}\right)$ and $\tilde{b}_i = \left(b_i^1, b_i^2, b_i^3\right) = \left(b_i^2; \bar{b}_i^{13}\right)$. The steps of our method for solving the semi-fully fuzzy linear programming problem with Triangular fuzzy numbers as follows:

Step 1. Solving *Eq. (6)* via *Eq. (11).* We have $p = q = \frac{t+1}{2} = 2$. We get $\bar{x}_j^{22} = \left[x_j^2, x_j^2\right] = x_j^2$ for $j = 1,\ldots,n$ and $Max / Min\,\bar{Z}\left(\bar{Z}^2\left(\bar{x}^2\right)\right) = \sum_{j=1}^{n} c_j^2 x_j^2$ subject to the constraints $\sum_{j=1}^{n} a_{ij} x_j^2 \begin{pmatrix} = \\ \leq \\ \geq \end{pmatrix} b_i^2, x_j^2 \geq 0$.

Step 2. Solving *Eq. (6)* via *Eq. (11).* We have $p = 1,\ q = 3$.
We get $\bar{x}_j^{13} = \left[x_j^{13} - w\left(\bar{x}_j^{13}\right), x_j^{13} + w\left(\bar{x}_j^{13}\right)\right] = \left[x_j^1, x_j^3\right]$ for $j = 1,\ldots,n$ and

$$Max / Min\,\bar{Z}\left(\bar{Z}^{13}\left(\bar{x}^{13}\right)\right) = \sum_{j=1}^{n} c_j^{13} x_j^{13} \text{ subject to the constraints } \sum_{j=1}^{n} a_{ij} x_j^{13} \begin{pmatrix} = \\ \leq \\ \geq \end{pmatrix} \frac{b_i^3 + b_i^1}{2}, x_j^{13} \geq 0.$$

Step 3. The optimal solution according to the choice of the decision maker is Max/ $Min\,\tilde{Z}\left(\tilde{x}\right) \approx \sum_{j=1}^{n} \tilde{c}_j \tilde{x}_j$ with $\tilde{x}_j = \left(x_j^2; \bar{x}_j^{13}\right) = \left(x_j^1, x_j^2, x_j^3\right)$.

3.3 | Solution Procedure for Semi-Fully Fuzzy Linear Programming Problem with Trapezoidal Fuzzy Numbers (t= 4)

For all the rest of this paper, we will consider the following semi-fully fuzzy linear programming problem with Trapezoidal fuzzy numbers as follows *Eq. (12)* and $(t = 4)$ [3] and [4] where $\tilde{x}_j = \left(x_j^1, x_j^2, x_j^3, x_j^4\right) = \left(\bar{x}_j^{23}; \bar{x}_j^{14}\right)$ and $\tilde{b}_i = \left(b_i^1, b_i^2, b_i^3, b_i^4\right) = \left(\bar{b}_i^{23}; \bar{b}_i^{14}\right)$. The steps of our method for solving the semi-fully fuzzy linear programming problem with Trapezoidal fuzzy numbers as follows:

Step 1. Solving *Eq. (6)* via *Eq. (11).* We have $p = \frac{t}{2} = 2, q = \frac{t+2}{2} = 3$. We get
$\bar{x}_j^{23} = \left[x_j^{23} - w\left(\bar{x}_j^{23}\right), x_j^{23} + w\left(\bar{x}_j^{23}\right)\right] = \left[x_j^2, x_j^3\right]$ for $j = 1,\ldots,n$ and

$$Max / Min\,\bar{Z}\left(\bar{Z}^{23}\left(\bar{x}^{23}\right)\right) = \sum_{j=1}^{n} c_j^{23} x_j^{23} \text{subject to the constraints } \sum_{j=1}^{n} a_{ij} x_j^{23} \begin{pmatrix} = \\ \leq \\ \geq \end{pmatrix} \frac{b_i^3 + b_i^2}{2}, x_j^{23} \geq 0.$$

Step 2. Solving *Eq. (6)* via *Eq. (11).* We have $p = 1,\ q = 4$. We get
$\bar{x}_j^{14} = \left[x_j^{14} - w\left(\bar{x}_j^{14}\right), x_j^{14} + w\left(\bar{x}_j^{14}\right)\right] = \left[x_j^1, x_j^4\right]$ for $j = 1,\ldots,n$ and

$$Max / Min\,\bar{Z}\left(\bar{Z}^{14}\left(\bar{x}^{14}\right)\right) = \sum_{j=1}^{n} c_j^{14} x_j^{14} \text{subject to the constraints } \sum_{j=1}^{n} a_{ij} x_j^{14} \begin{pmatrix} = \\ \leq \\ \geq \end{pmatrix} \frac{b_i^4 + b_i^1}{2}, x_j^{14} \geq 0.$$

Step 3. The optimal solution according to the choice of the decision maker is $Max / Min\,\tilde{Z}\left(\tilde{x}\right) \approx \sum_{j=1}^{n} \tilde{c}_j \tilde{x}_j$ with $\tilde{x}_j = \left(\bar{x}_j^{23}; \bar{x}_j^{14}\right) = \left(x_j^1, x_j^2, x_j^3, x_j^4\right)$.

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3.4 | Solution Procedure for Semi-Fully Fuzzy Linear Programming Problem with Pentagonal Fuzzy Numbers (t= 5)

For all the rest of this paper, we will consider the following semi-fully fuzzy linear programming problem with Pentagonal fuzzy numbers as follows *Eq. (12)* and $(t=5)$ [5] where $\tilde{x}_j=\left(x_j^1,x_j^2,x_j^3,x_j^4,x_j^5\right)=\left(x_j^3;\bar{x}_j^{24};\bar{x}_j^{15}\right)$ and $\tilde{b}_i=\left(b_i^1,b_i^2,b_i^3,b_i^4,b_i^5\right)=\left(b_i^3;\bar{b}_i^{24};\bar{b}_i^{15}\right)$. The steps of our method for solving the semi-fully fuzzy linear programming problem with Pentagonal fuzzy numbers as follows:

Step 1. Solving *Eq. (6)* via *Eq. (11)*. We have $p=q=\frac{t+1}{2}=3$. We get $\bar{x}_j^3=\left[x_j^3,x_j^3\right]=x_j^3$ for $j=1,\dots,n$ and $Max/Min\,\bar{Z}^3\left(\bar{x}^3\right)=\sum_{j=1}^{n}c_j^3x_j^3$ subject to the constraints $\sum_{j=1}^{n}a_{ij}x_j^3\begin{pmatrix}=\\ \le\\ \ge\end{pmatrix}b_i^3,\ x_j^3\ge 0$.

Step 2. Solving *Eq. (6)* via *Eq. (11)*. We have $p=2$ and $q=4$. We get $\bar{x}_j^{24}=\left[x_j^{24}-w\left(\bar{x}_j^{24}\right),x_j^{24}+w\left(\bar{x}_j^{24}\right)\right]=\left[x_j^2,x_j^4\right]$ for $j=1,\dots,n$ and $Max/Min\,\bar{Z}^{24}\left(\bar{x}^{24}\right)=\sum_{j=1}^{n}c_j^{24}x_j^{24}$ subject to the constraints $\sum_{j=1}^{n}a_{ij}x_j^{24}\begin{pmatrix}=\\ \le\\ \ge\end{pmatrix}\frac{b_i^4+b_i^2}{2},\ x_j^{24}\ge 0$.

Step 3. Solving *Eq. (6)* via *Eq. (11)*. We have $p=1$ and $q=5$. We get $\bar{x}_j^{15}=\left[x_j^{15}-w\left(\bar{x}_j^{15}\right),x_j^{15}+w\left(\bar{x}_j^{15}\right)\right]=\left[x_j^1,x_j^5\right]$ for $j=1,\dots,n$ and $Max/Min\,\bar{Z}^{15}\left(\bar{x}^{15}\right)=\sum_{j=1}^{n}c_j^{15}x_j^{15}$ subject to the constraints $\sum_{j=1}^{n}a_{ij}x_j^{15}\begin{pmatrix}=\\ \le\\ \ge\end{pmatrix}\frac{b_i^5+b_i^1}{2},\ x_j^{15}\ge 0$.

Step 4. The optimal solution according to the choice of the decision maker is $Max/Min\,\tilde{Z}\left(\tilde{x}\right)\approx\sum_{j=1}^{n}\tilde{c}_j\tilde{x}_j$ with $\tilde{x}_j=\left(x_j^1,x_j^2,x_j^3,x_j^4,x_j^5\right)=\left(x_j^3;\bar{x}_j^{24};\bar{x}_j^{15}\right)$.

3.5 | Solution Procedure for Semi-Fully Fuzzy Linear Programming Problem with Hexagonal Fuzzy Numbers (t= 6)

For all the rest of this paper, we will consider the following semi-fully fuzzy linear programming problem with Hexagonal fuzzy numbers as follows *Eq. (12)* and $(t=6)$ [6] where $\tilde{x}_j=\left(x_j^1,x_j^2,x_j^3,x_j^4,x_j^5,x_j^6\right)=\left(\bar{x}_{ij}^{34};\bar{x}_{ij}^{25};\bar{x}_{ij}^{16}\right)$ and $\tilde{b}_i=\left(b_i^1,b_i^2,b_i^3,b_i^4,b_i^5,b_i^6\right)=\left(\bar{b}_i^{34};\bar{b}_i^{25};\bar{b}_i^{16}\right)$. The steps of our method for solving the semi-fully fuzzy linear programming problem with Hexagonal fuzzy numbers as follows:

Step 1. Solving *Eq. (6)* via *Eq. (11)*. We have $p=\frac{t}{2}=3$ and $q=\frac{t+2}{2}=4$. We get $\bar{x}_j^{34}=\left[x_j^{34}-w\left(\bar{x}_j^{34}\right),x_j^{34}+w\left(\bar{x}_j^{34}\right)\right]=\left[x_j^3,x_j^4\right]$ for $j=1,\ldots,n$ and

$$Max/Min\left(\bar{Z}^{34}\left(\bar{x}^{34}\right)\right)=\sum_{j=1}^{n}c_j^{34}x_j^{34} \text{ subject to the constraints } \sum_{j=1}^{n}a_{ij}x_j^{34}\begin{pmatrix}=\\ \leq\\ \geq\end{pmatrix}\frac{b_i^4+b_i^3}{2},x_j^{34}\geq 0.$$

Step 2. Solving *Eq. (6)* via *Eq. (11)*. We have $p=2$ and $q=5$. We get $\bar{x}_j^{25}=\left[x_j^{25}-w\left(\bar{x}_j^{25}\right),x_j^{25}+w\left(\bar{x}_j^{25}\right)\right]=\left[x_j^2,x_j^5\right]$ for $j=1,\ldots,n$ and

$$Max/Min\left(\bar{Z}^{25}\left(\bar{x}^{25}\right)\right)=\sum_{j=1}^{n}c_j^{25}x_j^{25} \text{ subject to the constraints } \sum_{j=1}^{n}a_{ij}x_j^{25}\begin{pmatrix}=\\ \leq\\ \geq\end{pmatrix}\frac{b_i^5+b_i^2}{2},x_j^{25}\geq 0.$$

Step 3. Solving *Eq. (6)* via *Eq. (11)*. We have $p=1$ and $q=6$. We get $\bar{x}_j^{16}=\left[x_j^{16}-w\left(\bar{x}_j^{16}\right),x_j^{16}+w\left(\bar{x}_j^{16}\right)\right]=\left[x_j^1,x_j^6\right]$ for $j=1,\ldots,n$ and

$$Max/Min\left(\bar{Z}^{16}\left(\bar{x}^{16}\right)\right)=\sum_{j=1}^{n}c_j^{16}x_j^{16} \text{ subject to the constraints } \sum_{j=1}^{n}a_{ij}x_j^{16}\begin{pmatrix}=\\ \leq\\ \geq\end{pmatrix}\frac{b_i^6+b_i^1}{2},x_j^{16}\geq 0.$$

Step 4. The optimal solution according to the choice of the decision maker is $Max/Min\tilde{Z}\left(\tilde{x}\right)\approx\sum_{j=1}^{n}\tilde{c}_j\tilde{x}_j$ with $\tilde{x}_j=\left(x_j^1,x_j^2,x_j^3,x_j^4,x_j^5,x_j^6\right)=\left(\bar{x}_{ij}^{34};\bar{x}_{ij}^{25};\bar{x}_{ij}^{16}\right)$.

