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Technical Analysis of Petrochemical Industries of Iran Using a Network Data Envelopment Analysis Model

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Abstract

The data envelopment analysis method is commonly used to measure efficiency. An estimate of the relative efficiency of this model is derived by calculating the ratio between inputs and outputs. Data envelopment analysis models can also be applied to network structures due to the extension of these models. Supply Chain Management (SCM) is a novel approach that governed production management in recent years. In complex and dynamic environments, the petrochemical industry requires an investigation system similar to those used by other organizations to inform about its activity's desirability, especially in complex and dynamic environments. This research focused on the petrochemical company supply chain. Laboratory studies, experts, and visits to petrochemical sites were used to identify production processes and determine indicators. After that, they were evaluated with an envelopment model and a coefficient corresponding to the identified petrochemical supply chain structure. The aggregate and componentwise efficiency of the studied units in petrochemical were also examined from 2016 to 2019.

Keywords: Performance evaluation, Network data envelopment analysis, Aggregate efficiency, Componentwise efficiency.

1 | Introduction

The data envelopment analysis method is commonly used to measure efficiency. An estimate of the relative efficiency of this model is derived by calculating the ratio between inputs and outputs [1]. After pioneering work of Charnes and Cooper [2] many scholars and researchers entered fuzzy set theory in DEA [2]-[4]. For instance, Bagherzadeh et al. [6] proposed a novel ranking method for DMUs based on fuzzy DEA. Nojehdehi et al. [7] proposed an approach to measure the production possibility based on fuzzy efficient frontier in DEA. In real world problem production systems have a network structure and the output of each stage is used as an input for the next stage, the data envelopment analysis network method is used to measure the efficiency of all model's components [8]. Therefore, unlike classic data envelopment analysis models, it helps to model organization and measure the efficiency of model components [9]. A significant challenge in the development of performance evaluation based on data envelopment analysis is distinguishing the model validation from a wide range of input and output indices [10].

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Based on Yang [11], whereas early DEA researches concentrated mostly on theoretical and methodological progrecess, the number of DEA research incorporating real-world applications has grown. Several network structures can be found in current systems, including series, parallel, and mixed structures [10]-[14]. The importance of each dimension of the network model should also be considered [17]. In this study, the main objective is to select a proper model by considering proper variables and defining the correct relationship between model dimensions, in addition to considering the right weight for each dimension of data envelopment analysis to assess the efficiency of different units in petrochemical companies and to analyze progress and regression in units. A crucial issue in evaluating the petrochemical industry's performance is the activity nature and the dividing their business. Data envelopment analysis is a comprehensive procedure that entered the petrochemical industry to study performance evaluation, and it still was not accepted by managers. The popularity of this model is due to the existence of multiple inputs and outputs in this model and its proportionality in the study of nonlinear equations in analyses. In the absence of consideration for sub-processes, a superficial assessment of performance was conducted. A few studies divided overall efficiency into partial efficiencies to analyze subsidiary processes and resources of inefficiency [18]. Therefore, the network data envelopment analysis method specifies the efficiency of the entire system and services provision process and calculates the efficiency of each part of the model. In the petrochemical industry, it allows managers to make strategic decisions to enhance each sub-process.

Many of the Decision-Making Units (DMUs) have more than one stage. By evaluating the performance of these units using data envelopment analysis, the entire DMU cannot be viewed as a black box. Rather, the internal equations should also be taken into consideration. Different methods were presented to study the efficiency of multistage units. In organizations, it is particularly challenging to calculate the efficiency of sub-sets that have a cause-and-effect relationship. Furthermore, the time factor affects their performance to a great extent. As a result, organizational analysis requires developing a plan based on dynamic models, considering the time factor. Supply chains are among units that have multiple stages, and reversible factors exist in some of them. Thus, providing models to evaluate the efficiency of multistage units in the presence of reversible factors is crucial. The main question of this research is how to create a mathematical model in a network to measure the performance of an organization so that the overall and component functions can be presented. This study evaluates some petrochemicals in the country based on their information modeled at three levels. A review of data envelopment analysis and supply chain introductions was the focus of the second section. A data envelopment analysis model is presented in Section 3 for evaluating the aggregate and componentwise efficiency of chains within the petrochemical industry. Section 4 presents a functional example and demonstrates how the models are implemented. Section 5 includes the conclusion and suggestions.

2 | Introductions of Data Envelopment Analysis and Supply Chain

2.1 | Concepts and Fundamentals of DEA

Assume a unit that consumes the X input and creates the Y output. The relative efficiency is defined as below:

$$\text{Efficiency} = \frac{\text{Output}}{\text{Input}} = \frac{Y}{X}. \tag{1}$$

This definition is practical when the DMU has one input and one output. It is assumed that the output is equal to y for specific DMUs of the global standard. If the DMU consumes one unit of input and produces y_o units of output, the absolute efficiency will be as below:

$$\frac{y_\circ}{y^*}. \tag{2}$$

The reasons for using relative efficiency in the performance evaluation of DMUs are:

First, in developing countries such as Iran, real unit performance often falls short of international standards, and no method can be presented to enable units to reach the standard level, or if presented, it would cause disappointment. Second, there are no standards for most organizations, and considering international standards is not reasonable for organizations.

Assume that the j[th] DMUs consume the xj input and create yj output. The relative efficiency for the p[th] unit, which is shown with REp, is defined below:

$$RE_p = \frac{\frac{y_p}{x_p}}{\text{Max}\left\{\frac{y_j}{x_j}: j=1,\cdots,n\right\}}. \tag{3}$$

A DMU is a unit that receives the input vector, such as (x1,..., xm), to create the output vector, such as (y1,..., ys). A congruous DMU consists of units with similar performance and create similar outputs by receiving similar inputs. For example, branches of a bank are congruous units.

Consider a DMU that consumes an input vector of (x1, ..., xm) to create the output vector of (y1..., ys). The efficiency of such a unit is defined below:

$$\text{Efficiency} = \frac{u_1 y_1 + \cdots + u_s y_s}{v_1 x_1 + \cdots + v_m x_m}, \tag{4}$$

where ur is the price of the rth output, i.e. yr (r=1,..., s), and vi is the price of xi (i=1,...,m). This efficiency is known as economic efficiency. The x vector is dominant to the y vector if and only if $X \geq Y$ and $X \neq Y$, in which one can say that the Y vector has been conqured by the X vector.

Assume that we have n DMU, and each DMUj (j=1,..., n) uses m input of Xij (i=1,...,m) to create S output Yrj(r=1,..., s). DEA calculated the performance for DMUj as below:

$$h_j = \frac{\sum_{r=1}^{s} u_r y_{rj}}{\sum_{i=1}^{m} v_i x_{ij}}, \tag{5}$$

where vi (i=1,…,m) and ur (r= 1,…,s) are the weight of the relative input and output of DMUj. Weights in *Eq. (5)* are determined by the below programming problem:

$$\begin{aligned} & h_0 = \max\ h_0, \\ & \text{s.t.} \\ & h_j \leq 1,\ j=1,2,\ldots,n, \\ & v_i, u_r \geq 0. \end{aligned} \tag{6}$$

The CCR in *Model (7)* is known to have an input orientaion in the envelope form.

$$\begin{aligned} & \text{Min} \quad \theta \\ & \text{s.t.} \\ & \quad \sum_{j=1}^{n} \lambda_j x_{ij} \leq \theta x_{io}, \qquad i=1,2,\ldots,m, \\ & \quad \sum_{j=1}^{n} \lambda_j y_{rj} \geq y_{ro}, \qquad r=1,2,\ldots,s, \\ & \qquad \lambda_j \geq 0, \qquad j=1,2,\ldots,n. \end{aligned} \tag{7}$$

Note that the model is always feasible and $0 < \theta^* \leq 1.$

If $\theta^* = 1$, MU is practical; otherwise, it is not. The dual envelopment form, known as the multiplication form, is as below:

$$
\begin{aligned}
&\text{Max} \sum_{r=1}^{s} u_r y_{r\circ}, \\
&\text{s.t.} \\
&\qquad \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{s} v_i x_{ij} \leq \circ, \quad j = 1, \cdots, n, \\
&\qquad \sum_{i=1}^{m} v_i x_{i\circ} = 1, \\
&\qquad u_r \geq \circ, \quad r = 1, \cdots, s, \\
&\qquad v_i \geq \circ, \quad i = 1, \cdots, m.
\end{aligned} \tag{8}
$$

The CRR model in the output orientaion is as below:

$$
\begin{aligned}
&\text{Max} \quad \varphi \\
&\text{s.t.} \quad \sum_{j=1}^{n} \lambda_j X_j \leq X_\circ, \\
&\qquad \sum_{j=1}^{n} \lambda_j Y_j \geq \varphi Y_\circ, \\
&\qquad \lambda_j \geq \circ, \quad j = 1, \cdots, n, \\
&\qquad v_i \geq \circ, \quad i = 1, \cdots, m.
\end{aligned} \tag{9}
$$

The above model is the CRR model in the output orientaion. This model is always feasible and φ*=1. If φ*=1, DMU0 is practical; otherwise, it is not. The two above models are called multiplication models with the output orientaion of CCR, which is as below:

$$
\begin{aligned}
&\text{Max} \qquad V^t X_o, \\
&\text{s.t.} \\
&\qquad U^t Y_o = 1, \\
&\qquad V^t X_j - U^t Y_j \leq 0, \quad j = 1, 2, \ldots, n, \\
&\qquad U \geq 0, \;\; V \geq 0.
\end{aligned} \tag{10}
$$

DMU0 is practical only and only if after solving the multiplication form of the CCR model in the optimal input orientaion (V*, U*), $U^{*t}Y_o=1$, (U^*, V^*).