3.6 | Solution Procedure for Semi-Fully Fuzzy Linear Programming Problem with Heptagonal Fuzzy Numbers (t= 7)

For all the rest of this paper, we will consider the following semi-fully fuzzy linear programming problem with Heptagonal fuzzy numbers as follows *Eq. (12)* and $(t=7)$ [8] where $\tilde{x}_j=\left(x_j^1,x_j^2,x_j^3,x_j^4,x_j^5,x_j^6,x_j^7\right)=\left(x_j^4;\bar{x}_j^{35};\bar{x}_j^{26};\bar{x}_j^{17}\right)$ and

$\tilde{b}_i=\left(b_i^1,b_i^2,b_i^3,b_i^4,b_i^5,b_i^6,b_i^7\right)=\left(b_i^4;\bar{b}_i^{35};\bar{b}_i^{26};\bar{b}_i^{17}\right)$. The steps of our method for solving the semi-fully fuzzy linear programming problem with Heptagonal fuzzy numbers as follows:

Step 1. Solving *Eq. (6)* via *Eq. (11)*. We have $p=q=\frac{t+1}{2}=4$. We get $\bar{x}_j^{44}=\left[x_j^4,x_j^4\right]=x_j^4$ for $j=1,\ldots,n$ and $Max/Min\left(\bar{Z}^4\left(\bar{x}^4\right)\right)=\sum_{j=1}^{n}c_j^4x_j^4$ subject to the constraints $\sum_{j=1}^{n}a_{ij}x_j^4\begin{pmatrix}=\\ \leq\\ \geq\end{pmatrix}b_i^4,x_j^4\geq 0$.

Step 2. Solving *Eq. (6)* via *Eq. (11)*. We have $p = 3$ and $q = 5$. We get $\bar{x}_j^{35} = \left[x_j^{35} - w\left(\bar{x}_j^{35}\right), x_j^{35} + w\left(\bar{x}_j^{35}\right)\right] = \left[x_j^3, x_j^5\right]$ for $j = 1,\dots,n$ and $Max/Min\left(\bar{Z}^{35}\left(\bar{x}^{35}\right)\right) = \sum_{j=1}^{n} c_j^{35} x_j^{35}$ subject to the constraints $\sum_{j=1}^{n} a_{ij} x_j^{35} \begin{pmatrix} = \\ \le \\ \ge \end{pmatrix} \frac{b_i^5 + b_i^3}{2}$, $x_j^{35} \ge 0$.

Step 3. Solving *Eq. (6)* via *Eq. (11)*. We have $p = 2$ and $q = 6$. We get $\bar{x}_j^{26} = \left[x_j^{26} - w\left(\bar{x}_j^{26}\right), x_j^{26} + w\left(\bar{x}_j^{26}\right)\right] = \left[x_j^2, x_j^6\right]$ for $j = 1,\dots,n$ and $Max/Min\left(\bar{Z}^{26}\left(\bar{x}^{26}\right)\right) = \sum_{j=1}^{n} c_j^{26} x_j^{26}$ subject to the constraints $\sum_{j=1}^{n} a_{ij} x_j^{26} \begin{pmatrix} = \\ \le \\ \ge \end{pmatrix} \frac{b_i^6 + b_i^2}{2}$, $x_j^{26} \ge 0$.

Step 4. Solving *Eq. (6)* via *Eq. (11)*. We have $p = 1$ and $q = 7$. We get $\bar{x}_j^{17} = \left[x_j^{17} - w\left(\bar{x}_j^{17}\right), x_j^{17} + w\left(\bar{x}_j^{17}\right)\right] = \left[x_j^1, x_j^7\right]$ for $j = 1,\dots,n$ and

$Max/Min\left(\bar{Z}^{17}\left(\bar{x}^{17}\right)\right) = \sum_{j=1}^{n} c_j^{17} x_j^{17}$ subject to the constraints $\sum_{j=1}^{n} a_{ij} x_j^{17} \begin{pmatrix} = \\ \le \\ \ge \end{pmatrix} \frac{b_i^7 + b_i^1}{2}$, $x_j^{17} \ge 0$.

Step 5. The optimal solution according to the choice of the decision maker is $Max/Min\tilde{Z}\left(\tilde{x}\right) \approx \sum_{j=1}^{n} \tilde{c}_j \tilde{x}_j$ with $\tilde{x}_j = \left(x_j^1, x_j^2, x_j^3, x_j^4, x_j^5, x_j^6, x_j^7\right) = \left(x_j^4; \bar{x}_j^{35}; \bar{x}_j^{26}; \bar{x}_j^{17}\right)$.

3.7 | Solution Procedure for Semi-Fully Fuzzy Linear Programming Problem with Octagonal Fuzzy Numbers (t= 8)

For all the rest of this paper, we will consider the following semi-fully fuzzy linear programming problem with Octagonal fuzzy numbers as follows *Eq. (12)* and $(t = 8)$ [7] where $\tilde{x}_j = \left(x_j^1, x_j^2, x_j^3, x_j^4, x_j^5, x_j^6, x_j^7, x_j^8\right) = \left(\bar{x}_j^{45}; \bar{x}_j^{36}; \bar{x}_j^{27}; \bar{x}_j^{18}\right)$ and

$\tilde{b}_i = \left(b_i^1, b_i^2, b_i^3, b_i^4, b_i^5, b_i^6, b_i^7, b_i^8\right) = \left(\bar{b}_i^{45}; \bar{b}_i^{36}; \bar{b}_i^{27}; \bar{b}_i^{18}\right)$. The steps of our method for solving the semi-fully fuzzy linear programming problem with Octagonal fuzzy numbers as follows:

Step 1. Solving *Eq. (6)* via *Eq. (11)*. We have $p = \frac{t}{2} = 4$ and $q = \frac{t+2}{2} = 5$. We get $\bar{x}_j^{45} = \left[x_j^{45} - w\left(\bar{x}_j^{45}\right), x_j^{45} + w\left(\bar{x}_j^{45}\right)\right] = \left[x_j^4, x_j^5\right]$ for $j = 1,\dots,n$ and $Max/Min\left(\bar{Z}^{45}\left(\bar{x}^{45}\right)\right) = \sum_{j=1}^{n} c_j^{45} x_j^{45}$ subject to the constraints $\sum_{j=1}^{n} a_{ij} x_j^{45} \begin{pmatrix} = \\ \le \\ \ge \end{pmatrix} \frac{b_i^5 + b_i^4}{2}$, $x_j^{45} \ge 0$.

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Step 2. Solving *Eq. (6)* via *Eq. (11)*. We have $p = 3$ and $q = 6$. We get $\bar{x}_j^{36} = \left[x_j^{36} - w\left(\bar{x}_j^{36}\right), x_j^{36} + w\left(\bar{x}_j^{36}\right)\right] = \left[x_j^3, x_j^6\right]$ for $j = 1,\ldots,n$ and

$$Max/Min\,\bar{Z}^{36}\left(\bar{x}^{36}\right) = \sum_{j=1}^{n} c_j^{36} x_j^{36} \text{ subject to the constraints } \sum_{j=1}^{n} a_{ij} x_j^{36} \begin{pmatrix} = \\ \leq \\ \geq \end{pmatrix} \frac{b_i^6 + b_i^3}{2}, x_j^{36} \geq 0.$$

Step 3. Solving *Eq. (6)* via *Eq. (11)*. We have $p = 2$ and $q = 7$. We get $\bar{x}_j^{27} = \left[x_j^{27} - w\left(\bar{x}_j^{27}\right), x_j^{27} + w\left(\bar{x}_j^{27}\right)\right] = \left[x_j^2, x_j^7\right]$ for $j = 1,\ldots,n$ and

$$Max/Min\,\bar{Z}^{27}\left(\bar{x}^{27}\right) = \sum_{j=1}^{n} c_j^{27} x_j^{27} \text{ subject to the constraints } \sum_{j=1}^{n} a_{ij} x_j^{27} \begin{pmatrix} = \\ \leq \\ \geq \end{pmatrix} \frac{b_i^7 + b_i^2}{2}, x_j^{27} \geq 0.$$

Step 4. Solving *Eq. Eq. (6)* via *Eq. (11)*. We have $p = 1$ and $q = 8$. We get $\bar{x}_j^{18} = \left[x_j^{18} - w\left(\bar{x}_j^{18}\right), x_j^{18} + w\left(\bar{x}_j^{18}\right)\right] = \left[x_j^1, x_j^8\right]$ for $j = 1,\ldots,n$ and $Max/Min\,\bar{Z}^{18}\left(\bar{x}^{18}\right) = \sum_{j=1}^{n} c_j^{18} x_j^{18}$

$$\text{subject to the constraints } \sum_{j=1}^{n} a_{ij} x_j^{18} \begin{pmatrix} = \\ \leq \\ \geq \end{pmatrix} \frac{b_i^8 + b_i^1}{2}, x_j^{18} \geq 0.$$

Step 5. The optimal solution according to the choice of the decision maker is $Max/Min\,\tilde{Z}\left(\tilde{x}\right) \approx \sum_{j=1}^{n} \tilde{c}_j \tilde{x}_j$ with $\tilde{x}_j = \left(x_j^1, x_j^2, x_j^3, x_j^4, x_j^5, x_j^6, x_j^7, x_j^8\right) = \left(\bar{x}_j^{45}; \bar{x}_j^{36}; \bar{x}_j^{27}; \bar{x}_j^{18}\right)$.

3.8 | Solution Procedure for Semi-Fully Fuzzy Linear Programming Problem with Nonagonal Fuzzy Numbers (t= 9)

For all the rest of this paper, we will consider the following semi-fully fuzzy linear programming problem with Nonagonal fuzzy numbers as follows *Eq. (12)* and $\left(t = 9\right)$ [9] where $\tilde{x}_j = \left(x_j^1, x_j^2, x_j^3, x_j^4, x_j^5, x_j^6, x_j^7, x_j^8, x_j^9\right) = \left(x_j^5; \bar{x}_j^{46}; \bar{x}_j^{37}; \bar{x}_j^{28}; \bar{x}_j^{19}\right)$ and

$\tilde{b}_i = \left(b_i^1, b_i^2, b_i^3, b_i^4, b_i^5, b_i^6, b_i^7, b_i^8, b_i^9\right) = \left(b_i^5; \bar{b}_i^{46}; \bar{b}_i^{37}; \bar{b}_i^{28}; \bar{b}_i^{19}\right)$. The steps of our method for solving the semi-fully fuzzy linear programming problem with Nonagonal fuzzy numbers as follows:

Step 1. Solving *Eq. (6)* via *Eq. (11)*. We have $p = q = \frac{t+1}{2} = 5$. We get $\bar{x}_j^{55} = \left[x_j^5, x_j^5\right] = x_j^5$ for $j = 1,\ldots,n$

$$\text{and } Max/Min\,\bar{Z}^{5}\left(\bar{x}^{5}\right) = \sum_{j=1}^{n} c_j^{5} x_j^{5} \text{ subject to the constraints } \sum_{j=1}^{n} a_{ij} x_j^{5} \begin{pmatrix} = \\ \leq \\ \geq \end{pmatrix} b_i^5, x_j^5 \geq 0.$$

Step 2. Solving *Eq. (6)* via *Eq. (11)*. We have $p=4$ and $q=6$. We get $\bar{x}_j^{46}=\left[x_j^{46}-w\left(\bar{x}_j^{46}\right),x_j^{46}+w\left(\bar{x}_j^{46}\right)\right]=\left[x_j^{4},x_j^{6}\right]$ for $j=1,\ldots,n$ and $Max/Min\left(\bar{Z}^{46}\left(\bar{x}^{46}\right)\right)=\sum_{j=1}^{n}c_j^{46}x_j^{46}$

subject to the constraints $\sum_{j=1}^{n}a_{ij}x_j^{46}\begin{pmatrix}=\\ \le\\ \ge\end{pmatrix}\dfrac{b_i^{6}+b_i^{4}}{2},\ x_j^{46}\ge 0.$

Step 3. Solving *Eq. (6)* via *Eq. (11)*. We have $p=3$ and $q=7$. We get $\bar{x}_j^{37}=\left[x_j^{37}-w\left(\bar{x}_j^{37}\right),x_j^{37}+w\left(\bar{x}_j^{37}\right)\right]=\left[x_j^{3},x_j^{7}\right]$ for $j=1,\ldots,n$ and

$Max/Min\left(\bar{Z}^{37}\left(\bar{x}^{37}\right)\right)=\sum_{j=1}^{n}c_j^{37}x_j^{37}$ subject to the constraints $\sum_{j=1}^{n}a_{ij}x_j^{37}\begin{pmatrix}=\\ \le\\ \ge\end{pmatrix}\dfrac{b_i^{7}+b_i^{3}}{2},\ x_j^{37}\ge 0.$

Step 4. Solving *Eq. (6)* via *Eq. (11)*. We have $p=2$ and $q=8$. We get $\bar{x}_j^{28}=\left[x_j^{28}-w\left(\bar{x}_j^{28}\right),x_j^{28}+w\left(\bar{x}_j^{28}\right)\right]=\left[x_j^{2},x_j^{8}\right]$ for $j=1,\ldots,n$ and $Max/Min\left(\bar{Z}^{28}\left(\bar{x}^{28}\right)\right)=\sum_{j=1}^{n}c_j^{28}x_j^{28}$

subject to the constraints $\sum_{j=1}^{n}a_{ij}x_j^{28}\begin{pmatrix}=\\ \le\\ \ge\end{pmatrix}\dfrac{b_i^{8}+b_i^{2}}{2},\ x_j^{28}\ge 0.$

Step 5. Solving *Eq. (6)* via *Eq. (11)*. We have $p=1$ and $q=9$. We get $\bar{x}_j^{19}=\left[x_j^{19}-w\left(\bar{x}_j^{19}\right),x_j^{19}+w\left(\bar{x}_j^{19}\right)\right]=\left[x_j^{1},x_j^{9}\right]$ for $j=1,\ldots,n$ and $Max/Min\left(\bar{Z}^{19}\left(\bar{x}^{19}\right)\right)=\sum_{j=1}^{n}c_j^{19}x_j^{19}$

subject to the constraints $\sum_{j=1}^{n}a_{ij}x_j^{19}\begin{pmatrix}=\\ \le\\ \ge\end{pmatrix}\dfrac{b_i^{9}+b_i^{1}}{2},\ x_j^{19}\ge 0.$

Step 6. The optimal solution according to the choice of the decision maker is $Max/Min\tilde{Z}\left(\tilde{x}\right)\approx\sum_{j=1}^{n}\tilde{c}_j\tilde{x}_j$ with $\tilde{x}_j=\left(x_j^{1},x_j^{2},x_j^{3},x_j^{4},x_j^{5},x_j^{6},x_j^{7},x_j^{8},x_j^{9}\right)=\left(x_j^{5};\bar{x}_j^{46};\bar{x}_j^{37};\bar{x}_j^{28};\bar{x}_j^{19}\right)$.

3.9 | Solution Procedure for Semi-Fully Fuzzy Linear Programming Problem with Decagonal Fuzzy Numbers (t= 10)

For all the rest of this paper, we will consider the following semi-fully fuzzy linear programming problem with Decagonal fuzzy numbers as follows *Eq. (12)* and $(t=10)$ [10] where $\tilde{x}_j=\left(x_j^{1},x_j^{2},x_j^{3},x_j^{4},x_j^{5},x_j^{6},x_j^{7},x_j^{8},x_j^{9},x_j^{10}\right)=\left(\bar{x}_j^{56};\bar{x}_j^{47};\bar{x}_j^{38};\bar{x}_j^{29};\bar{x}_j^{110}\right)$ and

$\tilde{b}_i=\left(b_i^{1},b_i^{2},b_i^{3},b_i^{4},b_i^{5},b_i^{6},b_i^{7},b_i^{8},b_i^{9},b_i^{10}\right)=\left(\bar{b}_i^{56};\bar{b}_i^{47};\bar{b}_i^{38};\bar{b}_i^{29};\bar{b}_i^{110}\right)$. The steps of our method for solving the semi-fully fuzzy linear programming problem with Decagonal fuzzy numbers as follows:

Step 1. Solving *Eq. (6)* via *Eq. (11)*. We have $p=\frac{t}{2}=5$ and $q=\frac{t+2}{2}=6$. We get $\bar{x}_j^{56}=\left[x_j^{56}-w\left(\bar{x}_j^{56}\right),x_j^{56}+w\left(\bar{x}_j^{56}\right)\right]=\left[x_j^5,x_j^6\right]$ for $j=1,\ldots,n$ and

$$Max/Min\left(\bar{Z}^{56}\left(\bar{x}^{56}\right)\right)=\sum_{j=1}^{n}c_j^{56}x_j^{56} \text{ subject to the constraints } \sum_{j=1}^{n}a_{ij}x_j^{56}\begin{pmatrix}=\\ \leq\\ \geq\end{pmatrix}\frac{b_i^6+b_i^5}{2},\ x_j^{56}\geq 0.$$

Step 2. Solving *Eq. (6)* via *Eq. (11)*. We have $p=4$ and $q=7$. We get $\bar{x}_j^{47}=\left[x_j^{47}-w\left(\bar{x}_j^{47}\right),x_j^{47}+w\left(\bar{x}_j^{47}\right)\right]=\left[x_j^4,x_j^7\right]$ for $j=1,\ldots,n$ and

$$Max/Min\left(\bar{Z}^{47}\left(\bar{x}^{47}\right)\right)=\sum_{j=1}^{n}c_j^{47}x_j^{47} \text{ subject to the constraints } \sum_{j=1}^{n}a_{ij}x_j^{47}\begin{pmatrix}=\\ \leq\\ \geq\end{pmatrix}\frac{b_i^7+b_i^4}{2},\ x_j^{47}\geq 0.$$

Step 3. Solving *Eq. (6)* via *Eq. (11)*. We have $p=3$ and $q=8$. We get $\bar{x}_j^{38}=\left[x_j^{38}-w\left(\bar{x}_j^{38}\right),x_j^{38}+w\left(\bar{x}_j^{38}\right)\right]=\left[x_j^3,x_j^8\right]$ for $j=1,\ldots,n$ and

$$Max/Min\left(\bar{Z}^{38}\left(\bar{x}^{38}\right)\right)=\sum_{j=1}^{n}c_j^{38}x_j^{38} \text{ subject to the constraints } \sum_{j=1}^{n}a_{ij}x_j^{38}\begin{pmatrix}=\\ \leq\\ \geq\end{pmatrix}\frac{b_i^8+b_i^3}{2},\ x_j^{38}\geq 0.$$

Step 4. Solving *Eq. (6)* via *Eq. (11)*. We have $p=2$ and $q=9$. We get $\bar{x}_j^{29}=\left[x_j^{29}-w\left(\bar{x}_j^{29}\right),x_j^{29}+w\left(\bar{x}_j^{29}\right)\right]=\left[x_j^2,x_j^9\right]$ for $j=1,\ldots,n$ and

$$Max/Min\left(\bar{Z}^{29}\left(\bar{x}^{29}\right)\right)=\sum_{j=1}^{n}c_j^{29}x_j^{29} \text{ subject to the constraints } \sum_{j=1}^{n}a_{ij}x_j^{29}\begin{pmatrix}=\\ \leq\\ \geq\end{pmatrix}\frac{b_i^9+b_i^2}{2},\ x_j^{29}\geq 0.$$

Step 5. Solving *Eq. (6)* via *Eq. (11)*. We have $p=1$ and $q=10$. We get $\bar{x}_j^{110}=\left[x_j^{110}-w\left(\bar{x}_j^{110}\right),x_j^{110}+w\left(\bar{x}_j^{110}\right)\right]=\left[x_j^1,x_j^{10}\right]$ for $j=1,\ldots,n$ and

$$Max/Min\left(\bar{Z}^{110}\left(\bar{x}^{110}\right)\right)=\sum_{j=1}^{n}c_j^{110}x_j^{110} \text{ subject toconstraints } \sum_{j=1}^{n}a_{ij}x_j^{110}\begin{pmatrix}=\\ \leq\\ \geq\end{pmatrix}\frac{b_i^{10}+b_i^1}{2},\ x_j^{110}\geq 0.$$

Step 6. The optimal solution according to the choice of the decision maker $Max/Min\tilde{Z}\left(\tilde{x}\right)\approx\sum_{j=1}^{n}\tilde{c}_j\tilde{x}_j$ with $\tilde{x}_j=\left(x_j^1,x_j^2,x_j^3,x_j^4,x_j^5,x_j^6,x_j^7,x_j^8,x_j^9,x_j^{10}\right)=\left(\bar{x}_j^{56};\bar{x}_j^{47};\bar{x}_j^{38};\bar{x}_j^{29};\bar{x}_j^{110}\right)$.

3.10 | Solution Procedure for Semi-Fully Fuzzy Linear Programming Problem with Decagonal Fuzzy Numbers (t= 11)

For all the rest of this paper, we will consider the following semi-fully fuzzy linear programming problem with Hendecagonal fuzzy numbers as follows *Eq. (12)* and $(t=11)$ [11] where $\tilde{x}_j=\left(x_j^1,x_j^2,x_j^3,x_j^4,x_j^5,x_j^6,x_j^7,x_j^8,x_j^9,x_j^{10},x_j^{11}\right)=\left(x_j^6;\bar{x}_j^{57};\bar{x}_j^{48};\bar{x}_j^{39};\bar{x}_j^{210};\bar{x}_j^{111}\right)$ and

$\tilde{b}_i=\left(b_i^1,b_i^2,b_i^3,b_i^4,b_i^5,b_i^6,b_i^7,b_i^8,b_i^9,b_i^{10},b_i^{11}\right)=\left(b_i^6;\bar{b}_i^{57};\bar{b}_i^{48};\bar{b}_i^{39};\bar{b}_i^{210};\bar{b}_i^{111}\right)$. The steps of our method for solving the semi-fully fuzzy linear programming problem with Hendecagonal fuzzy numbers as follows:

Step 1. Solving *Eq. (6)* via *Eq. (11)*. We have $p=q=\dfrac{t+1}{2}=6$. We get $\bar{x}_j^{66}=\left[x_j^6,x_j^6\right]=x_j^6$ for $j=1,\ldots,n$ and $Max/Min\,h\left(\bar{Z}^6\left(\bar{x}^6\right)\right)=\sum_{j=1}^{n}c_j^6x_j^6$ subject to the constraints $\sum_{j=1}^{n}a_{ij}x_j^6\begin{pmatrix}=\\\le\\\ge\end{pmatrix}b_i^6,\ x_j^6\ge 0$.

Step 2. Solving *Eq. (6)* via *Eq. (11)*. We have $p=5$ and $q=7$. We get $\bar{x}_j^{57}=\left[x_j^{57}-w\left(\bar{x}_j^{57}\right),x_j^{57}+w\left(\bar{x}_j^{57}\right)\right]=\left[x_j^5,x_j^7\right]$ for $j=1,\ldots,n$ and

$Max/Min\,h\left(\bar{Z}^{57}\left(\bar{x}^{57}\right)\right)=\sum_{j=1}^{n}c_j^{57}x_j^{57}$ subject to the constraints $\sum_{j=1}^{n}a_{ij}x_j^{57}\begin{pmatrix}=\\\le\\\ge\end{pmatrix}\dfrac{b_i^7+b_i^5}{2},\ x_j^{57}\ge 0$.

Step 3. Solving *Eq. (6)* via *Eq. (11)*. We have $p=4$ and $q=8$. We get $\bar{x}_j^{48}=\left[x_j^{48}-w\left(\bar{x}_j^{48}\right),x_j^{48}+w\left(\bar{x}_j^{48}\right)\right]=\left[x_j^4,x_j^8\right]$ for $j=1,\ldots,n$ and $Max/Min\,h\left(\bar{Z}^{48}\left(\bar{x}^{48}\right)\right)=\sum_{j=1}^{n}c_j^{48}x_j^{48}$ subject to the constraints $\sum_{j=1}^{n}a_{ij}x_j^{48}\begin{pmatrix}=\\\le\\\ge\end{pmatrix}\dfrac{b_i^8+b_i^4}{2},\ x_j^{48}\ge 0$.

Step 4. Solving *Eq. (6)* via *Eq. (11)*. We have $p=3$ and $q=9$. We get $\bar{x}_j^{39}=\left[x_j^{39}-w\left(\bar{x}_j^{39}\right),x_j^{39}+w\left(\bar{x}_j^{39}\right)\right]=\left[x_j^3,x_j^9\right]$ for $j=1,\ldots,n$ and $Max/Min\,h\left(\bar{Z}^{39}\left(\bar{x}^{39}\right)\right)=\sum_{j=1}^{n}c_j^{39}x_j^{39}$ subject to the constraints $\sum_{j=1}^{n}a_{ij}x_j^{39}\begin{pmatrix}=\\\le\\\ge\end{pmatrix}\dfrac{b_i^9+b_i^3}{2},\ x_j^{39}\ge 0$.

Step 5. Solving *Eq. (6)* via *Eq. (11)*. We have $p=2$ and $q=10$. We get $\bar{x}_j^{210}=\left[x_j^{210}-w\left(\bar{x}_j^{210}\right),x_j^{210}+w\left(\bar{x}_j^{210}\right)\right]=\left[x_j^2,x_j^{10}\right]$ for $j=1,\ldots,n$ and

$Max/Min\,h\left(\bar{Z}^{210}\left(\bar{x}^{210}\right)\right)=\sum_{j=1}^{n}c_j^{210}x_j^{210}$ subject to constraints $\sum_{j=1}^{n}a_{ij}x_j^{210}\begin{pmatrix}=\\\le\\\ge\end{pmatrix}\dfrac{b_i^{10}+b_i^2}{2},\ x_j^{210}\ge 0$.