By considering return technology to the changing scale of the BCC model, the input orientaion in the envelopment form is defined as below:

$$
\begin{aligned}
&\text{Min} \quad \theta, \\
&\text{s.t.} \\
&\quad \sum_{j=1}^{n} \lambda_j x_{ij} \le \theta x_{io}, \quad i = 1,2,\dots,m, \\
&\quad \sum_{j=1}^{n} \lambda_j y_{rj} \ge y_{ro}, \quad r = 1,2,\dots,s, \\
&\quad \sum_{j=1}^{n} \lambda_j = 1, \quad j = 1,2,\dots,n, \\
&\quad \lambda_j \ge 0, \quad j = 1,2,\dots,n.
\end{aligned} \tag{11}
$$

This LP model is always practical and has a finite optimum, and always $0<\theta^*\le 1$.

The DMU of DMU0 in the BCC model is the efficiency of Pareto. If $\theta_B=1$, all subsidiary variables are zero in all optimal responses. The dual of this model is known as the multiplication form of BCC and is as below:

$$
\begin{aligned}
&\text{Max} \quad U^t Y_o + u_o, \\
&\text{s.t.} \\
&\quad V^t X_o = 1, \\
&\quad U^t Y_j - V^t X_j + u_o \le 0, \quad j = 1,2,\dots,n, \\
&\quad U \ge 0, V \ge 0.
\end{aligned} \tag{12}
$$

2.2 | Network Data Envelopment Analysis

The network data envelopment analysis of DEA conventional models assumes DMUs to be a black box and neglects their internal structure. Färe [19], and Färe and Geraskove [20], [21] proposed network data envelopment analysis to overcome this problem as well as the problem of neglecting efficiency calculations. They believe that DEA conventional models overlook the organizational processes of the DMUs in their investigations and consider them as a black box, in which inputs are transformed into outputs without considering their internal structure. To improve performance, however, it is required to study different processes of the organization at different levels and divide successful parts from failed ones [22]. There are two common methods among the conventional DEA models to measure the efficiency of multiple parts organizations.

Accumulation (black box): As shown in *Fig. 1*, in a simple procedure, sections are accumulated and considered as a company. This procedure overlooks internal activities interaction and cannot calculate the impact of the inefficiency of sections on the entire efficiency of the company. In addition, this state can result in the improper selection of inputs, outputs, and non-logical evaluation of the DMU.

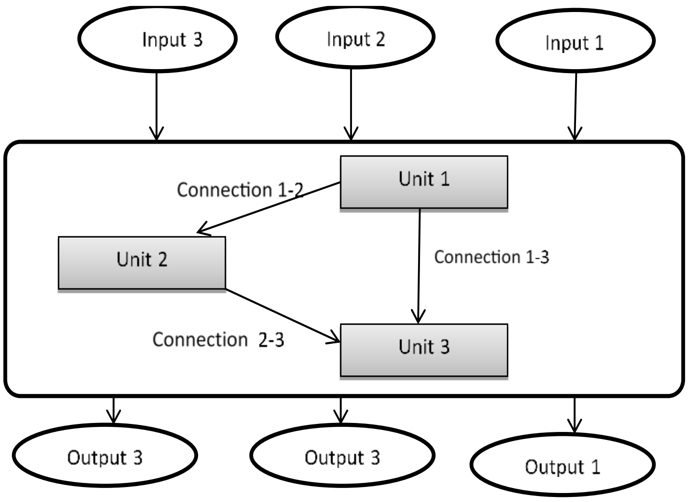

Fig. 1. Accumulation of the organization's units in the form of a black box.

Division: the second procedure involves measuring the efficiency of individual parts. Using this method, it is possible to evaluate the efficiency of each unit of the company among the DMUs. The procedure, however, is not practical for maintaining connectivity between units (see *Fig. 2*).

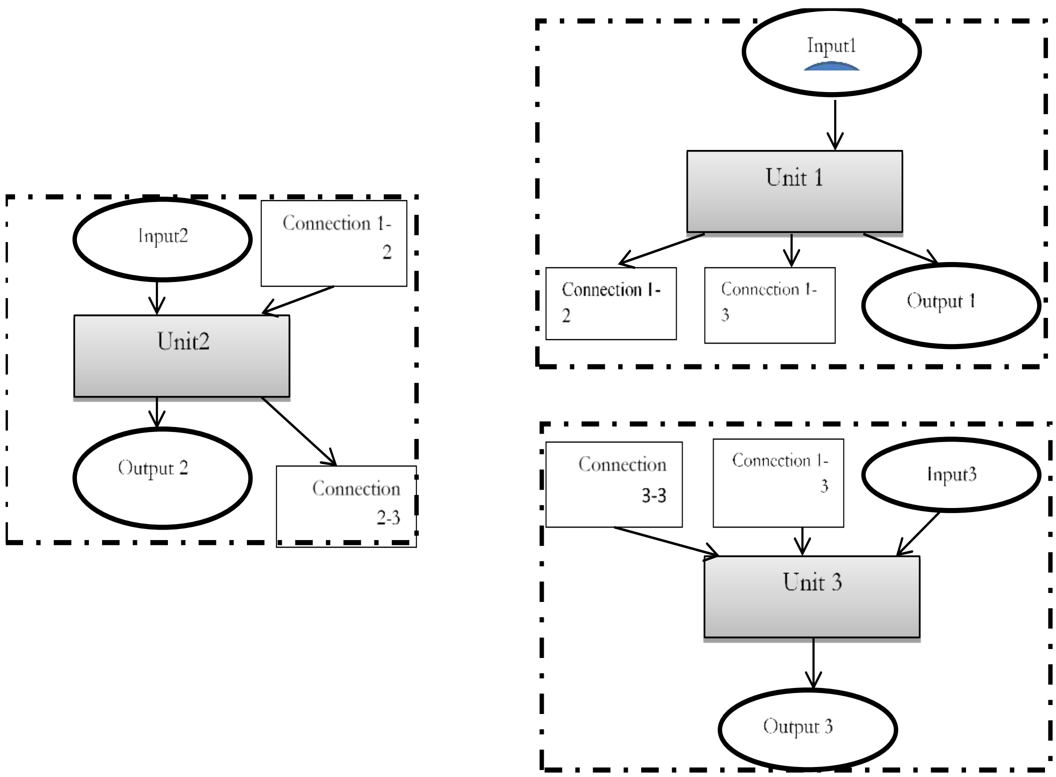

Fig. 2. Division of units of the organization.

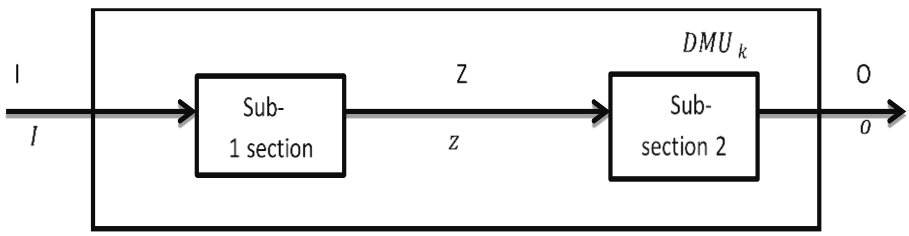

Fig. 3. Two-stage system.

The overall form of the two-stage system is shown in *Fig. 3*, in which I, O, and Z are the input, the output of the DMUs, and the interconnection between sub-sections, respectively. The output of the first subsection is the input of the second subsection. The second subsection does not consume any exogenous input, and the first subsection does not create any exogenous output. Therefore, in 2008, the following model was presented for calculating the efficiency of a DMU with a two-stage structure, as shown in *Fig. 3* [12].

$$E_k^s = \max {\sum_{r=1}^{s} u_r y_{rk}} \Big/ {\sum_{i=1}^{m} v_i x_{ik}},$$

$$\begin{aligned} \text{s.t.} \quad & \\ & {\sum_{r=1}^{s} u_r y_{rk}} \Big/ {\sum_{i=1}^{m} v_i x_{ik}} \leq 1. \ \ j = 1. \dots . n, \\ & {\sum_{p=1}^{q} w_p z_{pj}} \Big/ {\sum_{i=1}^{m} v_i x_{ij}} \leq 1. \ \ j = 1. \dots . n, \\ & {\sum_{r=1}^{s} u_r y_{rj}} \Big/ {\sum_{p=1}^{q} w_p z_{pj}} \leq 1. \ \ j = 1. \dots . n, \\ & u_r . v_i . w_p \geq \varepsilon \ r = 1. \dots . s. i = 1. \dots . m. p = 1. \dots . q. \end{aligned} \tag{13}$$

2.3 | Supply Chain

In the global competition of the current era, various products should be provided regarding the customer's needs. The customer's desire for high quality and quick service has created pressure that has never been experienced before. In conclusion, companies cannot do everything alone. In the present competitive market, economic and production companies need management and monitoring resources and respective members outside the organization, in addition to considering internal resources. The reason is to achieve competitive advantages aiming to have a larger share of the market. Accordingly, activities such as supply and demand management, material provision, production management, good maintenance service, availability control, distribution, delivery, and service to the customer, which have already been noted in the country, were elevated to the supply chain level. The key factor in a supply chain is managing and controlling all these activities. Supply Chain Management (SCM) is a phenomenon to do this, and customers can receive reliable and fast service with a high quality and low price. In the 1960-70 decades, organizations paid more attention to developing market strategies focused on satisfying customers. They found that strong engineering, design, and harmonic production operation are required to achieve market demands and, consequently larger share of the market. It is thus imperative that designers incorporate the ideals and requirements of customers when designing productions and present them to the market at a minimum price while maintaining the maximum possible level of quality. In 1990, along with improving production abilities, industrial managers found that receiving materials and services from different providers significantly affected increasing the abilities of organizations to meet customer requirements. It influences the organization's focus, supply bases, and resource-finding strategies. Managers also found that merely producing a production is not enough. In fact, the provision of products with criteria of the customers (when, where, how) and their required cost and quality created new challenges. In this circumstance, they found from the above changes that these changes are not sufficient in the long-term to manage their organization. They should have been involved in the network management of all factories and companies that provided the input of their organization directly and indirectly, and in companies related to delivery and after-sale services. Regarding this vision, supply chain procedures and management appeared.