Step 6. Solving *Eq. (6)* via *Eq. (11)*. We have $p = 1$ and $q = 11$. We get

$\bar{x}_j^{111} = \left[x_j^{111} - w\left(\bar{x}_j^{111}\right), x_j^{111} + w\left(\bar{x}_j^{111}\right)\right] = \left[x_j^{1}, x_j^{11}\right]$ for $j = 1, \ldots, n$ and

$$Max/Min\left(\bar{Z}^{111}\left(\bar{x}^{111}\right)\right) = \sum_{j=1}^{n} c_j^{111} x_j^{111} \text{ subject to constraints } \sum_{j=1}^{n} a_{ij} x_j^{111} \begin{pmatrix} = \\ \le \\ \ge \end{pmatrix} \frac{b_i^{111} + b_i^{111}}{2}, x_j^{111} \ge 0.$$

Step 7. The optimal solution according to the choice of the decision maker $Max/Min\tilde{Z}\left(\tilde{x}\right) \approx \sum_{j=1}^{n} \tilde{c}_j \tilde{x}_j$ with $\tilde{x}_j = \left(x_j^1, x_j^2, x_j^3, x_j^4, x_j^5, x_j^6, x_j^7, x_j^8, x_j^9, x_j^{10}, x_j^{11}\right) = \left(x_j^6; \bar{x}_j^{57}; \bar{x}_j^{48}; \bar{x}_j^{39}; \bar{x}_j^{210}; \bar{x}_j^{111}\right)$.

3.11 | Solution Procedure for Semi-Fully Fuzzy Linear Programming Problem with Dodecagonal Fuzzy Numbers (t= 12)

For all the rest of this paper, we will consider the following semi-fully fuzzy linear programming problem with Dodecagonal fuzzy numbers as follows *Eq. (12)* and $(t = 12)$ [11] where $\tilde{x}_j = \left(x_j^1, x_j^2, x_j^3, x_j^4, x_j^5, x_j^6, x_j^7, x_j^8, x_j^9, x_j^{10}, x_j^{11}, x_j^{12}\right) = \left(\bar{x}_j^{67}; \bar{x}_j^{58}; \bar{x}_j^{49}; \bar{x}_j^{310}; \bar{x}_j^{211}; \bar{x}_j^{112}\right)$ and

$\tilde{b}_i = \left(b_i^1, b_i^2, b_i^3, b_i^4, b_i^5, b_i^6, b_i^7, b_i^8, b_i^9, b_i^{10}, b_i^{11}, b_i^{12}\right) = \left(\bar{b}_i^{67}; \bar{b}_i^{58}; \bar{b}_i^{49}; \bar{b}_i^{310}; \bar{b}_i^{211}; \bar{b}_i^{112}\right)$. The steps of our method for solving the semi-fully fuzzy linear programming problem with Dodecagonal fuzzy numbers as follows:

Step 1. Solving *Eq. (6)* via *Eq. (11)*. We have $p = \frac{t}{2} = 6$ and $q = \frac{t+2}{2} = 7$. We get

$\bar{x}_j^{67} = \left[x_j^{67} - w\left(\bar{x}_j^{67}\right), x_j^{67} + w\left(\bar{x}_j^{67}\right)\right] = \left[x_j^{6}, x_j^{7}\right]$ for $j = 1, \ldots, n$ and

$$Max/Min\left(\bar{Z}^{67}\left(\bar{x}^{67}\right)\right) = \sum_{j=1}^{n} c_j^{67} x_j^{67} \text{ subject to the constraints } \sum_{j=1}^{n} a_{ij} x_j^{67} \begin{pmatrix} = \\ \le \\ \ge \end{pmatrix} \frac{b_i^{7} + b_i^{6}}{2}, x_j^{76} \ge 0.$$

Step 2. Solving *Eq. (6)* via *Eq. (11)*. We have $p = 5$ and $q = 8$. We get

$\bar{x}_j^{58} = \left[x_j^{58} - w\left(\bar{x}_j^{58}\right), x_j^{58} + w\left(\bar{x}_j^{58}\right)\right] = \left[x_j^{5}, x_j^{8}\right]$ for $j = 1, \ldots, n$ and

$$Max/Min\left(\bar{Z}^{58}\left(\bar{x}^{58}\right)\right) = \sum_{j=1}^{n} c_j^{58} x_j^{58} \text{ subject to the constraints } \sum_{j=1}^{n} a_{ij} x_j^{58} \begin{pmatrix} = \\ \le \\ \ge \end{pmatrix} \frac{b_i^{8} + b_i^{5}}{2}, x_j^{58} \ge 0.$$

Step 3. Solving *Eq. (6)* via *Eq. (11)*. We have $p = 4$ and $q = 9$. We get

$\bar{x}_j^{49} = \left[x_j^{49} - w\left(\bar{x}_j^{49}\right), x_j^{49} + w\left(\bar{x}_j^{49}\right)\right] = \left[x_j^{4}, x_j^{9}\right]$ for $j = 1, \ldots, n$ and

$$Max/Min\left(\bar{Z}^{49}\left(\bar{x}^{49}\right)\right) = \sum_{j=1}^{n} c_j^{49} x_j^{49} \text{ subject to the constraints } \sum_{j=1}^{n} a_{ij} x_j^{49} \begin{pmatrix} = \\ \le \\ \ge \end{pmatrix} \frac{b_i^{9} + b_i^{4}}{2}, x_j^{49} \ge 0.$$

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Step 4. Solving *Eq. (6)* via *Eq. (11)*. We have $p = 3$ and $q = 10$. We get $\bar{x}_j^{310} = \left[x_j^{310} - w\left(\bar{x}_j^{310}\right), x_j^{310} + w\left(\bar{x}_j^{310}\right)\right] = \left[x_j^{3}, x_j^{10}\right]$ for $j = 1,\dots,n$ and

$$Max / Min\left(\bar{Z}^{310}\left(\bar{x}^{310}\right)\right) = \sum_{j=1}^{n} c_j^{310} x_j^{310} \text{ subject to constraints } \sum_{j=1}^{n} a_{ij} x_j^{310} \begin{pmatrix} = \\ \le \\ \ge \end{pmatrix} \frac{b_i^{10} + b_i^{3}}{2},$$

Step 5. Solving *Eq. (6)* via *Eq. (11)*. We have $p = 2$ and $q = 11$. We get $\bar{x}_j^{211} = \left[x_j^{211} - w\left(\bar{x}_j^{211}\right), x_j^{211} + w\left(\bar{x}_j^{211}\right)\right] = \left[x_j^{2}, x_j^{11}\right]$ for $j = 1,\dots,n$ and

$$Max / Min\left(\bar{Z}^{211}\left(\bar{x}^{211}\right)\right) = \sum_{j=1}^{n} c_j^{211} x_j^{211} \text{ subject to constraints } \sum_{j=1}^{n} a_{ij} x_j^{211} \begin{pmatrix} = \\ \le \\ \ge \end{pmatrix} \frac{b_i^{11} + b_i^{2}}{2}, x_j^{211} \ge 0$$

Step 6. Solving *Eq. (6)* via *Eq. (11)*. We have $p = 1$ and $q = 12$. We get $\bar{x}_j^{112} = \left[x_j^{112} - w\left(\bar{x}_j^{112}\right), x_j^{112} + w\left(\bar{x}_j^{112}\right)\right] = \left[x_j^{1}, x_j^{12}\right]$ for $j = 1,\dots,n$ and

$$Max / Min\left(\bar{Z}^{112}\left(\bar{x}^{112}\right)\right) = \sum_{j=1}^{n} c_j^{112} x_j^{112} \text{ subject to constraints } \sum_{j=1}^{n} a_{ij} x_j^{112} \begin{pmatrix} = \\ \le \\ \ge \end{pmatrix} \frac{b_i^{12} + b_i^{1}}{2}, x_j^{112} \ge 0$$

Step 7. The optimal solution according to the choice of the decision maker $Max / Min\,\tilde{Z}\left(\tilde{x}\right) \approx \sum_{j=1}^{n} \tilde{c}_j \tilde{x}_j$ with $\tilde{x}_j = \left(x_j^1, x_j^2, x_j^3, x_j^4, x_j^5, x_j^6, x_j^7, x_j^8, x_j^9, x_j^{10}, x_j^{11}, x_j^{12}\right) = \left(\bar{x}_j^{67}; \bar{x}_j^{58}; \bar{x}_j^{49}; \bar{x}_j^{310}; \bar{x}_j^{211}; \bar{x}_j^{112}\right)$.

4 | Examples

Example 1. Consider the following semi-fully interval linear programming problem [1]

Min $\bar{Z}^{12}\left(\bar{x}^{12}\right) \approx \left[25, 27\right]\bar{x}_1^{12} + \left[6, 8\right]\bar{x}_2^{12}$ subject to the constraints $6\bar{x}_1^{12} + 4\bar{x}_2^{12} \quad \left[29, 31\right]$, $5\bar{x}_1^{12} + 2\bar{x}_2^{12} \quad \left[22, 24\right]$ and $3\bar{x}_1^{12} + 5\bar{x}_2^{12} \quad \left[28, 30\right]$.

Step 1. Solving $Eq.\left(6\right)$ via $Eq.\left(10\right)$. We have $p = 1$, $q = 2$. We get Min $Z^{12}\left(x^{12}\right) = 26x_1^{12} + 7x_2^{12}$ subject to the constraints $6x_1^{12} + 4x_2^{12} \ge 30$, $5x_1^{12} + 2x_2^{12} \ge 23$ and $3x_1^{12} + 5x_2^{12} \ge 29$. Optimal solution: $x_1^{12} = 0$ and $x_2^{12} = \frac{23}{2}$. Slack variables values: $x_3^{12} = 16$, $x_4^{12} = 0$ and $x_5^{12} = \frac{57}{2}$.

Very important decision: For $x_4^{12} = 0$, we have $5w\left(\bar{x}_1^{12}\right) + 2w\left(\bar{x}_2^{12}\right) = 1.$ We get $w\left(\bar{x}_1^{12}\right) = 0$, ($x_1^{12} = 0$) and $w\left(\bar{x}_2^{12}\right) = \frac{1}{2}$. Therefore, we get $\bar{x}_1^{12} = \left[0, 0\right]$ and $\bar{x}_2^{12} = \left[11, 12\right]$.

Step 2. The optimal solution according to the choice of the decision maker is $Min\,\bar{Z}^{12}\left(\bar{x}^{12}\right)\approx\left[66,96\right]$ where $\bar{x}_1^{12}=\left[0,0\right]$ and $\bar{x}_2^{12}=\left[11,12\right]$. Then the corresponding dual problem is given by: $Max\,\bar{W}^{12}\left(\bar{y}^{12}\right)\approx\left[66,96\right]$ where $\bar{y}_1^{12}=\left[0,0\right]$, $\bar{y}_2^{12}=\left[3,4\right]$ and $\bar{y}_3^{12}=\left[0,0\right]$.

Remark 3. We see that both primal and dual problems have interval optimal solutions and the two interval optimal values are equal. In contrast to most existing approaches [1], the centers of all constraints are saturated and some constraint is saturated.

Example 2. Consider the following linear programming problem with variables given as Triangular fuzzy numbers [2]:

Max $\tilde{Z}\left(\tilde{x}\right)\approx\left(5,6,8\right)\tilde{x}_1+\left(4,4,4\right)\tilde{x}_2$ Subject to the constraints $3\tilde{x}_1+2\tilde{x}_2\quad\left(140,150,150\right)$ and $4\tilde{x}_1+3\tilde{x}_2\quad\left(155,160,165\right)$.

Step 1. Solving $Eq.\left(6\right)$ via $Eq.\left(11\right)$. We have $p=2$, $q=2$. We get Max $Z^2\left(x^2\right)=6x_1^2+4x_2^2$ subject to the constraints $3x_1^2+2x_2^2\le 150$ and $4x_1^2+3x_2^2\le 160$. Optimal solution: $x_1^2=40$ and $x_2^2=0$. Slack variables values: $x_3^2=30$ and $x_4^2=0$.

Step 2. Solving $Eq.\left(6\right)$ via $Eq.\left(11\right)$. We have $p=1, q=3$. We get Max $Z^{13}\left(x^{13}\right)=\frac{13}{2}x_1^{13}+4x_2^{13}$ subject to the constraints $3x_1^{13}+2x_2^{13}\le 145$ and $4x_1^{13}+3x_2^{13}\le 160$. Optimal solution: $x_1^{13}=40$ and $x_2^{13}=0$. Slack variables values: $x_3^{13}=25$ and $x_4^{13}=0$. Very important decision: For $x_4^{13}=0$, we have $4w\left(\bar{x}_1^{13}\right)+3w\left(\bar{x}_2^{13}\right)=w\left(\bar{b}_1^{13}\right)=5$. We get $w\left(\bar{x}_1^{13}\right)=\frac{5}{4}$ and $w\left(\bar{x}_2^{13}\right)=0$ with $\left|x_1^{13}-x_1^2\right|\le\frac{5}{4}$. Therefore, we get $\bar{x}_1^{13}=\left[\frac{155}{4},\frac{165}{4}\right]$ and $\bar{x}_2^{13}=\left[0,0\right]$.