Therefore, one can say that the supply chain includes all stages which directly and indirectly affect meeting the requirements of a customer. In an ordinary supply chain, raw materials are sent from providers to factories. Then, products are delivered to central and distributor warehouses to get to the final customers or consumers. Then, the good passes through different steps of a chain to get to the consumer. In some of these stages, the good is stored, and in others, it is transported. It means the supply chain is a set of storage and transportation. Members of an ordinary supply chain are providers, ingredient warehouses, production centers, distributors, retailers, and final customers. Each commercial organization is a unit of the supply chain, and many organizations are units of several supply chains. A supply chain's number and type are determined by specifying which organization is the producer or beneficiary. Traditionally, a supply chain consists of the below stages or cycles (see *Fig. 4*).

Fig. 4. Supply chain.

3 | Performance Evaluation of the Supply Chain of Petrochemical Using Data Envelopment Analysis

Fig. 5 shows a three-stage chain (network). As we will discuss in the next section, this structure is derived from the petrochemical chain. Despite being part of the petrochemical chain, we know of many petrochemical units that share similar production characteristics, so we can compare them. In this figure, the first stage has an independent input of X1. Also, a returned output from the second stage, Y, will enter this stage, exit from the first stage of the Z output, which is a mediator production, and enters the second stage. The first stage has another output, D, which exit as the final output. The mediator production of Z is the input of the second stage. Additionally, this stage has another input stage, X2, which enters from outside the system. Y represents the mediator stage of the second stage of production. Mediator productions of the second stage return to the first stage, and another proceeds to the third stage. The third stage has three other independent inputs, X3, which enters the system from outside. Finally, an output, D, exits the third stage as the final output.

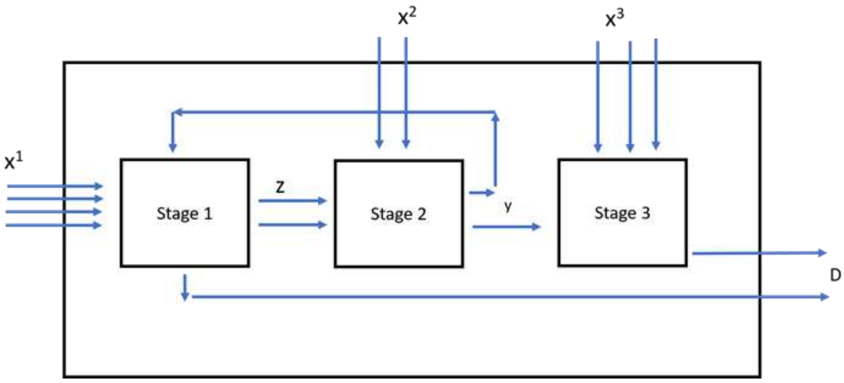

Fig. 5. Three-stage network (chain) based on the petrochemical process structure.

Analysis and investigation of network units in the data envelopment analysis attracted attention after presenting the DEA classic models, which considered DMUs as a black box. In this procedure, the impact of relationships between stages and considering this equation in the numerical modeling for evaluation is required. Because considering the internal structure of a DMU, composed of different components, affects the network evaluation considerably. This research involved modeling and evaluation, as shown in *Fig. 1*. We examine modeling for efficiency evaluation, finding the pattern, and obtaining improved activities for the entire network and each stage. We follow this numerical modeling in DEA with both envelopment and multiplication forms, and we try to consider different states of modeling to achieve overall efficiency, aggregate efficiency, and stage efficiency.

3.1 | Modeling in the Envelopment form of the Data Envelopment Analysis

I. Envelopment form model of the input orientaion

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Consider the input orientaion envelopment form of the above network. In this model, efficiency has been minimized in terms of inputs. Note that all external inputs that enter the three-stages of thenetwork were minimized.

$$
\begin{aligned}
&\text{Min} \quad \theta, \\
&\text{s.t.} \\
&\qquad \sum_{j=1}^{n} \lambda_j^1 x_{ij}^1 \le \theta x_{io}^1, \qquad i = 1,\dots,4, \\
&\qquad \sum_{j=1}^{n} \lambda_j^1 d_{1j} \ge d_{1o}, \\
&\qquad \sum_{j=1}^{n} \lambda_j^1 z_{fj} \ge z_{fo}, \qquad f = 1,2, \\
&\qquad \sum_{j=1}^{n} \lambda_j^1 y_{1j} \ge y_{1o}, \\
&\qquad \sum_{j=1}^{n} \lambda_j^2 x_{lj}^2 \le \theta x_{lo}^2, \qquad l = 1,2, \\
&\qquad \sum_{j=1}^{n} \lambda_j^2 z_{fj} \le z_{fo}, \qquad f = 1,2, \\
&\qquad \sum_{j=1}^{n} \lambda_j^2 y_{rj} \ge y_{ro}, \qquad r = 1,2, \\
&\qquad \sum_{j=1}^{n} \lambda_j^3 x_{pj}^3 \le \theta x_{po}^3, \qquad l = 1,2,3, \\
&\qquad \sum_{j=1}^{n} \lambda_j^3 d_{2j} \le d_{2o}, \\
&\qquad \sum_{j=1}^{n} \lambda_j^3 y_{2j}^3 \ge y_{2o}, \\
&\qquad \sum_{j=1}^{n} (\lambda_j^s) = 1, \qquad s=1,2,3, \\
&\qquad \lambda_j^s \ge 0, \qquad j = 1,\dots,n, s = 1,2,3.
\end{aligned}
\tag{14}
$$

This model is used to determine efficiency and improved inputs. If we accept that θ^* is the value of the overall efficiency of the supply chain system, the corresponding formula of the improved inputs is:

$$\text{improved inputs} = (\theta^* x_o^1,\ \theta^* x_o^2,\ \theta^* x_o^3).$$

We prove two below theorems to analyze the noted model.

Theorem 1. From *Model (1)*, $\theta^* \le 1$.

Theorem 2. Improved inputs in the DMUo evaluation unit, which is calculated from $(\theta^* x_o^1, \theta^* x_o^2, \theta^* x_o^3)$, are on the weak efficiency border.

Proving Theorem 2. From *Model (14)*, $\theta^* \leq 1$. Since there is a practical $\theta = 1$, answer for *Model (15)*, and this model is from the minimization type, then $\theta^* \leq 1$.

Theorem 3. Improved inputs in the DMUo unit are calculated from $(\theta^* x_o^1, \theta^* x_o^2, \theta^* x_o^3)$, and are in the weak efficiency border.

Proving Theorem 3. By absurd hypothesis, the point is not on the weak efficiency border, so a smaller value than θ^*, such as $\bar{\theta}^*$,$\theta^* < \theta^*$can be found, to $\left(\theta^* x_o^1, \theta^* x_o^2, \theta^* x_o^3\right)$ be on the weak efficiency border, which is against the assumption of θ^* being optimum. Then the absurd hypothesis is wrong, and the verdict is valid.

Consider the output orientaion of the envelopment form corresponding to the above network. In this model, the efficiency was maximized in terms of outputs. Note that all outputs quitting the triple stages of the network were maximized.

II. Envelopment form model of the output orientaion.

Consider the output orientaion of the envelopment form corresponding to the above network. In this model, the efficiency was maximized in terms of outputs. Note that all external outputs entering the triple stages of the network were maximized.

$$
\begin{aligned}
&\text{Max} \quad \varphi \\
&\text{s.t.} \\
&\sum_{j=1}^{n} \lambda_j^1 x_{ij}^1 \leq x_{io}^1, && i = 1,\ldots,4, \\
&\sum_{j=1}^{n} \lambda_j^1 d_{1j} \geq \varphi d_{1o}, \\
&\sum_{j=1}^{n} \lambda_j^1 z_{fj} \geq z_{fo}, && f = 1,2, \\
&\sum_{j=1}^{n} \lambda_j^1 y_{1j} \geq y_{1o}, \\
&\sum_{j=1}^{n} \lambda_j^2 x_{lj}^2 \leq x_{lo}^2, && l = 1,2, \qquad (15) \\
&\sum_{j=1}^{n} \lambda_j^2 z_{fj} \leq z_{fo}, && f = 1,2, \\
&\sum_{j=1}^{n} \lambda_j^2 y_{rj} \geq y_{ro}, && r = 1,2, \\
&\sum_{j=1}^{n} \lambda_j^3 x_{pj}^3 \leq x_{po}^3, && l = 1,2,3, \\
&\sum_{j=1}^{n} \lambda_j^3 d_{2j} \leq \varphi d_{2o},
\end{aligned}
$$

$$\sum_{j=1}^{n}\lambda_j^3 y_{2j}^3 \geq y_{2o},$$

$$\sum_{j=1}^{n}(\lambda_j^s)=1, \qquad s=1,2,3,$$

$$\lambda_j^s \geq 0, \qquad j=1,\dots,n, s=1,2,3.$$

This model is used to determine efficiency and improved inputs. The corresponding formula of the improved inputs is:

$$\text{Improved Outputs}=(\varphi^* d_o).$$

We prove two below theorems to analyze the noted model:

Theorem 4. From *Model (15)*, $\varphi^* \geq 1$.

Theorem 5. Improved inputs in the DMUo evaluation unit, which is calculated from ($\varphi^* d_o$), are on the weak efficiency border.

Theorem 6. From *Model (15)*, $\varphi^* \geq 1$.

Proving Theorem 6. From *Model (14)*, $\theta^* \leq 1$. Since there is a practical $\varphi = 1$ answer for *Model (15)*, and this model is from the maximizing type, then $\varphi^* \geq 1$.

Theorem 7. Improved inputs in the DMUo unit are calculated from ($\varphi^* d_o$), and are in the weak efficiency border.

Proving Theorem 7. By absurd hypothesis, the point is not on the weak efficiency border, so a larger value than φ^* , such as $\overline{\varphi}^*$, φ^* can be found, to ($\varphi^* d_o$) be on the weak efficiency border, which is against the assumption of φ^* being optimum. Then the absurd hypothesis is wrong, and the verdict is valid.