Step 3. The optimal solution according to the choice of the decision maker is $Max\,\tilde{Z}\left(\tilde{x}\right)\approx\sum_{j=1}^{n}c_j\tilde{x}_j$ with $\tilde{x}_j=\left(x_j^2;\bar{x}_j^{13}\right)=\left(x_j^1,x_j^2,x_j^3\right)$: $Max\,\tilde{Z}\left(\tilde{x}\right)\approx\left(\frac{775}{4},240,330\right)$ where $\tilde{x}_1=\left(\frac{155}{4},40,\frac{165}{4}\right)$ and $\tilde{x}_2=\left(0,0,0\right)$. Then the corresponding dual problem is given by: $Min\,\tilde{W}\left(y\right)\approx\left(\frac{775}{4},240,330\right)$ where $\tilde{y}_1=\tilde{0}$ and $\tilde{y}_2=\left(\frac{5}{4},\frac{3}{2},2\right)$.

Remark 4. We see that both primal and dual problems have fuzzy optimal solutions and the two fuzzy optimal values are equal. In contrast to most existing approaches [2], the centers of all constraints are saturated and some constraint is saturated.

Example 3. Consider the following linear programming problem with variables given as Trapezoidal fuzzy numbers [3] and [4]:

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Min $\tilde{Z} = (1,4,8,11)\tilde{x}_1 + (2,4,6,8)\tilde{x}_2 + (1,2,4,5)\tilde{x}_3$ Subject to the constraints $3\tilde{x}_1 + 4\tilde{x}_2 + 2\tilde{x}_3 \quad (3,6,10,13)$, $4\tilde{x}_1 + 2\tilde{x}_2 + \tilde{x}_3 \quad (2,4,6,8)$ and $2\tilde{x}_1 + \tilde{x}_2 + 3\tilde{x}_3 \quad (1,2,6,7)$.

Step 1. Solving $Eq.(6)$ via $Eq.(10)$. We have $p = 2$, $q = 3$. We get Min $Z^{23}(x^{23}) = 6x_1^{23} + 5x_2^{23} + 3x_3^{23}$ subject to the constraints $3x_1^{23} + 4x_2^{23} + 2x_3^{23} \geq 8$, $4x_1^{23} + 2x_2^{23} + x_3^{23} \geq 5$ and $2x_1^{23} + x_2^{23} + 3x_3^{23} \geq 4$. Optimal solution: $x_1^{23} = \frac{2}{5}$, $x_2^{23} = \frac{7}{5}$ and $x_3^{23} = \frac{3}{5}$. Slack variables values: $x_4^{23} = 0$, $x_5^{23} = 0$ and $x_6^{23} = 0$. Very important decision: For $x_4^{23} = 0$, $x_5^{23} = 0$ and $x_6^{23} = 0$, we have $3w(\bar{x}_1^{23}) + 4w(\bar{x}_2^{23}) + 2w(\bar{x}_3^{23}) = 2$, $4w(\bar{x}_1^{23}) + 2w(\bar{x}_2^{23}) + w(\bar{x}_3^{23}) = 1$ and $2w(\bar{x}_1^{23}) + w(\bar{x}_2^{23}) + 3w(\bar{x}_3^{23}) = 2$. We get $w(\bar{x}_1^{23}) = 0$, $w(\bar{x}_2^{23}) = \frac{1}{5}$ and $w(\bar{x}_3^{23}) = \frac{3}{5}$. Therefore, we get $\bar{x}_1^{23} = \left[\frac{2}{5}, \frac{2}{5}\right]$, $\bar{x}_2^{23} = \left[\frac{6}{5}, \frac{8}{5}\right]$ and $\bar{x}_3^{23} = \left[0, \frac{6}{5}\right]$.

Step 2. Solving $Eq.(6)$ via $Eq.(10)$. We have $p = 1$, $q = 4$. We get Min $Z^{14}(x^{14}) = 6x_1^{14} + 5x_2^{14} + 3x_3^{14}$ subject to the constraints $3x_1^{14} + 4x_2^{14} + 2x_3^{14} \geq 8$, $4x_1^{14} + 2x_2^{14} + x_3^{14} \geq 5$ and $2x_1^{14} + x_2^{14} + 3x_3^{14} \geq 4$. Optimal solution: $x_1^{14} = \frac{2}{5}$, $x_2^{14} = \frac{7}{5}$ and $x_3^{14} = \frac{3}{5}$. Slack variables values: $x_4^{14} = 0$, $x_5^{14} = 0$ and $x_6^{14} = 0$. Very important decision: For $x_4^{14} = 0$, $x_5^{14} = 0$ and $x_6^{14} = 0$, we have $3w(\bar{x}_1^{14}) + 4w(\bar{x}_2^{14}) + 2w(\bar{x}_3^{14}) = 5$, $4w(\bar{x}_1^{14}) + 2w(\bar{x}_2^{14}) + w(\bar{x}_3^{14}) = 3$ and $2w(\bar{x}_1^{14}) + w(\bar{x}_2^{14}) + 3w(\bar{x}_3^{14}) = 3$. We get $w(\bar{x}_1^{14}) = \frac{1}{5}$, $w(\bar{x}_2^{14}) = \frac{4}{5}$ and $w(\bar{x}_3^{14}) = \frac{3}{5}$ with $|x_1^{14} - x_1^{23}| + 0 = 0 \leq \frac{1}{5}$, $|x_2^{14} - x_2^{23}| + \frac{1}{5} = \frac{1}{5} \leq \frac{4}{5}$ and $|x_3^{14} - x_3^{23}| + \frac{3}{5} = \frac{3}{5} \leq \frac{3}{5}$. Therefore, we get $\bar{x}_1^{14} = \left[\frac{1}{5}, \frac{3}{5}\right]$, $\bar{x}_2^{14} = \left[\frac{3}{5}, \frac{11}{5}\right]$ and $\bar{x}_3^{14} = \left[0, \frac{6}{5}\right]$.

Step 3. The optimal solution according to the choice of the decision maker is $Min\tilde{Z}(\tilde{x}) \approx \sum_{j=1}^{n} c_j \tilde{x}_j$ with $\tilde{x}_j = (\bar{x}_j^{23}; \bar{x}_j^{13}) = (x_j^1, x_j^2, x_j^3, x_j^4)$. $Min\tilde{Z}(\tilde{x}) \approx \left(\frac{7}{5}, \frac{32}{5}, \frac{88}{5}, \frac{151}{5}\right)$ where $\tilde{x}_1 = \left(\frac{1}{5}, \frac{2}{5}, \frac{2}{5}, \frac{3}{5}\right)$, $\tilde{x}_2 = \left(\frac{3}{5}, \frac{6}{5}, \frac{8}{5}, \frac{11}{5}\right)$ and $\tilde{x}_3 = \left(0, 0, \frac{6}{5}, \frac{6}{5}\right)$. Then the corresponding dual problem is given by: $Max\tilde{W}(y) \approx \left(\frac{1}{5}, \frac{32}{5}, \frac{88}{5}, \frac{151}{5}\right)$ where $\tilde{y}_1 = \left(\frac{3}{5}, \frac{4}{5}, \frac{4}{5}, 1\right)$, $\tilde{y}_2 = \left(\frac{-1}{5}, \frac{2}{5}, \frac{6}{5}, \frac{9}{5}\right)$ and $\tilde{y}_3 = \left(0, 0, \frac{2}{5}, \frac{2}{5}\right)$

Remark 5. We see that both primal and dual problems have fuzzy optimal solutions and the two fuzzy optimal values are equal. In contrast to most existing approaches [3] and [4], the centers of all constraints are saturated and some constraint is saturated.

Example 4. Consider the following linear programming problem with variables given as Trapezoidal fuzzy numbers [3] and [4]:

Max $\tilde{Z} = (11,13,15,17)\tilde{x}_1 + (9,12,14,17)\tilde{x}_2 + (13,15,17,19)\tilde{x}_3$ Subject to the constraints

$12\tilde{x}_1 + 13\tilde{x}_2 + 12\tilde{x}_3 \quad (469,475,505,511)$, $14\tilde{x}_1 + 13\tilde{x}_3 \quad (452,460,480,488)$ and

$12\tilde{x}_1 + 15\tilde{x}_2 \quad (460,465,495,500)$.

Step 1. Solving $Eq.(6)$ via $Eq.(10)$.We have $p=2$, $q=3$. We get Max $Z^{23}(x^{23}) = 14x_1^{23} + 13x_2^{23} + 16x_3^{23}$ subject to the constraints $12x_1^{23} + 13x_2^{23} + 12x_3^{23} \le 490$, $14x_1^{23} + 13x_3^{23} \le 470$ and $12x_1^{23} + 15x_2^{23} \le 480$. Optimal solution: $x_1^{23} = 0$, $x_2^{23} = \frac{730}{169}$ and $x_3^{23} = \frac{470}{13} = \frac{6110}{169}$. Slack variables values: $x_4^{23} = 0$, $x_5^{23} = 0$ and $x_6^{23} = \frac{70170}{169}$. Very important decision: For $x_4^{23} = 0$ and $x_5^{23} = 0$, we have $12w(\bar{x}_1^{23}) + 13w(\bar{x}_2^{23}) + 12w(\bar{x}_3^{23}) = w(\bar{b}_1^{23}) = 15$ and $14w(\bar{x}_1^{23}) + 13w(\bar{x}_3^{23}) = w(\bar{b}_2^{23}) = 10$. We get $w(\bar{x}_1^{23}) = 0, w(\bar{x}_2^{23}) = \frac{75}{169}$ and $w(\bar{x}_3^{23}) = \frac{10}{13}$. Therefore, we get $\bar{x}_1^{23} = \bar{0}$, $\bar{x}_2^{23} = \left[\frac{655}{169}, \frac{805}{169}\right]$ and $\bar{x}_3^{23} = \left[\frac{460}{13}, \frac{480}{13}\right]$.

Step 2. Solving $Eq.(6)$ via $Eq.(10)$. We have $p=1$, $q=4$. We get Max $Z^{14}(x^{14}) = 14x_1^{14} + 13x_2^{14} + 16x_3^{14}$ subject to the constraints $12x_1^{14} + 13x_2^{14} + 12x_3^{14} \le 490$, $14x_1^{14} + 13x_3^{14} \le 470$ and $12x_1^{14} + 15x_2^{14} \le 480$. Optimal solution: $x_1^{14} = 0$, $x_2^{14} = \frac{730}{169}$ and $x_3^{14} = \frac{470}{13} = \frac{6110}{169}$. Slack variables values: $x_4^{14} = 0$, $x_5^{14} = 0$ and $x_6^{14} = \frac{70170}{169}$. Important decision: For $x_4^{14} = 0$, we have $12w(\bar{x}_1^{14}) + 13w(\bar{x}_2^{14}) + 12w(\bar{x}_3^{14}) = w(\bar{b}_1^{14}) = 21$. We get $w(\bar{x}_1^{14}) = 0$, $w(\bar{x}_2^{14}) = \frac{21}{26}$ and $w(\bar{x}_3^{14}) = \frac{21}{24}$ with $|x_2^{14} - x_2^{23}| + \frac{75}{169} = \frac{75}{169} \le \frac{21}{26}$ and $|x_3^{14} - x_3^{23}| + \frac{10}{13} = \frac{10}{13} \le \frac{21}{24}$. Therefore, we get $\bar{x}_1^{14} = \bar{0}$, $\bar{x}_2^{14} = \left[\frac{15431}{4394}, \frac{22529}{4394}\right]$ and $\bar{x}_3^{14} = \left[\frac{143091}{4056}, \frac{150189}{4056}\right]$.

Step 3. The optimal solution according to the choice of the decision maker is $Max\,\tilde{Z}(\tilde{x}) \approx \left(\frac{51697854}{105456}, \frac{97560}{169}, \frac{117350}{169}, \frac{83385198}{105456}\right)$ where $\tilde{x}_1 = \tilde{0}$, $\tilde{x}_2 = \left(\frac{15431}{4394}, \frac{655}{169}, \frac{805}{169}, \frac{22529}{4394}\right)$ and $\tilde{x}_3 = \left(\frac{143091}{4056}, \frac{460}{13}, \frac{480}{13}, \frac{150189}{4056}\right)$. Then the corresponding dual problem is given by: $Min\,\tilde{W}(y) \approx \left(\frac{52443066}{105456}, \frac{97560}{169}, \frac{117350}{169}, \frac{82396626}{105456}\right)$ where $\tilde{y}_1 = \left(\frac{21}{24}, \frac{12}{23}, \frac{14}{13}, \frac{27}{24}\right)$, $\tilde{y}_2 = \left(\frac{5}{26}, \frac{51}{169}, \frac{53}{169}, \frac{11}{26}\right)$ and $\tilde{y}_3 = \tilde{0}$.

Remark 6. We see that both primal and dual problems have fuzzy optimal solutions and the two fuzzy optimal values are equal. In contrast to most existing approaches [3] and [4], the centers of all constraints are saturated and some constraint is saturated.

Example 5. Consider the following linear programming problem with variables given as Hexagonal fuzzy numbers [6]: Max $\tilde{Z}(\tilde{x}) \approx (11, 13, 15, 17, 19, 21)\tilde{x}_1 + (31, 33, 35, 37, 39, 41)\tilde{x}_2$ subject to the constraints $69\tilde{x}_1 + 99\tilde{x}_2 \quad (151, 153, 155, 157, 159, 161)$ and

$129\tilde{x}_1 + 159\tilde{x}_2 \quad (271, 273, 275, 277, 279, 281)$.

Step 1. Solving $Eq.(6)$ via $Eq.(10)$. We have $p=3$, $q=4$. We get Max $Z^{34}\left(x^{34}\right)=16x_1^{34}+36x_2^{34}$ subject to the constraints $69x_1^{34}+99x_2^{34}\leq 156$ and $129x_1^{34}+159x_2^{34}\leq 276$. Optimal solution: $x_1^{34}=0$ and $x_2^{34}=\frac{52}{33}$. Slack variables values: $x_3^{34}=0$ and $x_4^{34}=\frac{280}{11}$. Very important decision: For $x_3^{34}=0$, we have $66w\left(\bar{x}_1^{34}\right)+99w\left(\bar{x}_2^{34}\right)=w\left(\bar{b}_2^{34}\right)=1$. We get $w\left(\bar{x}_1^{34}\right)=0$ and $w\left(\bar{x}_2^{34}\right)=\frac{1}{99}$. Therefore, we get $\bar{x}_1^{34}=[0,0]$ and $\bar{x}_2^{34}=\left[\frac{155}{99},\frac{157}{99}\right]$.

Step 2. Solving $Eq.(6)$ via $Eq.(10)$. We have $p=2$, $q=5$. We get Max $Z^{25}\left(x^{25}\right)=16x_1^{25}+36x_2^{25}$ subject to the constraints $69x_1^{25}+99x_2^{25}\leq 156$ and $129x_1^{25}+159x_2^{25}\leq 276$. Optimal solution: $x_1^{25}=0$ and $x_2^{25}=\frac{52}{33}$. Slack variables values: $x_3^{25}=0$ and $x_4^{25}=\frac{280}{11}$. We get $w\left(\bar{x}_1^{25}\right)=0$ and $w\left(\bar{x}_2^{25}\right)=\frac{1}{33}$ with $\left|x_2^{25}-x_2^{32}\right|=0\leq\frac{1}{33}$. Therefore, we get $\bar{x}_1^{25}=[0,0]$ and $\bar{x}_2^{25}=\left[\frac{17}{11},\frac{53}{33}\right]$.

Step 3. Solving $Eq.(6)$ via $Eq.(10)$. We have $p=1$, $q=6$. We get Max $Z^{16}\left(x^{16}\right)=16x_1^{16}+36x_2^{16}$ subject to the constraints $69x_1^{16}+99x_2^{16}\leq 156$ and $129x_1^{16}+159x_2^{16}\leq 276$. Optimal solution: $x_1^{16}=0$ and $x_2^{16}=\frac{52}{33}$. Slack variables values: $x_3^{16}=0$ and $x_4^{16}=\frac{280}{11}$. Very important decision: For $x_3^{16}=0$, we have $66w\left(\bar{x}_1^{16}\right)+99w\left(\bar{x}_2^{16}\right)=w\left(\bar{b}_1^{16}\right)=5$. We get $w\left(\bar{x}_1^{16}\right)=0$ and $w\left(\bar{x}_2^{16}\right)=\frac{5}{99}$ with $\left|x_2^{16}-x_2^{25}\right|+\frac{1}{33}=\frac{1}{33}\leq\frac{5}{99}$. Therefore, we get $\bar{x}_1^{16}=[0,0]$ and $\bar{x}_2^{16}=\left[\frac{151}{99},\frac{161}{99}\right]$.

Step 4. The optimal solution according to the choice of the decision maker is $Max\tilde{Z}\left(\tilde{x}\right)\approx\left(\frac{4681}{99},\frac{5049}{99},\frac{5425}{99},\frac{5809}{99},\frac{6201}{99},\frac{6601}{99}\right)$ with $\tilde{x}_1=\tilde{0}$ and $\tilde{x}_2=\left(\frac{151}{99},\frac{153}{99},\frac{155}{99},\frac{157}{99},\frac{159}{99},\frac{161}{99}\right)$. Then the corresponding dual problem is given by: $Min\tilde{W}\left(y\right)\approx\left(\frac{4681}{99},\frac{5049}{99},\frac{5425}{99},\frac{5809}{99},\frac{6201}{99},\frac{6601}{99}\right)$ where $\tilde{y}_1=\left(\frac{31}{99},\frac{33}{99},\frac{35}{99},\frac{37}{99},\frac{39}{99},\frac{41}{99}\right)$ and $y_2=\tilde{0}$.

Remark 7. We see that both primal and dual problems have fuzzy optimal solutions and the two fuzzy optimal values are equal. In contrast to most existing approaches [6], the centers of all constraints are saturated and some constraint is saturated.

Example 6. Consider the following linear programming problem with variables given as Octagonal fuzzy numbers [19]: Min $\tilde{Z}\left(\tilde{x}\right)\approx\left(2,3,4,5,7,8,9,10\right)\tilde{x}_1+\left(3,4,5,7,9,11,12,13\right)\tilde{x}_2$ subject to the constraint

$20\tilde{x}_1+30\tilde{x}_2\quad\left(885,886,888,890,910,912,914,915\right)$ and

$40\tilde{x}_1+30\tilde{x}_2\quad\left(1190,1191,1193,1195,1205,1207,1209,1210\right)$.

Step 1. Solving $Eq.(6)$ via $Eq.(10)$. We have $p=4$, $q=5$. We get Min $Z^{45}(x^{45})=6x_1^{45}+8x_2^{45}$ subject to the constraints $20x_1^{45}+30x_2^{45}\geq 900$ and $40x_1^{45}+30x_2^{45}\geq 1200$. Optimal solution: $x_1^{45}=15$ and $x_2^{45}=20$. Slack variables values: $x_3^{45}=0 \triangleright$ and $x_4^{45}=0$. Important decision: For $x_4^{45}=0$, we have $40w(\bar{x}_1^{45})+30w(\bar{x}_2^{45})=w(\bar{b}_2^{45})=5$. We get $w(\bar{x}_1^{45})=\frac{1}{20}$ and $w(\bar{x}_2^{45})=\frac{2}{20}$. Therefore, we get $\bar{x}_1^{45}=\left[\frac{299}{20},\frac{301}{20}\right]$ and $\bar{x}_2^{45}=\left[\frac{398}{20},\frac{402}{20}\right]$.

Step 2. Solving $Eq.(6)$ via $Eq.(10)$. We have $p=3$, $q=6$. We get Min $Z^{36}(x^{36})=6x_1^{36}+8x_2^{36}$ subject to the constraints $20x_1^{36}+30x_2^{36}\geq 900$ and $40x_1^{36}+30x_2^{36}\geq 1200$. Optimal solution: $x_1^{36}=15$ and $x_2^{36}=20$. Slack variables values: $x_3^{36}=0$ and $x_4^{36}=0$. Important decision: For $x_4^{36}=0$, we have $40w(\bar{x}_1^{36})+30w(\bar{x}_2^{36})=w(\bar{b}_2^{36})=7$. We get $w(\bar{x}_1^{36})=\frac{2}{20}$ and $w(\bar{x}_2^{36})=\frac{2}{20}$ with $\left|x_1^{36}-x_1^{45}\right|+\frac{1}{20}=\frac{1}{20}\leq\frac{2}{20}$ and $\left|x_2^{36}-x_2^{45}\right|+\frac{2}{20}=\frac{2}{20}\leq\frac{2}{20}$. Therefore, we get $\bar{x}_1^{36}=\left[\frac{298}{20},\frac{302}{20}\right]$ and $\bar{x}_2^{36}=\left[\frac{398}{20},\frac{402}{20}\right]$.

Step 3. Solving $Eq.(6)$ via $Eq.(10)$. We have $p=2$, $q=7$. We get Min $Z^{27}(x^{27})=6x_1^{27}+8x_2^{27}$ subject to the constraints $20x_1^{27}+30x_2^{27}\geq 900$ and $40x_1^{27}+30x_2^{27}\geq 1200$. Optimal solution: $x_1^{27}=15$ and $x_2^{27}=20$. Slack variables values: $x_3^{27}=0$ and $x_4^{27}=0$. Important decision: For $x_4^{27}=0$, we have $40w(\bar{x}_1^{27})+30w(\bar{x}_2^{27})=w(\bar{b}_2^{27})=9$. We get $w(\bar{x}_1^{27})=\frac{2}{20}$ and $w(\bar{x}_2^{27})=\frac{1}{6}$ with $\left|x_1^{27}-x_1^{36}\right|+\frac{2}{20}=\frac{2}{20}\leq\frac{2}{20}$ and $\left|x_2^{27}-x_2^{36}\right|+\frac{2}{20}=\frac{2}{20}\leq\frac{1}{6}$. Therefore, we get $\bar{x}_1^{27}=\left[\frac{298}{20},\frac{302}{20}\right]$ and $\bar{x}_2^{27}=\left[\frac{1190}{60},\frac{1210}{60}\right]$.

Step 4. Solving $Eq.(6)$ via $Eq.(10)$. We have $p=1$, $q=8$. We get Min $Z^{18}(x^{18})=6x_1^{18}+8x_2^{18}$ subject to the constraints $20x_1^{18}+30x_2^{18}\geq 900$ and $40x_1^{18}+30x_2^{18}\geq 1200$. Optimal solution: $x_1^{18}=15$ and $x_2^{18}=20$. Slack variables values: $x_3^{18}=0$ and $x_4^{18}=0$. Important decision: For $x_4^{18}=0$, we have $40w(\bar{x}_1^{18})+30w(\bar{x}_2^{18})=w(\bar{b}_2^{18})=10$. We get $w(\bar{x}_1^{18})=\frac{5}{40}$ and $w(\bar{x}_2^{18})=\frac{1}{6}$ with $\left|x_1^{18}-x_1^{27}\right|+\frac{2}{20}=\frac{2}{20}\leq\frac{5}{40}$ and $\left|x_2^{18}-x_2^{27}\right|+\frac{1}{6}=\frac{1}{6}\leq\frac{1}{6}$. Therefore, we get $\bar{x}_1^{18}=\left[\frac{595}{40},\frac{605}{40}\right]$ and $\bar{x}_2^{18}=\left[\frac{119}{6},\frac{121}{6}\right]$.

Step 5. The optimal solution according to the choice of the decision maker is $Min\tilde{\mathbb{Z}}(\tilde{x})\approx\left(\frac{10710}{120},\frac{14884}{120},\frac{19116}{120},\frac{25686}{120},\frac{34350}{120},\frac{41028}{120},\frac{45348}{120},\frac{49610}{120}\right)$ with $\tilde{x}_1=\left(\frac{1785}{120},\frac{1788}{120},\frac{1794}{120},\frac{1794}{120},\frac{1806}{120},\frac{1812}{120},\frac{1812}{120},\frac{1815}{120}\right)$ and

$\tilde{x}_2 = \left(\frac{2380}{120}, \frac{2380}{120}, \frac{2388}{120}, \frac{2388}{120}, \frac{2412}{120}, \frac{2412}{120}, \frac{2420}{120}, \frac{2420}{120}\right)$. Then the corresponding dual problem is given by: $Max\,\tilde{W}(y) \approx \left(\frac{9320}{120}, \frac{13530}{120}, \frac{17760}{120}, \frac{25530}{120}, \frac{34530}{120}, \frac{42488}{120}, \frac{46822}{120}, \frac{51120}{120}\right)$ where $\tilde{y}_1 = \left(\frac{16}{120}, \frac{18}{120}, \frac{20}{120}, \frac{26}{120}, \frac{30}{120}, \frac{36}{120}, \frac{38}{120}, \frac{40}{120}\right)$ and

$$\tilde{y}_2 = \left(\frac{-4}{120}, \frac{-2}{120}, 0, \frac{2}{120}, \frac{6}{120}, \frac{8}{120}, \frac{10}{120}, \frac{12}{120}\right).$$

Remark 8. We see that both primal and dual problems have fuzzy optimal solutions and the two fuzzy optimal values are equal. In contrast to most existing approaches [7], the centers of all constraints are saturated and some constraint is saturated.