An index can be introduced considering *Models (15)* and *(16)* by combining the optimum responses of these models as below. If the input efficiency of the chain axis is θ^* and the output efficiency of its axis is φ^*, the combined equation is as below:

$$\text{Efficiency Index (E.I)}=\frac{1}{w_1\theta^* + w_2\varphi^*},$$

where $w_1, w_2 \in R^+, w_1 + w_2 = 1$.

In other words, w1 and w2 weights are determined by the system manager or expert and represent the importance of $\theta^*$and φ^*concerning each other. It is evident that $0 \prec E.\,I \leq 1$.

The supply chain efficiency is within the range of 0 and 1. According to the envelopment analysis fundamentals, if the efficiency is 1, then the structure is efficient. To do this, the below theorem can be proved.

Theorem 8. If E. I=1 in the DMUo evaluation, the unit is efficient, and if E. I<1, the evaluated unit is inefficient.

Proving Theorem 8. If E. I=1, we have $w_1\theta^* + w_2\varphi^* = 1$. since $w_1, w_2 \in R^+, w_1 + w_2 = 1$ and $\theta^* \leq 1, \varphi^* \geq 1$.

It was found that $\theta^* = \varphi^* = 1$. Then the efficiency unit is efficient. Because there is no suggested change in the input or outputs of the model.

If $E.I < 1$, then $w_1\theta^* + w_2\varphi^* > 1$, since $w_1, w_2 \in R^+, w_1 + w_2 = 1$, it was concluded that $\varphi^* > 1$ or $\theta^* < 1$. If one or both noted equations are validated, we conclude that the evaluation unit can reduce its inputs or increase its outputs. Since there is a dominant unit for it, it is inefficient.

3.2 | Modeling in the Envelopment form of the Data Envelopment Analysis

We consider the noted network (1) to write the multiplication form model in two input and output orientaions.

I. Multiplication form model in the input orientation.

In this model, the corresponding optimum weight of the input and outputs of each stage is determined, in addition to the efficiency. Note that the stage efficiency should be equal to or lower than 1.

$$
\begin{aligned}
&\text{Max} \quad \sum_{r=1}^{2} k_r d_{ro},\\
&\text{s.t.}\\
&\quad \sum_{i=1}^{3} v_i^1 x_{io}^1 + \sum_{l=1}^{2} v_l^2 x_{lo}^2 + \sum_{p=1}^{3} v_p^3 x_{po}^3 = 1,\\
&\quad \sum_{f=1}^{2} w_f z_{fj} + u_1 d_{1j} - \sum_{i=1}^{3} v_i^1 x_{ij}^1 - k_1 y_{1j} + q_0^1 \le 0, \qquad j=1,\dots,n,\\
&\quad \sum_{r=1}^{2} k_r y_{rj} - \sum_{l=1}^{2} v_l^2 x_{lj}^2 - \sum_{f=1}^{2} w_f z_{fj} + q_0^2 \le 0, \qquad j=1,\dots,n,\\
&\quad u_2 d_{2j} - \sum_{p=1}^{3} v_p^3 x_{pj}^3 - k_2 y_{2j} + q_0^3 \le 0, \qquad j=1,\dots,n,\\
&\quad \sum_{r=1}^{2} k_r d_{rj} - \sum_{i=1}^{3} v_i^1 x_{ij}^1 - \sum_{l=1}^{2} v_l^2 x_{lj}^2 - \sum_{p=1}^{3} v_p^3 x_{pj}^3 \le 0, \qquad j=1,\dots,n,\\
&\quad k_r \ge 0, v_i^1 \ge 0, v_l^2 \ge 0, v_p^3 \ge 0, w_f \ge 0, u_s \ge 0 \quad \text{for all} \quad r,i,l,p,f,s,\\
&\quad q_0^1,\ q_0^2,\ q_0^3 \quad \text{Free.}
\end{aligned}
\tag{16}
$$

Theorem 9. It is proved that $\sum_{r=1}^{2} k_r^* d_{ro} \le 1$.

Proving Theorem 9. Considering that *Model (16)* is the dual of *Model (14)*, the finite optimum response of these two problems has a similar value of the goal function. Therefore: $\sum_{r=1}^{2} k_r^* d_{ro} \le 1$.

II. Multiplication form model in the output orientation.

We write the Multiplication form model in the input orientaion for *Fig. 1* to be in accordance with the *Eq. (4)*. In this model, the corresponding optimum weight of the input and outputs of each stage is determined, in addition to the efficiency. Note that the stage efficiency should be equal to or larger than 1.

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$$
\begin{aligned}
\text{Min} \quad & \sum_{i=1}^{3} v_i^1 x_{io}^1 + \sum_{l=1}^{2} v_l^2 x_{lo}^2 + \sum_{p=1}^{3} v_p^3 x_{po}^3 \\
\text{s.t.} \quad & \\
& \sum_{r=1}^{2} k_r d_{ro} = 1, \\
& \sum_{i=1}^{3} v_i^1 x_{ij}^1 + k_1 y_{1j} - \sum_{f=1}^{2} w_f z_{fj} - u_1 d_{1j} + t_0^1 \geq 0, \qquad j=1,\dots,n, \\
& \sum_{l=1}^{2} v_l^2 x_{lj}^2 + \sum_{f=1}^{2} w_f z_{fj} - \sum_{r=1}^{2} k_r y_{rj} + t_0^2 \geq 0, \qquad j=1,\dots,n, \\
& \sum_{p=1}^{3} v_p^3 x_{pj}^3 + k_2 y_{2j} - u_2 d_{2j} + t_0^3 \geq 0, \qquad j=1,\dots,n, \\
& \sum_{i=1}^{3} v_i^1 x_{ij}^1 + \sum_{l=1}^{2} v_l^2 x_{lj}^2 + \sum_{p=1}^{3} v_p^3 x_{pj}^3 - \sum_{r=1}^{2} k_r d_{rj} \geq 0, \qquad j=1,\dots,n, \\
& k_r \geq 0, v_i^1 \geq 0, v_l^2 \geq 0, v_p^3 \geq 0, w_f \geq 0, u_s \geq 0 \quad \text{for all } r,i,l,p,f,s, \\
& t_0^1, t_0^2, t_0^2 \ \text{Free.}
\end{aligned}
\tag{17}
$$

Theorem 10. It is proved that:

$$\sum_{i=1}^{3} v_i^{1*} x_{io}^1 + \sum_{l=1}^{2} v_l^{2*} x_{lo}^2 + \sum_{p=1}^{3} v_p^{3*} x_{po}^3 \geq 1.$$

Proving Theorem 10. By considering that *Model (17)* is the dual of *Model (15)*, the finite optimum response of these two problems has a goal function equal to $\sum_{i=1}^{3} v_i^{1*} x_{io}^1 + \sum_{l=1}^{2} v_l^{2*} x_{lo}^2 + \sum_{p=1}^{3} v_p^{3*} x_{po}^3 \geq 1$. Therefore, one can change modeling in *Models (16)* to obtain the stage efficiency. The mathematical equations are shown below to calculate the efficiency of various stages. If ei is the value of the efficiency of different stages of this supply chain, we can calculate the values of these efficiencies according to the below equations:

$$\text{Efficiency Stage } 1 = e_1 = \frac{\sum_{f=1}^{2} w_f z_{fj} + u_1 d_{1j} + q_0^1}{\sum_{i=1}^{3} v_i^1 x_{ij}^1 + k_1 y_{1j}} \leq 1, \qquad j=1,\dots,n,$$

$$\text{Efficiency Stage } 2 = e_2 = \frac{\sum_{r=1}^{2} k_r y_{rj} - \sum_{l=1}^{2} v_l^2 x_{lj}^2 + q_0^1}{\sum_{f=1}^{2} w_f z_{fj}} \leq 1, \qquad j=1,\dots,n,$$

$$\text{Efficiency Stage } 3 = e_3 = \frac{u_2 d_{2j} + q_0^3}{\sum_{p=1}^{3} v_p^3 x_{pj}^3 - k_2 y_{2j}} \leq 1, \qquad j=1,\dots,n,$$

$$\text{Efficiency Overall} = e_o = \frac{\sum_{r=1}^{2} u_r d_{rj}}{\sum_{i=1}^{3} v_i^1 x_{ij}^1 + \sum_{l=1}^{2} v_l^2 x_{lj}^2 + \sum_{p=1}^{3} v_p^3 x_{pj}^3} \leq 1, \qquad j=1,\dots,n.$$

Consider *Model (17)*. An important principle of this model is to determine if the input efficiency of the entire supply chain can be obtained based on input reduction. If we change *Model (16)* as described below, we can interpret the efficiency of the stage based on the model. Below is a formula that describes the stage efficiency and the entire network following the presented model.