5 | Advantages of the Proposed Method over the Existing Methods

To be more specific, we will concentrate on showing the advantages of the proposed method over the well-known existing methods existing methods proposed by [1], [2], [3], [4], [6], [7], [18].

The advantages of the new method proposed over the existing methods proposed by [1], [2], [3], [4], [6], [7], [18] can be summarized as follows:

I. The new method improves the existing methods for solving the semi-fully interval and semi-fully fuzzy linear programming problems.

II. The new method improves the existing methods for solving the interval Transportation Problems and Fully Fuzzy Transportation Problems [6] and [15].

III. In contrast to most existing approaches, our method of transforming a fuzzy number into interval numbers is the first. So, our method proposed is the first.

IV. The proposed technique does not use the goal and parametric approaches which are difficult to apply in real life situations. These difficulties (or limitations) are overcome by the new proposed method.

V. To solve the *Eq. (12)* by using the existing method, there is need to use arithmetic operations of generalized fuzzy numbers. While, if the proposed technique is used for the same then there is need to use arithmetic operations of real numbers. This proves that it is much easy to apply the proposed method as compared to the existing method.

VI. In contrast to most existing approaches, which provide an optimal solution using ranking function, the proposed method provides a fuzzy optimal solution without using ranking function. Similarly, to the competing methods in the literature, the proposed method is applicable for all types of fuzzy numbers.

VII. Also, the fuzzy optimal solution, obtained by using the new method mentioned, will always exactly satisfy the centers of all the constraints and some constraints.

6 | Concluding Remarks and Future Research Directions

6.1 | Concluding Remarks

The present paper proposes an alternative solution approach for solving the semi-fully fuzzy linear programming problem where the coefficients in the objective function, the right-hand side vector and the decision variables are a kind of fuzzy numbers, simultaneity. Firstly, the Semi-fully Fuzzy Linear Programming Problem is transformed into equivalent semi-fully interval linear programming problems. After that, the solutions to these interval linear programming problems are then obtained with the help of linear programming technique. The comparisons numerical examples show that in all problems the proposed method provides a better solution than the existing methods [1], [2], [3], [4], [6], [7], [18]. So, the proposed approach can be considered as an alternative approach for solving the Semi-fully Fuzzy Linear Programming Problems if decision maker is interested in finding the fuzzy optimal solution with minimum uncertainty.

6.2 | Future Research Directions

Finally, we feel that, there are many other points of research and should be studied later on interval numbers or fuzzy numbers. Some of these points are below:

- *Linear programming problem with generalized interval-valued fuzzy numbers.*
- *Interval-valued intuitionistic semi-fully fuzzy linear programming problem.*
- *Semi-Fully fuzzy linear fractional programming problems with fuzzy numbers and intuitionistic fuzzy.*

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Conflicts of Interest

All authors have contributed equally in this work. The authors declare that there is no conflict of interest for this publication.

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Some Results on $(c)-$Mapping

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Abstract

Compared with [1], in this paper, we will give first some sufficient conditions under which a $(c)-$mapping possesses an Approximate Fixed Point Sequence (AFPS). And then, we will prove that $(c)-$mapping has a fixed point. Finally, we will check some special properties of the fixed point sets of these mappings, such as closedness, convexity.

Keywords: $(c)-$ mapping, Fixed point, Closedness, Convexity.

1 | Introduction

It is well known that various nonlinear generalizations of the contraction mapping are of great significance in the literature. Nonexpansive mappings, asymptotically nonexpansive mappings are some examples of such generalizations. We know that every nonexpansive mapping or asymptotically nonexpansive mapping on a non-empty closed, bounded, convex subset of a uniformly convex Banach space has at least one fixed point, see [2], [3] and [4]. Subsequently, many authors have introduced several kinds of nonlinear mappings generalizing the class of nonexpansive mappings such as asymptotically pseudocontractive mappings, uniformly asymptotically regular mappings, uniformly asymptotically regular mappings with sequence, uniformly $L-$Lipschitzian mappings, etc.

In proving the existence of fixed point of the above mentioned mappings, many authors took the help of Approximate Fixed Point Sequence (AFPS). Here, we mention a number of iterative AFPS.

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I. The Mann iteration [5]: sequence $\{x_n\}$ is defined by

$$x_{n+1} = (1-\lambda_n)x_n + \lambda_n Tx_n,$$

where (λ_n) is a sequence of real numbers satisfying $0 \le \lambda_n < 1$ for all $n \in N$.

II. The Krasnoselskij iteration [6]: sequence $\{x_n\}$ is defined by

$$x_{n+1} = \frac{1}{2}(x_n + Tx_n).$$

III. The Halpern iteration [7]: sequence $\{x_n\}$ is defined by

$$x_{n+1} = (1-\lambda_n)Tx_n + \lambda_n u,$$

where (λ_n) is a sequence in $[0,1]$ and $u \in X$.

IV. The modified Mann iteration [8]: sequence $\{x_n\}$ is defined by

$$x_{n+1} = (1-\lambda_n)x_n + \lambda_n T^n x_n,$$

where (λ_n) is a sequence in $[0,1]$.

In this paper, we will extend these results to $(c)-$mapping and we will first give some sufficient conditions under which a $(c)-$mapping possesses an AFPS.

2 | Preliminaries

In this section, we collect some necessary definitions, which will be used in next section.

Definition 1. [1], [9]. Let X be a normed linear space, C a non-empty subset of X and $T:C \to C$ be a mapping. The mapping T is said to be a Reich type nonexpansive mapping if there exists non-negative real numbers a,b,c with $a+b+c=1$, such that the condition.

$$\|Tx-Ty\| \le a\|x-y\| + b\|x-Tx\| + c\|y-Ty\|. \tag{1}$$

Holds for all $x,y \in C$. The mapping T is said to be a Chatterjea type nonexpansive mapping if there exists non-negative real numbers a,b,c with $a+b+c=1$, such that the condition

$$\|Tx-Ty\| \le a\|x-y\| + b\|x-Ty\| + c\|y-Tx\|. \tag{2}$$

Holds for all $x,y \in C$. In both cases, we say that T is a Reich type nonexpansive (Chatterjea type nonexpansive) mapping with coefficients (a,b,c).

Definition 2. [10], [11]. Let (C,d) be a metric space. A self-mapping $T:C\rightarrow C$ is said to be a generalized nonexpansive mapping if there exist a,b and $c\in[0,1]$, such that $a+2b+2c\leq 1$ and:

$$\begin{aligned} d(Tx_1,Tx_2) \leq\, & ad(x_1,x_2)+b(d(x_1,Tx_1)+d(x_2,Tx_2)) \\ & +c(d(x_1,Tx_2)+d(x_2,Tx_1)), \end{aligned} \tag{3}$$

For all $x_1,x_2\in C$.

The most important case is when $a+2b+2c=1$, which covers in particular the following situations:

If $b=0$ and $c>0$, then T is called $(c)-$ mapping [12] and [13].

Definition 3. [14], [15]. A self-mapping T of a metric space (X,d) is said to be asymptotically regular if $\lim_{n\rightarrow\infty} d(T^n x,T^{n+1}x)=0$ for all $x\in X$.

Definition 4. [1]. A sequence (x_n) in a normed linear space is said to be an AFPS if

$$\lim_{n\rightarrow\infty}\|x_n-Tx_n\|=0.$$

Proposition 1. [16]. Let (z_n) and (w_n) be two bounded sequences in a Banach space X and let $\lambda\in(0,1)$. Let $z_{n+1}=\lambda w_n+(1-\lambda)z_n$ and suppose $\|w_{n+1}-w_n\|\leq\|z_{n+1}-z_n\|$ for all $n\in N$. Then $\lim_{n\rightarrow\infty}\|w_n-z_n\|=0$.

3 | Main Results

In this section, we compare Reich type nonexpansive, Chatterjea type nonexpansive mappings with $(c)-$mapping, therefore, we extend some results in [1] to $(c)-$mapping. We will give first some sufficient conditions under which a $(c)-$mapping possesses AFPS.

Theorem 1. Let X be a Banach space and C be a non-empty closed, convex, bounded subset of X. Let $T:C\rightarrow C$ be a $(c)-$mapping with coefficients (a, b, c), such that $a+2c=1, 0<c<1$. Also assume that for $x,y\in C$

$$\frac{1-c}{5}\|x-Ty\|\leq\|x-y\|\Rightarrow\|Tx-Ty\|\leq\|x-y\|.$$

Then T has an AFPS in C. Moreover, the AFPS is asymptotically regular.

Proof. Since T is a $(c)-$mapping with $a+2c=1, 0<c<1$ and

$$\|Tx-Ty\|\leq a\|x-y\|+c\|x-Ty\|+c\|y-Tx\|. \tag{4}$$

For all $x,y\in C$. Let $x_0\in C$ be arbitrary but fixed. We consider the sequence (x_n) in X defined by

$x_{n+1} = \lambda Tx_n + (1-\lambda)x_n$ for all $n \geq 2$ where $\frac{1}{2} \leq \lambda < 1$. Since C is convex and bounded, it follows that (x_n) is a bounded sequence in C. Now putting $x = x_n, y = x_{n+1}$ in *Eq. (4)*, we get

$$\begin{aligned}\|Tx_n - Tx_{n+1}\| &\leq a\|x_n - x_{n+1}\| + c\|x_n - Tx_{n+1}\| + c\|x_{n+1} - Tx_n\| \\ &\leq a\|x_n - x_{n+1}\| + c\|x_n - x_{n+1}\| + c\|x_{n+1} - Tx_{n+1}\| \\ &\quad + c\|x_{n+1} - x_n\| + c\|x_n - Tx_n\| \\ &= \|x_n - x_{n+1}\| + c\|x_{n+1} - Tx_{n+1}\| + c\|x_n - Tx_n\|.\end{aligned} \tag{5}$$

Since $x_{n+1} = \lambda Tx_n + (1-\lambda)x_n$ for all $n \geq 2$, we have $\|x_n - Tx_n\| = \frac{1}{\lambda}\|x_n - x_{n+1}\|$ and $\|x_{n+1} - Tx_{n+1}\| = \frac{1}{\lambda}\|x_{n+1} - x_{n+2}\|$. Using these in *Eq. (5)*, we get

$$\lambda\|Tx_n - Tx_{n+1}\| \leq \lambda\|x_n - x_{n+1}\| + c\|x_{n+1} - x_{n+2}\| + c\|x_n - x_{n+1}\|. \tag{6}$$

Now

$$\begin{aligned}x_{n+1} - x_{n+2} &= \lambda Tx_n + (1-\lambda)x_n - \left[\lambda Tx_{n+1} + (1-\lambda)x_{n+1}\right] \\ &= \lambda(Tx_n - Tx_{n+1}) + (1-\lambda)(x_n - x_{n+1}).\end{aligned}$$

$$\begin{aligned}\|x_{n+1} - x_{n+2}\| &\leq \lambda\|Tx_n - Tx_{n+1}\| + (1-\lambda)\|x_n - x_{n+1}\| \\ &\leq \lambda\|x_n - x_{n+1}\| + c\|x_{n+1} - x_{n+2}\| + c\|x_n - x_{n+1}\| \\ &\quad + (1-\lambda)\|x_n - x_{n+1}\|.\end{aligned}$$

$$\Rightarrow \|x_{n+1} - x_{n+2}\| \leq \frac{1+c}{1-c}\|x_n - x_{n+1}\|. \tag{7}$$

Again, we have

$$\begin{aligned}&\lambda(Tx_n - Tx_{n+1}) = (x_{n+1} - x_{n+2}) + (\lambda - 1)(x_n - x_{n+1}) \\ &\Rightarrow \lambda\|Tx_n - Tx_{n+1}\| = \|x_{n+1} - x_{n+2}\| + (1-\lambda)\|x_n - x_{n+1}\|.\end{aligned} \tag{8}$$

Therefore,

$$\begin{aligned}\|x_n - Tx_{n+1}\| &\leq \|x_n - Tx_n\| + \|Tx_n - Tx_{n+1}\| \\ &= \frac{1}{\lambda}\|x_n - x_{n+1}\| + \|Tx_n - Tx_{n+1}\|,\end{aligned}$$

$$\begin{aligned}\lambda\|x_n - Tx_{n+1}\| &\leq \|x_n - x_{n+1}\| + \lambda\|Tx_n - Tx_{n+1}\| \\ &\leq \|x_n - x_{n+1}\| + \|x_{n+1} - x_{n+2}\| + (1-\lambda)\|x_n - x_{n+1}\| \\ &\leq (2-\lambda)\|x_n - x_{n+1}\| + \frac{1+c}{1-c}\|x_n - x_{n+1}\| \\ &< \left(\frac{3}{2} + \frac{1+c}{1-c}\right)\|x_n - x_{n+1}\|.\end{aligned}$$

Thus

$$\frac{1}{2}\left\|x_n - Tx_{n+1}\right\| \le \lambda\left\|x_n - Tx_{n+1}\right\|$$
$$\le \left(\frac{3}{2}+\frac{1+c}{1-c}\right)\left\|x_n - x_{n+1}\right\|.$$

Hence,

$$\frac{1-c}{5}\left\|x_n - Tx_{n+1}\right\| < \left\|x_n - x_{n+1}\right\|.$$

Therefore, by given hypothesis, we get

$$\left\|Tx_n - Tx_{n+1}\right\| \le \left\|x_n - x_{n+1}\right\|.$$

Thus, by Proposition 1, we have $\left\|x_n - Tx_n\right\| \to 0$ as $n \to \infty$. So $\left(x_n\right)$ is an AFPS of T. Further, we have $n \to \infty$,

$$\left\|x_n - x_{n+1}\right\| = \lambda\left\|x_n - Tx_n\right\| \to 0.$$

Therefore, the AFPS $\left(x_n\right)$ is asymptotically regular also.