Notably, a crucial equation in calculating the entire system efficiency is using an aggregate relationship. The efficiency of the first, second, and third stages are e1, e2, and e3, respectively, and by considering the aggregate equation, the overall efficiency is defined as below:

$$\text{Efficiency Aggregate } = e^a = \mu_1 e_1 + \mu_2 e_2 + \mu_3 e_3.$$

Substitution of their equivalent defined mathematical equations gives us the aggregate efficiency as below:

$$\text{Efficiency Aggregate} = e^a = \mu_1 e_1 + \mu_2 e_2 + \mu_3 e_3,$$

$$e^a = \mu_1 \frac{\sum_{f=1}^{2} w_f z_{fj} + u_1 d_{1j} + q_0^1}{\sum_{i=1}^{3} v_i^1 x_{ij}^1 + k_1 y_{1j}} + \mu_2 \frac{\sum_{r=1}^{2} k_r y_{rj} + \sum_{l=1}^{2} v_l^2 x_{lj}^2 + q_0^2}{\sum_{f=1}^{2} w_f z_{fj}} + \mu_3 \frac{u_2 d_{2j} + q_0^3}{\sum_{p=1}^{3} v_p^3 x_{pj}^3 + k_2 y_{2j}} \quad j=1,\dots,n.$$

Values of μ_i show the efficiency value weights of each stage. We define weights as below. Each fraction for each DMUj is the ratio of the utilized inputs in each stage to independent inputs (independent inputs enter each stage outside the system) of the system.

$$\mu_1 = \frac{\sum_{i=1}^{3} v_i^1 x_{ij}^1 + k_1 y_{1j}}{\sum_{i=1}^{3} v_i^1 x_{ij}^1 + \sum_{l=1}^{2} v_l^2 x_{lj}^2 + \sum_{p=1}^{3} v_p^3 x_{pj}^3} j=1,\dots,n,$$

$$\mu_2 = \frac{\sum_{f=1}^{2} w_f z_{fj}}{\sum_{i=1}^{3} v_i^1 x_{ij}^1 + \sum_{l=1}^{2} v_l^2 x_{lj}^2 + \sum_{p=1}^{3} v_p^3 x_{pj}^3} j=1,\dots,n,$$

$$\mu_3 = \frac{\sum_{p=1}^{3} v_p^3 x_{pj}^3 + k_2 y_{2j}}{\sum_{i=1}^{3} v_i^1 x_{ij}^1 + \sum_{l=1}^{2} v_l^2 x_{lj}^2 + \sum_{p=1}^{3} v_p^3 x_{pj}^3} =1,\dots,n.$$

Therefore, the simple form of the defined state for the introduced network system aggregate efficiency in *Fig. 1* is as below:

$$e^a = \mu_1 e_1 + \mu_2 e_2 + \mu_3 e_3 = \frac{\sum_{f=1}^{2} w_f z_{fo} + u_1 d_{1o} + u_2 d_{2o} + \sum_{r=1}^{2} k_r y_{rj}}{\sum_{i=1}^{3} v_i^1 x_{io}^1 + \sum_{l=1}^{2} v_l^2 x_{lo}^2 + \sum_{p=1}^{3} v_p^3 x_{po}^3}, \quad j=1,\dots,n.$$

We overlooked the free variable in the numerator of the fraction after simplifying the introduced state in the equation above. *Model (18)* is derived from the above equation. As shown in the following model, the goal is to maximize the aggregate efficiency of the network if both the overall efficiency and the stage efficiency are less than 1. See the following model:

$$\text{Max} \quad \frac{\sum_{f=1}^{2} w_f z_{fo} + u_1 d_{1o} + u_2 d_{2o} + \sum_{r=1}^{2} k_r y_{ro}}{\sum_{i=1}^{3} v_i^1 x_{io}^1 + \sum_{l=1}^{2} v_l^2 x_{lo}^2 + \sum_{p=1}^{3} v_p^3 x_{po}^3},$$

s.t.

$$\frac{\sum_{f=1}^{2} w_f z_{fj} + u_1 d_{1j} + q_0^1}{\sum_{i=1}^{3} v_i^1 x_{ij}^1 + k_1 y_{1j}} \le 1, \qquad j = 1, \dots, n,$$

$$\frac{\sum_{r=1}^{2} k_r y_{rj} + \sum_{l=1}^{2} v_l^2 x_{lj}^2 + q_0^2}{\sum_{f=1}^{2} w_f z_{fj}} \le 1, \qquad j = 1, \dots, n,$$

$$\frac{u_2 d_{2j} + q_0^3}{\sum_{p=1}^{3} v_p^3 x_{pj}^3 + k_2 y_{2j}} \le 1, \qquad j = 1, \dots, n, \tag{18}$$

$$\frac{\sum_{r=1}^{2} u_r d_{rj}}{\sum_{i=1}^{3} v_i^1 x_{ij}^1 + \sum_{l=1}^{2} v_l^2 x_{lj}^2 + \sum_{p=1}^{3} v_p^3 x_{pj}^3} \le 1, \qquad j = 1, \dots, n,$$

$$\frac{\sum_{f=1}^{2} w_f z_{fj} + u_1 d_{1j} + u_2 d_{2j} + \sum_{r=1}^{2} k_r y_{rj}}{\sum_{i=1}^{3} v_i^1 x_{ij}^1 + \sum_{l=1}^{2} v_l^2 x_{lj}^2 + \sum_{p=1}^{3} v_p^3 x_{pj}^3} \le 1, \qquad j = 1, \dots, n,$$

$$k_r \ge 0, v_i^1 \ge 0, v_l^2 \ge 0, v_p^3 \ge 0, w_f \ge 0, u_s \ge 0 \quad \text{for all } r,i,l,p,f,s,$$

$$q_0^1,\ q_0^2,\ q_0^3 \ \text{Free.}$$

After the application of the variable transformation, $h = \frac{1}{\sum_{i=1}^{3} v_i^1 x_{io}^1 + \sum_{l=1}^{2} v_l^2 x_{lo}^2 + \sum_{p=1}^{3} v_p^3 x_{po}^3}$ we transform the above model, which has a nonlinear goal function, to a linear problem. So, by applying the above variable transformation we will have:

$$hk_r = k_r, hv_i^1 = v_i^1, hv_l^2 = v_l^2, hv_p^3 = v_p^3, hw_f = w_f, hu_s = u_s \quad \text{for all } r,i,l,p,f,s$$
$$hq_0^1 = q_0^1,\ hq_0^2 = q_0^2,\ hq_0^3 = q_0^3, hq = q.$$

To simplify formulation, we used the previous variable name after the variable transformation.

$$hk_r = k_r, hv_i^1 = v_i^1, hv_l^2 = v_l^2, hv_p^3 = v_p^3, hw_f = w_f, hu_s = u_s \quad \text{for all } r,i,l,p,f,s$$
$$hq_0^1 = q_0^1,\ hq_0^2 = q_0^2,\ hq_0^3 = q_0^3, hq = q.$$

Therefore, considering h>0, after applying variable transformation of all nonnegative variables, they remain nonnegative. On the other hand, all free variables in the sign remain free in the sign. After variable transformation, we will have:

$$
\begin{aligned}
\text{Max} \quad & \sum_{f=1}^{2} w_f z_{fo} + u_1 d_{1o} + u_2 d_{2o} + \sum_{r=1}^{2} k_r y_{ro}, \\
\text{s.t.} \quad & \\
& \sum_{i=1}^{3} v_i^1 x_{io}^1 + \sum_{l=1}^{2} v_l^2 x_{lo}^2 + \sum_{p=1}^{3} v_p^3 x_{po}^3 = 1, \\
& \sum_{f=1}^{2} w_f z_{fj} + u_1 d_{1j} - \sum_{i=1}^{3} v_i^1 x_{ij}^1 - k_1 y_{1j} + q_0^1 \le 0, \qquad j=1,\dots,n, \\
& \sum_{r=1}^{2} k_r y_{rj} - \sum_{l=1}^{2} v_l^2 x_{lj}^2 - \sum_{f=1}^{2} w_f z_{fj} + q_0^2 \le 0, \qquad j=1,\dots,n, \\
& u_2 d_{2j} - \sum_{p=1}^{3} v_p^3 x_{pj}^3 - k_2 y_{2j} + q_0^3 \le 0, \qquad j=1,\dots,n, \\
& \sum_{r=1}^{2} u_r d_{rj} - \sum_{i=1}^{3} v_i^1 x_{ij}^1 - \sum_{l=1}^{2} v_l^2 x_{lj}^2 - \sum_{p=1}^{3} v_p^3 x_{pj}^3 \le 0, \qquad j=1,\dots,n, \\
& \sum_{f=1}^{2} w_f z_{fj} + u_1 d_{1j} + u_2 d_{2j} + \sum_{r=1}^{2} k_r y_{rj} - \sum_{i=1}^{3} v_i^1 x_{ij}^1 - \sum_{l=1}^{2} v_l^2 x_{lj}^2 - \sum_{p=1}^{3} v_p^3 x_{pj}^3 \le 0, \qquad j=1,\dots,n, \\
& k_r \ge 0, v_i^1 \ge 0, v_l^2 \ge 0, v_p^3 \ge 0, w_f \ge 0, u_s \ge 0 \quad \text{for all } r,i,l,p,f,s, \\
& q_0^1,\ q_0^2,\ q_0^3 \ \text{Free.}
\end{aligned}
\tag{19}
$$

It is possible to calculate the overall efficiency, the stage efficiency, and the aggregate efficiency based on the optimum response of the above model. All these values were formulated from changing inputs. In other words, models are in the input orientation.

The stage efficiency can be calculated based on the output orientaion. The *Model (19)* can be modified to achieve efficiency at each stage. Consider the *Model (19)*. This model is based on an evaluation of the efficiency of the output of the entire supply chain if the output efficiency has been achieved. We assume that bi is the efficiency value of each stage. We can interpret the efficiency of stages from the model by using the below equations. Following is a formula for determining the stage efficiency and the overall efficiency of the network based on the presented model.

$$\text{Efficiency Stage 1} = b_1 = \frac{\sum_{i=1}^{3} v_i^1 x_{ij}^1 + k_1 y_{1j} + t_0^1}{\sum_{f=1}^{2} w_f z_{fj} + u_1 d_{1j}} \ge 1, \qquad j=1,\dots,n,$$

$$\text{Efficiency Stage 2} = b_2 = \frac{\sum_{f=1}^{2} w_f z_{fj} + t_0^2}{\sum_{r=1}^{2} k_r y_{rj} + \sum_{l=1}^{2} v_l^2 x_{lj}^2} \ge 1, \qquad j=1,\dots,n,$$

$$\text{Efficiency Stage 3} = b_3 = \frac{\sum_{p=1}^{3} v_p^3 x_{pj}^3 + k_2 y_{2j} + t_0^3}{u_2 d_{2j}} \ge 1, \qquad j=1,\dots,n,$$

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$$\text{Efficiency Overall} = b_o = \frac{\sum_{i=1}^{3} v_i^1 x_{ij}^1 + \sum_{l=1}^{2} v_l^2 x_{lj}^2 + \sum_{p=1}^{3} v_p^3 x_{pj}^3}{\sum_{r=1}^{2} u_r d_{rj}} \geq 1, \qquad j=1,\dots,n.$$

As previously discussed, the aggregate efficiency can be defined as follows. The aggregate efficiency is composed of the summation of the weighted efficiency of stages.

$$\text{Efficiency Aggregate} = b^a = \delta_1 b_1 + \delta_2 b_2 + \delta_3 b_3,$$

$$b^a = \delta_1 \frac{\sum_{i=1}^{3} v_i^1 x_{ij}^1 + k_1 y_{1j} + t_0^1}{\sum_{f=1}^{2} w_f z_{fj} + u_1 d_{1j}} + \delta_2 \frac{\sum_{f=1}^{2} w_f z_{fj} + \sum_{l=1}^{2} v_l^2 x_{lj}^2 + t_0^2}{\sum_{r=1}^{2} k_r y_{rj}} + \delta_3 \frac{\sum_{p=1}^{3} v_p^3 x_{pj}^3 + k_2 y_{2j} + t_0^3}{u_2 d_{2j}}, \quad j=1,\dots,n\ .$$

Values of μi show the efficiency value weights of each stage. We define weights as below. Each fraction for each DMUj is the ratio of the produced outputs in each stage to independent outputs (independent outputs exit as the final product from the entire system) in the entire system.

$$\mu_1 = \frac{\sum_{f=1}^{2} w_f z_{fj} + u_1 d_{1j}}{u_1 d_{1j} + u_2 d_{2j}} \quad j=1,\dots,n,$$

$$\mu_2 = \frac{\sum_{r=1}^{2} k_r y_{rj}}{u_1 d_{1j} + u_2 d_{2j}} \quad j=1,\dots,n,$$

$$\mu_3 = \frac{u_2 d_{2j}}{u_1 d_{1j} + u_2 d_{2j}} \quad j=1,\dots,n.$$

Therefore, the simple form of the defined state for the introduced network system aggregate efficiency in *Fig. 1* is as below:

$$b^a = \delta_1 b_1 + \delta_2 b_2 + \delta_3 b_3 = \frac{\sum_{i=1}^{3} v_i^1 x_{ij}^1 + k_1 y_{1j} + \sum_{f=1}^{2} w_f z_{fj} + \sum_{l=1}^{2} v_l^2 x_{lj}^2 + \sum_{p=1}^{3} v_p^3 x_{po}^3 + k_2 y_{2j}}{u_1 d_{1j} + u_2 d_{2j}} \quad j=1,\dots,n.$$

We know that in the above equation, the goal is to minimize the aggregate efficiency of the network if both the overall efficiency and the stage efficiency are less than 1. See the following model:

$$\text{Min} \quad \frac{\sum_{i=1}^{3} v_i^1 x_{io}^1 + k_1 y_{1o} + \sum_{f=1}^{2} w_f z_{fo} + \sum_{l=1}^{2} v_l^2 x_{lo}^2 + \sum_{p=1}^{3} v_p^3 x_{po}^3 + k_2 y_{2o}}{u_1 d_{1o} + u_2 d_{2o}},$$

s.t.

$$\frac{\sum_{i=1}^{3} v_i^1 x_{ij}^1 + k_1 y_{1j} + t_0^1}{\sum_{f=1}^{2} w_f z_{fj} + u_1 d_{1j}} \geq 1, \qquad j=1,\dots,n,$$

$$\frac{\sum_{f=1}^{2} w_f z_{fj} + t_0^2}{\sum_{r=1}^{2} k_r y_{rj} + \sum_{l=1}^{2} v_l^2 x_{lj}^2} \geq 1, \qquad j = 1,\ldots,n,$$

$$\frac{\sum_{p=1}^{3} v_p^3 x_{pj}^3 + k_2 y_{2j} + t_0^3}{u_2 d_{2j}} \geq 1, \qquad j = 1,\ldots,n,$$

$$\frac{\sum_{i=1}^{3} v_i^1 x_{ij}^1 + \sum_{l=1}^{2} v_l^2 x_{lj}^2 + \sum_{p=1}^{3} v_p^3 x_{pj}^3}{\sum_{r=1}^{2} u_r d_{rj}} \geq 1, \qquad j = 1,\ldots,n,$$

$$\frac{\sum_{i=1}^{3} v_i^1 x_{ij}^1 + k_1 y_{1j} + \sum_{f=1}^{2} w_f z_{fj} + \sum_{l=1}^{2} v_l^2 x_{lj}^2 + \sum_{p=1}^{3} v_p^3 x_{pj}^3 + k_2 y_{2j}}{u_1 d_{1j} + u_2 d_{2j}} \geq 1 \quad j = 1,\ldots,n,$$

$$k_r \geq 0, v_i^1 \geq 0, v_l^2 \geq 0, v_p^3 \geq 0, w_f \geq 0, u_s \geq 0, \quad \text{for all } r,i,l,p,f,s,$$
$$t_0^1,\ t_0^2,\ t_0^3 \ \text{Free.}$$

We overlooked the free variable in the numerator of the fraction, which was created after simplifying the introduced state in the equation above.

After the application of the variable transformation, $\frac{1}{u_1 d_{1j} + u_2 d_{2j}} = c$, we transform the above model, which has a nonlinear goal function, to a linear problem. So, by applying the above variable transformation we will have:

$$ck_r = k_r, cv_i^1 = v_i^1, cv_l^2 = v_l^2, cv_p^3 = v_p^3, cw_f = w_f, cu_s = u_s \quad \text{for all } r,i,l,p,f,s.$$
$$ct_0^1 = t_0^1,\ ct_0^2 = t_0^2,\ ct_0^3 = t_0^3,\ ct = t.$$

To simplify formulation, we used the previous variable name after the variable transformation.

To simplify formulation, we used the previous variable name after the variable transformation. Therefore, considering c>0, after applying variable transformation of all nonnegative variables, they remain nonnegative. On the other hand, all free variables in the sign remain free in the sign. After variable transformation, we will have:

$$k_r \geq 0, v_i^1 \geq 0, v_l^2 \geq 0, v_p^3 \geq 0, w_f \geq 0, u_s \geq 0 \quad \text{for all } r,i,l,p,f,s,$$
$$t_0^1,\ t_0^2,\ t_0^3,\ t \ \text{Free.}$$

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Therefore, the below model is driven after simplification:

$$\text{Min} \quad \sum_{i=1}^{3} v_i^1 x_{io}^1 + k_1 y_{1o} + \sum_{f=1}^{2} w_f z_{fo} + \sum_{l=1}^{2} v_l^2 x_{lo}^2 + \sum_{p=1}^{3} v_p^3 x_{po}^3 + k_2 y_{2o}$$

$$\begin{aligned}
\text{s.t.}\quad & u_1 d_{1o} + u_2 d_{2o} = 1, \\
& \sum_{i=1}^{3} v_i^1 x_{ij}^1 + k_1 y_{1j} - \sum_{f=1}^{2} w_f z_{fj} - u_1 d_{1j} + t_0^1 \geq 0, \quad j = 1,\dots,n, \\
& \sum_{f=1}^{2} w_f z_{fj} + \sum_{l=1}^{2} v_l^2 x_{lj}^2 - \sum_{r=1}^{2} k_r y_{rj} + t_0^2 \geq 0, \quad j = 1,\dots,n, \\
& \sum_{p=1}^{3} v_p^3 x_{pj}^3 + k_2 y_{2j} - u_2 d_{2j} + t_0^3 \geq 0, \quad j = 1,\dots,n, \\
& \sum_{i=1}^{3} v_i^1 x_{ij}^1 + \sum_{l=1}^{2} v_l^2 x_{lj}^2 + \sum_{p=1}^{3} v_p^3 x_{pj}^3 - \sum_{r=1}^{2} u_r d_{rj} \geq 0, \quad j = 1,\dots,n, \\
& \sum_{i=1}^{3} v_i^1 x_{ij}^1 + k_1 y_{1j} + \sum_{f=1}^{2} w_f z_{fj} + \sum_{l=1}^{2} v_l^2 x_{lj}^2 + \sum_{p=1}^{3} v_p^3 x_{pj}^3 + k_2 y_{2j} - u_1 d_{1j} + u_2 d_{2j} \geq 0 \quad j = 1,\dots,n, \\
& k_r \geq 0, v_i^1 \geq 0, v_l^2 \geq 0, v_p^3 \geq 0, w_f \geq 0, u_s \geq 0 \quad \text{for all } r,i,l,p,f,s, \\
& t_0^1,\ t_0^2,\ t_0^3 \ \text{Free.}
\end{aligned} \tag{20}$$

From the optimal response of the above model, one can calculate overall efficiency, the stage efficiency, and aggregate efficiency. All these values were formulated based on changing outputs. In other words, models are in the output orientaion. In Section 4, the above models were implemented.

4 | A Practical Example in the Petrochemical Industry

Many of the DMUs have more than one stage. By evaluating the performance of these units using data envelopment analysis, the entire DMU cannot be viewed as a black box. Rather, the internal equations should also be taken into consideration. Different methods were presented to study the efficiency of multistage units.

As noted in Section 3, two different methods were introduced considering the multiplication model to investigate a supply chain, including mediator and reversible relationships along with independent inputs and outputs of the system. One of these methods focused on investigating the aggregate efficiency of the chain, and the other one aimed to evaluate the overall efficiency of the system in terms of input and output orientaion. We evaluated the efficiency of the network stages in both states. Each of the methods has its unique theoretical properties.

In this section, we examine a practical example in the petrochemical industry and implement both introduced procedures in this practical example to analyze the obtained results. A part of the petrochemical industry has been extracted. In addition, we will examine the aggregate and overall efficiency in terms of input and output orientaion, considering the proposed model. Since our objective was to evaluate the results of input and output orientaion, we did not use the hybrid nature of data envelopment analysis.

4.1 | Different Forms of Implementing Decision-Making Units

A DMU transforms data into outputs. In the DEA model, DMUs should be homogenous and have similar tasks and goals. This method measures efficiency by considering the ratio of different inputs (or resources) to different produced outputs (services). Therefore, the variables of the problem can be divided into two overall groups of data and outputs. Determining data and output variables is crucial in implementing the DEA model because the results of this model are based on the selected data and outputs, and changing data or output will change the model results. Therefore, a correct definition of data and output variable gives a realistic efficiency of DMUs. The extraction of evaluation inputs and outputs, which were selected among a set of indices, is the most critical part of the research. It should be noted that considering different goals in evaluation results in selecting various input and output indices. However, the role of indices is to warn decision-makers about potential or hidden problems in specific fields or to continue the desired process in other fields.