Next, we prove a result concerning the existence of fixed points of such mappings using *Theorem 1*.

Theorem 2. Suppose that all the conditions of *Theorem 1* are satisfied. Further, assume that for any $\varepsilon > 0$, there exists $\delta > 0$ such that

$$\left\|x-y\right\| + \left\|x - Ty\right\| + \left\|y - Tx\right\| < 3\varepsilon + \delta \Rightarrow \left\|Tx - Ty\right\| \le \frac{\varepsilon}{2}. \quad (9)$$

Then T has a fixed point in C.

Proof. By *Theorem 1*, T has an AFPS $\left(x_n\right)$, where $x_{n+1} = \lambda Tx_n + \left(1-\lambda\right)x_n$ and $\frac{1}{2} \le \lambda < 1$. Here we take $\lambda = \frac{1}{2}$. So we get an AFPS $\left(x_n\right)$ given by $x_{n+1} = \frac{1}{2}\left(Tx_n + x_n\right)$, and this sequence is asymptotically regular also. Next, we show that $\left(x_n\right)$ is a Cauchy sequence. Let $\varepsilon > 0$ be arbitrary. So there exists $\delta > 0$ such that *Eq. (9)* holds. Without loss of generality, we take $\delta < \varepsilon$. Since, $\left(x_n\right)$ is asymptotically regular, there exists $N \in N$ such that

$$\left\|x_n - x_{n+1}\right\| < \frac{\delta}{4}.$$

For all $n \ge N$. Next, we show by induction on p that

$$\left\|x_N - x_{N+p}\right\| < \varepsilon \text{ for all } p \in N. \quad (10)$$

Clearly *Eq. (10)* is true for $p = 1$. Let *Eq. (10)* be true for some $p \in N$.

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Therefore,

$$\begin{aligned}&\|x_N - x_{N+p}\| + \|x_N - Tx_{N+p}\| + \|x_{N+p} - Tx_N\| \\ &\le \|x_N - x_{N+p}\| + \|x_N - x_{N+p}\| + \|x_{N+p} - Tx_{N+p}\| + \|x_{N+p} - x_N\| + \|x_N - Tx_N\| \\ &= 3\|x_N - x_{N+p}\| + 2\|x_{N+p} - x_{N+p+1}\| + 2\|x_N - x_{N+1}\| \\ &< 3\varepsilon + \delta.\end{aligned}$$

$$\|Tx_N - Tx_{N+p}\| \le \frac{\varepsilon}{2}.$$

From *Eq. (9)*, we get

Again by the formation of (x_n) , we get

$$\|x_{N+p+1} - x_{N+1}\| \le \frac{1}{2}\|Tx_{N+p} - Tx_N\| + \frac{1}{2}\|x_{N+p} - x_N\| < \frac{3\varepsilon}{4}.$$

Thus

$$\|x_N - x_{N+p+1}\| \le \|x_N - x_{N+1}\| + \|x_{N+1} - x_{N+p+1}\| < \frac{\delta}{4} + \frac{3\varepsilon}{4} < \varepsilon.$$

Therefore, *Eq. (10)* is true for $p+1$. So *Eq. (10)* is true for all p. Continuing in a similar manner, we can show that

$$\|x_n - x_{n+p}\| < \varepsilon. \ \text{ for all } \ n \in N \ \text{ and for all } \ p \in N.$$

Therefore, (x_n) is a Cauchy sequence and hence convergent to some $z \in C$.

Again,

$$\begin{aligned}\|Tx_n - Tx_m\| &\le a\|x_n - x_m\| + c\|x_n - Tx_m\| + c\|x_m - Tx_n\| \\ &\le a\|x_n - x_m\| + c\|x_n - x_m\| + c\|x_m - Tx_m\| \\ &\quad + c\|x_m - x_n\| + c\|x_n - Tx_n\| \\ &= \|x_n - x_m\| + c\|x_m - Tx_m\| + c\|x_n - Tx_n\| \to 0 \text{ as } n, m \to \infty.\end{aligned}$$

Therefore, (Tx_n) is a Cauchy sequence in C. Also, since $x_{n+1} = \frac{1}{2}(Tx_n + x_n)$, we have that $Tx_n = 2x_{n+1} - x_n \to z \text{ as } n \to \infty.$ Again,

$$\begin{aligned}\|z - Tz\| &\le \|z - x_n\| + \|x_n - Tx_n\| + \|Tx_n - Tz\| \\ &\le \|z - x_n\| + \|x_n - Tx_n\| \\ &\quad + \left(a\|x_n - z\| + c\|x_n - Tz\| + c\|z - Tx_n\|\right).\end{aligned}$$

Letting $n \to \infty$ in above inequality, we get

$$\|z - Tz\| \le c\|z - Tz\|.$$

Since $1 > c > 0$, which gives $z = Tz$, i.e., z is a fixed point of T.

The following theorem characterizes the fixed point set of $(c)-$ mapping.

Theorem 3. Let X be a Banach space and C be a non-empty subset of X. Let $T : C \to C$ be a $(c)-$ mapping with coefficients (a, b, c), such that $a + 2c = 1, 0 < c < 1$, then $Fix(T)$ is a closed subset of C.

Proof. Since T is $(c)-$ mapping with coefficients (a, b, c), we have $a + 2c = 1, 0 < c < 1$ and

$$\|Tx - Ty\| \le a\|x - y\| + c\|x - Ty\| + c\|y - Tx\|.$$

For all $x, y \in C$. Let (z_n) be a sequence in $Fix(T)$ converging to some $z \in C$.

Then, we have

$$\|Tz_n - Tz\| \le a\|z_n - z\| + c\|z_n - Tz\| + c\|z - Tz_n\|$$
$$\Rightarrow \|z_n - Tz\| \le a\|z_n - z\| + c\|z_n - Tz\| + c\|z - z_n\|.$$

Taking limit as $n \to \infty$ in above inequality, we get $\|z - Tz\| \le c\|z - Tz\|$. Since $0 < c < 1$, so $z = Tz$, i.e., $z \in Fix(T)$ and hence $Fix(T)$ is a closed set.

In the next theorem, we give another characterization of the fixed point set of $(c)-$ mapping by taking the underlying space as a Hilbert space in place of Banach space.

Theorem 4. Let X be a Hilbert space and C be a non-empty subset of X. Let $T : C \to C$ be a (c)- mapping with coefficients (a, b, c), such that $a + 2c = 1, 0 < c < 1$, then $Fix(T)$ is a convex subset of C

Proof. Let $x, y \in Fix(T)$ be any two points and take $z = \lambda x + (1 - \lambda)y$, where λ is a scalar with $0 \le \lambda \le 1$. Then we have

$$\|Tz - Tx\| \le a\|z - x\| + c\|z - Tx\| + c\|x - Tz\|$$
$$\Rightarrow \|Tz - x\| \le a\|z - x\| + c\|z - x\| + c\|x - Tz\|$$

$$\Rightarrow (1 - c)\|Tz - x\| \le (a + c)\|z - x\|$$

$$\Rightarrow \|Tz - x\| \le \|z - x\|.$$

Next, using parallelogram law we have

$$\left\|\frac{z - x}{2} + \frac{Tz - x}{2}\right\|^2 + \left\|\frac{z - x}{2} - \frac{Tz - x}{2}\right\|^2 = 2\left(\left\|\frac{z - x}{2}\right\|^2 + \left\|\frac{Tz - x}{2}\right\|^2\right)$$

$$\left\|\frac{z-x}{2}+\frac{Tz-x}{2}\right\|^2+\frac{1}{4}\|z-Tz\|^2=\frac{1}{2}\|z-x\|^2+\frac{1}{2}\|Tz-x\|^2$$
$$\leq\frac{1}{2}\|z-x\|^2+\frac{1}{2}\|z-x\|^2$$

$$\Rightarrow\left\|\frac{z-x}{2}+\frac{Tz-x}{2}\right\|^2\leq\|z-x\|^2-\frac{1}{4}\|z-Tz\|^2$$

$$\Rightarrow\left\|\frac{z+Tz}{2}-x\right\|^2\leq(1-\lambda)^2\|x-y\|^2-\frac{1}{4}\|z-Tz\|^2.$$

$$\left\|\frac{z+Tz}{2}-y\right\|^2\leq\lambda^2\|x-y\|^2-\frac{1}{4}\|z-Tz\|^2.$$

Similarly, we have

Now if $z\neq Tz$, then we have

$$\left\|\frac{z+Tz}{2}-x\right\|<(1-t)\|x-y\|, \text{ and}$$

$$\left\|\frac{z+Tz}{2}-y\right\|<t\|x-y\|.$$

Then, we get

$$\|x-y\|\leq\left\|\frac{z+Tz}{2}-x\right\|+\left\|\frac{z+Tz}{2}-y\right\|$$
$$<(1-t)\|x-y\|+t\|x-y\|$$
$$=\|x-y\|,$$

which gives a contradiction. So we must have $z=Tz$, i.e., $z\in Fix(T)$. Therefore, $Fix(T)$ is a convex set.

4 | Applications

In this section, we will give a concrete example to illustrate the theorem 2 and show the rationality of the obtained theorems.

Example 1. Let us consider the Banach space R equipped with the ususal norm, take $C=\left[0,\frac{3}{2}\right]$ and define a mapping $T:C\to C$ by

$$Tx=\begin{cases}\frac{3}{2} & \text{if } x<\frac{1}{3};\\ \frac{5}{4} & \text{if } x\geq\frac{1}{3}.\end{cases}$$

Choose $a = c = \frac{1}{3}$. Then for any $x, y \in C$, if $x, y < \frac{1}{3}$ or if $x, y \geq \frac{1}{3}$, it is obvious to check that

$$\|Tx - Ty\| \leq a\|x - y\| + c(\|x - Ty\| + \|y - Tx\|).$$

Next, suppose that $x < \frac{1}{3}$ and $y \geq \frac{1}{3}$. Then $\|Tx - Ty\| = \frac{1}{4}$ and

$$a\|x - y\| + c(\|x - Ty\| + \|y - Tx\|) = \frac{1}{3}\left(y - x + \frac{5}{4} - x + \frac{3}{2} - y\right) \geq \frac{1}{4}.$$

So

$$\|Tx - Ty\| \leq a\|x - y\| + c(\|x - Ty\| + \|y - Tx\|).$$

Therefore, T is a $(c)-$mapping with coefficients $a = c = \frac{1}{3}$. Also, T has an AFPS $\left(\frac{5}{4} + \frac{1}{n+2}\right)_{n \in N}$ and $Fix(T) = \left\{\frac{5}{4}\right\}$, which is obviously closed and convex.

Remark 1. *Example 1* shows that the class of $(c)-$mapping is larger than that of nonexpansive mapping.

5 | Conclusion

In this article, we notice the differences between Reich type nonexpansive, Chatterjea type nonexpansive mappings and $(c)-$mapping in reference [1] and [9]. We first proved a new theorem that some sufficient conditions under which a $(c)-$mapping possesses an AFPS. And then, we also proved that $(c)-$mapping has a fixed point. Finally, we checked some special properties of the fixed point sets of these mappings, such as closedness, convexity. In the end, we gave a concrete real example to show the theorem 2. This shows that our work is meaningful. Through this study, we advanced the work of researching fixed point theory in $(c)-$mapping.

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Conflicts of Interest

The authors declare no conflict of interest.

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