The overall process of different petrochemicals was examined to identify indices. Then, experts verified the process of the below figure (see *Fig. 6*).

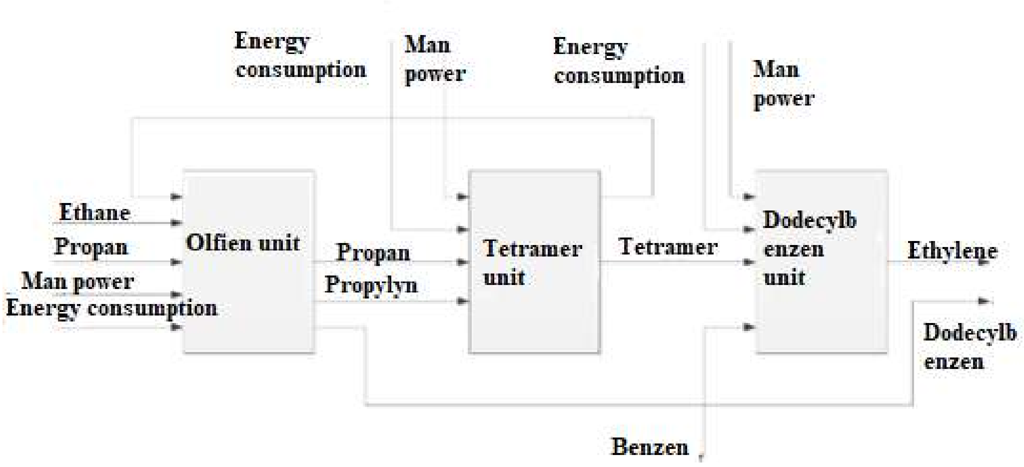

Fig. 6. Petrochemical process.

The figure shows processes in three stages of olefin, tetramer, and dodecylbenzene units. Each unit has specific tasks that are described below:

Industries that transform hydrocarbon of crude oil or natural gas is transformed to a new chemical substance called petrochemical. In some cases, in petrochemical production, a principal upper unit produces raw material for other units. For example, the olefin unit provides the demand for polyethylene and polypropylene units by creating ethylene and propylene. Therefore, the energy state in each unit is examined separately, considering the process difference and diversity.

However, in this industry, like the refinery industries, some units, such as olefine, consume fuel as feed. Consumable energy carriers in petrochemical complexes are often natural gas and fossil fuel. In Petrochemical production, a principal upper unit can produce raw materials for other units. For example, the olefin unit provides the demand for polyethylene, tetramer, and propyl units by creating ethylene and propyl.

After several investigations, indices were completed corresponding to the chain structure. Consultations were made with petrochemicals that have similar processes to extract data from 20 petrochemical units. *Tables 1* and *2* show indices:

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Table 1. Supply chain data in the first stage.

DMUs	Stage 1 Inputs x11 Ethane Value (Thousad Tons)	x12 Propane Value (Thousad Tons)	x13 Man Power Number	x14 Unit Cost	x15 Returned Propane (Thousand Tons)	Intermediate Z11 Propane Value (Thousad Tons)	Z12 Propylene Amount	Output d Ethylene Cost (in Thousands of Dollars)
1	620	325	64	4480	82	117	387	333650
2	540	280	73	5110	75	120	260	322500
3	720	460	123	8610	200	240	530	307500
4	120	1000	38	2660	122	147	650	81000
5	650	390	96	6720	135	170	290	43500
6	750	450	84	5880	180	260	370	427500
7	910	650	101	7070	215	370	440	562500
8	720	480	84	5880	190	320	280	450000
9	420	225	34	3480	132	217	387	433650
10	340	580	83	6110	95	320	460	622500
11	340	360	73	1610	100	140	130	207500
12	160	900	154	8660	202	127	750	61000
13	450	190	56	8720	235	270	390	83500
14	820	850	74	2880	120	160	870	327500
15	820	350	91	2070	195	470	940	862500
16	900	880	64	8880	290	120	580	750000
17	270	625	104	7480	172	470	790	733650
18	820	580	118	9110	175	460	270	522500
19	470	825	94	6480	182	570	940	807500
20	520	280	88	6110	145	120	880	765490

Table 2. Supply chain data in the second and third stages.

DMUs	Stage 2 Input x2 Man Power Tetramer Unit	x2 Unit Cost	Intermediate y Tetramer Value (Thousad Tons)	y	Stage 3 Inputs x3 Man Power Number	x3 Dodecylbenzene (Thousand Tons)	x3 Unit Cost	Output d Benzene Value (Thousand Tons)
1	47	3456	317	182	42	447	4536	143
2	53	3786	360	175	35	472	6543	120
3	68	4327	550	200	35	620	5674	135
4	34	5367	180	122	17	197	5413	137
5	57	4298	325	135	24	397	7654	105
6	48	3987	450	180	36	497	5672	142
7	65	4871	595	215	52	710	6437	187
8	75	5001	380	190	40	464	9254	175
9	37	5456	417	192	62	447	4536	153
10	73	2786	560	165	25	272	5543	160
11	88	8327	250	100	75	720	8674	175
12	74	2367	190	212	47	870	2413	157
13	27	8298	225	235	74	297	9654	125
14	88	9987	650	280	26	497	3672	132
15	16	6871	195	315	82	410	7437	127
16	95	2001	880	290	20	964	2254	195
17	26	9871	217	192	94	897	9654	173
18	95	8001	860	125	86	297	9672	190
19	56	2871	250	220	72	910	8437	185
20	75	4001	380	121	90	264	8254	172

We can analyze units based on data and analysis methods described in Section 3. *Table 3* shows the overall efficiency and stage efficiency based on the multiplication model of the input orientaion.

Table 3. Table results based on the multiplication model of the input orientation.

DMUS	s1 Olefin Unit	s2 Tetramer Unit	s3 Dodecylbenzene Unit	Overall Total Efficiency
DMU1	0.295500	0.668100	0.016600	0.202300
DMU2	0.255500	0.675200	0.009000	0.175400
DMU3	0.153400	0.813700	0.008500	0.126100
DMU4	0.130700	0.468300	0.018400	0.062600
DMU5	0.036200	0.563200	0.006200	0.021200
DMU6	0.255200	0.914800	0.012200	0.234200
DMU7	0.256000	0.636800	0.007600	0.168800
DMU8	0.311300	0.507500	0.006600	0.168400
DMU9	0.301400	1.000000	0.020200	0.299500
DMU10	0.322100	0.804400	0.011100	0.260200
DMU11	0.226300	0.272600	0.005900	0.066600
DMU12	0.087900	0.272200	0.018700	0.025800
DMU13	0.097300	0.707500	0.010300	0.071000
DMU14	0.146100	0.688500	0.009100	0.101400
DMU15	1.000000	1.000000	0. 17700	0.991900
DMU16	0.246700	1.000000	0.018100	0.245400
DMU17	0.909800	0.714100	0.014200	0.653200
DMU18	0.173200	0.881900	0.005400	0.152000
DMU19	1.000000	0.457800	1.574600	0.457000
DMU20	0. 00800	0..239200	0.009800	0.012500

Based on the findings, the studied units, 15 and 20, have the bests and worst conditions, respectively. *Fig. 7* shows the chain's overall efficiency and the stage efficiency in a graph.

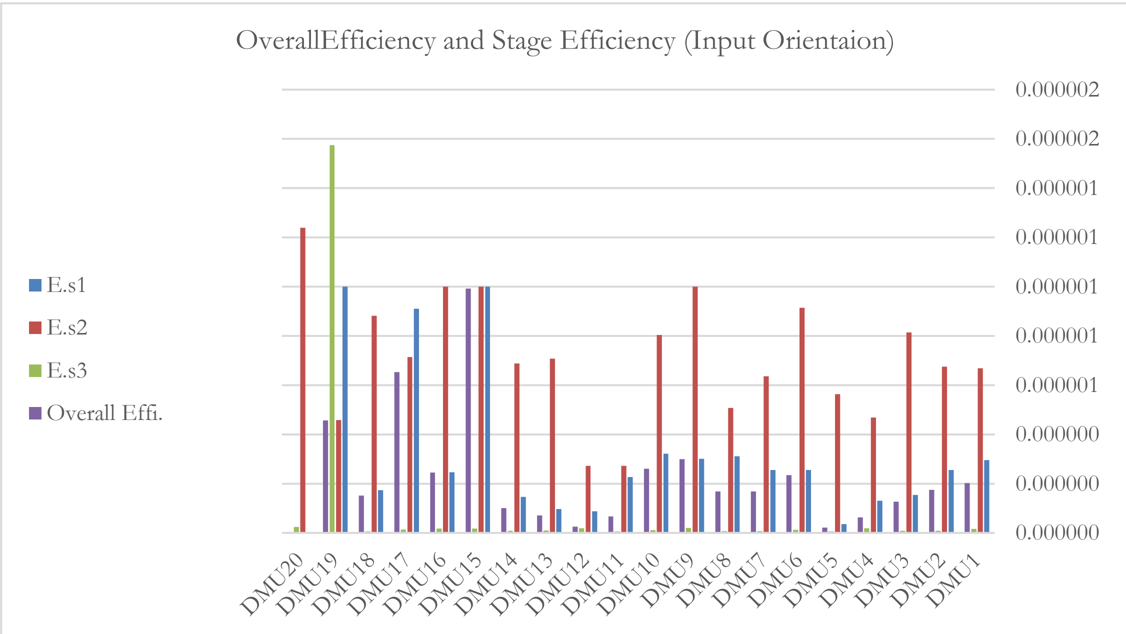

Fig. 7. Efficiency of the entire chain and stage efficiency.

By investigating units in the output orientaion according to the equations of Section 3, the overall efficiency and stage efficiency from the multiplication model of the output orientaion is:

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Table 4. Table results based on the multiplication model of the output orientation.

DMUS	s1 Olefin Unit	s2 Tetramer Unit	s3 Dodecylbenzene Unit	Overall Total Efficiency
DMU1	0.297053	0.52579	0.000242	0.105361
DMU2	0.257162	0.501555	0.000211	0.091121
DMU3	0.154257	0.554847	0.000208	0.067056
DMU4	0.130305	0.392881	0.000347	0.032143
DMU5	0.035933	0.398486	0.000241	0.011208
DMU6	0.256226	0.640369	0.000243	0.125873
DMU7	0.257838	0.433388	0.000267	0.089227
DMU8	0.313607	0.380474	0.000283	0.089066
DMU9	0.302115	0.730194	0.000246	0.155698
DMU10	0.324055	0.520806	0.000298	0.13488
DMU11	0.227211	0.190647	0.000538	0.034385
DMU12	0.08791	0.287836	0.000229	0.013317
DMU13	0.097619	0.722543	0.000164	0.038245
DMU14	0.146041	0.492417	0.000146	0.052429
DMU15	1	0.566669	0.00124	0.57501
DMU16	0.248806	0.664761	0.000207	0.127915
DMU17	0.915081	0.654236	0.000275	0.344542
DMU18	0.174679	0.504999	0.000464	0.080118
DMU19	1	0.430626	0.000257	0.228352
DMU20	0.16542	0.54652	0.000245	0.34210

According to the results in Stage 1 i.e., Olefin unit the DMU5 has the lowest score and DMU15 and DMU19 are efficient. In stage 2 DMU11 has the lowest scoreand DMU9 has the highest score but not efficient. The average score in stage 3 is lower than the other two stages and the highest score in this stage is for DMU11 and the lowest score is for DMU15. Based on the findings, the studied units, 15 and 12, have the bests and worst conditions, respectively (note that the values of the above table are the reverse of values obtained from the model. These values are between 0 and 1, and comparing them is easier, and they represent efficiency). See the below graph. *Fig. 8* shows the chain's overall efficiency and the stage efficiency.

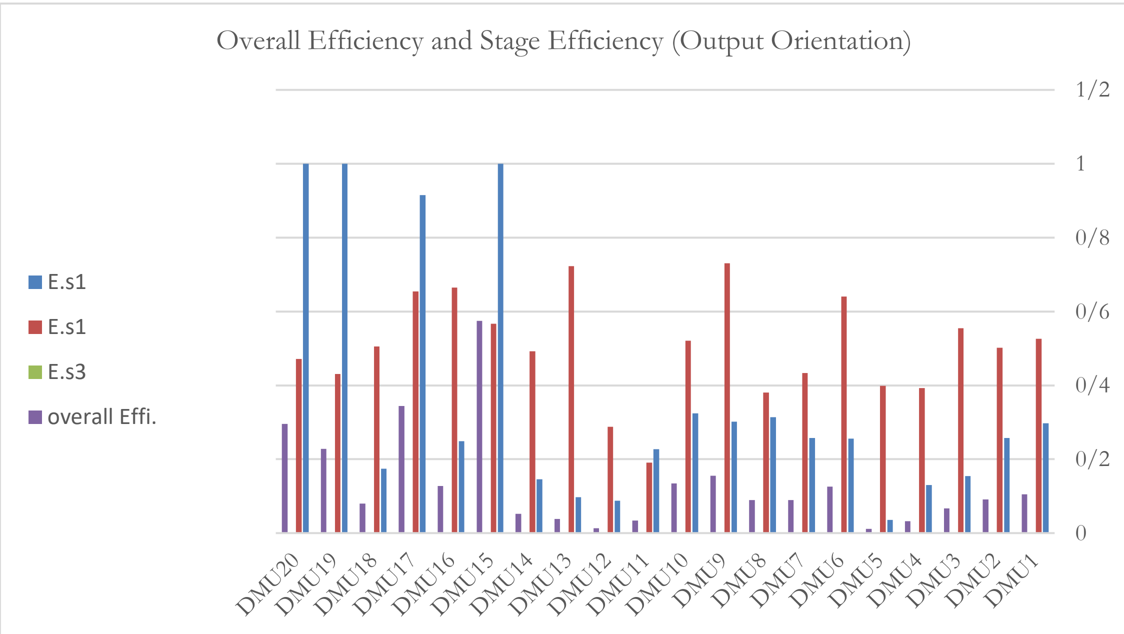

Fig. 8. Efficiency of the entire chain and stage efficiency.

If the evaluation is conducted based on the equations of aggregate efficiency, *Table 5* shows the aggregate efficiency and stage efficiency based on the multiplication model of the input orientaion.

Table 5. Table results based on the aggregate efficiency of the multiplication model of the input orientation.

DMUs	s1 Olefin Unit	s2 Tetramer Unit	s3 Dodecylbenzene Unit	Agregate Total Efficiency
DMU1	0.6682	0.2707	0.0545	0.6618
DMU2	0.6561	0.2845	0.0334	0.6485
DMU3	0.2895	0.1452	0.0192	0.2854
DMU4	0.2899	0.0487	0.0840	0.2861
DMU5	0.0563	0.2203	0.0162	0.0553
DMU6	0.5311	0.3133	0.0273	0.5247
DMU7	0.5825	0.1402	0.0257	0.5742
DMU8	0.5470	0.2241	0.0210	0.5381
DMU9	1.0000	0.2263	0.0666	0.9884
DMU10	1.0000	0.4836	0.0424	0.9898
DMU11	0.4829	0.0403	0.0418	0.4743
DMU12	0.0995	0.1212	0.0712	0.0982
DMU13	0.1543	0.0975	0.0220	0.1509
DMU14	0.5508	0.0837	0.0487	0.5414
DMU15	1.0000	0.0615	0.0175	0.9822
DMU16	0.9156	0.4185	0.0670	0.9094
DMU17	1.0000	0.0726	0.0213	0.9770
DMU18	0.5324	0.1116	0.0186	0.5218
DMU19	1.0000	0.2100	0.0234	0.9846
DMU20	1.0000	0.2370	0.0254	0.9854

Based on the findings, the studied units, 15 and 5, have the bests and worst conditions, respectively. *Fig. 9* shows the chain's overall efficiency and the stage efficiency in a graph.

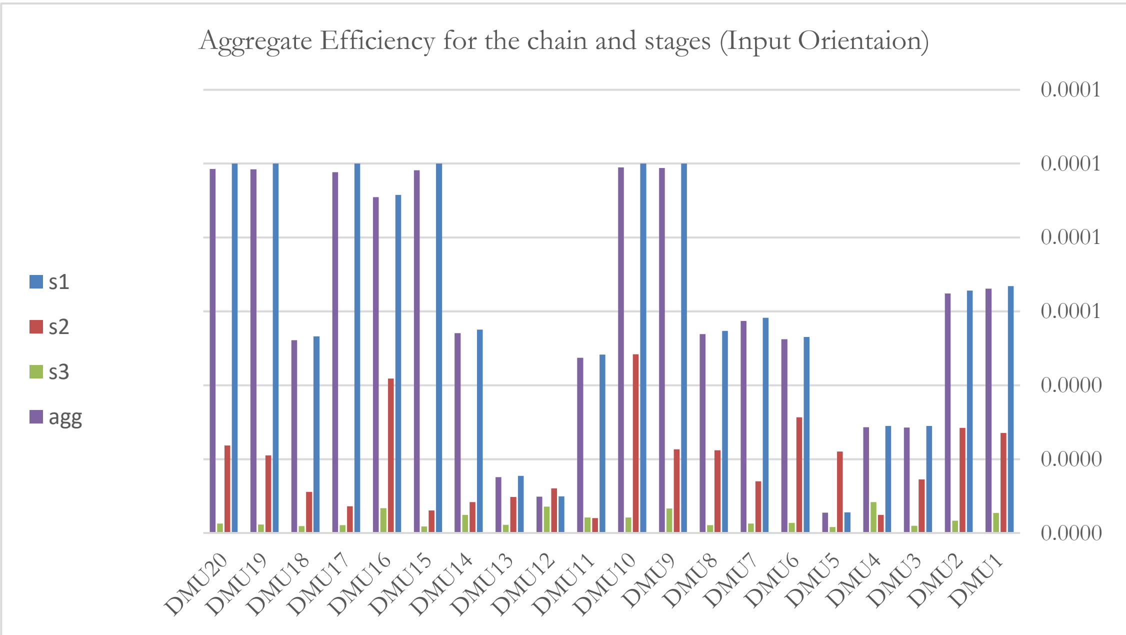

Fig. 9. Aggregate efficiency of the chain and stage efficiency.

If the evaluation is conducted based on the equations of aggregate efficiency, *Table 6* shows the aggregate efficiency and stage efficiency based on the multiplication model of the input orientaion.

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Table 6. Table results based on the aggregate efficiency of the multiplication model of the input orientation.

DMUs	s1 Olefin Unit	s2 Tetramer Unit	s3 Dodecylbenzene Unit	Aggregate Total Efficiency
DMU1	0.297053	0.668047	0.082275	0.125521
DMU2	0.257162	0.675037	0.051482	0.10824
DMU3	0.154257	0.813736	0.067213	0.073573
DMU4	0.130305	0.46834	0.293703	0.043523
DMU5	0.035933	0.563095	0.280041	0.014176
DMU6	0.256226	0.914829	0.052005	0.131082
DMU7	0.257838	0.636699	0.044825	0.108016
DMU8	0.313607	0.507331	0.039077	0.116572
DMU9	0.302115	1	0.067347	0.155674
DMU10	0.324055	0.804505	0.042791	0.14906
DMU11	0.227211	0.272361	0.088062	0.053503
DMU12	0.08791	0.272087	0.724795	0.020727
DMU13	0.097619	0.706864	0.14569	0.04436
DMU14	0.146041	0.688326	0.090032	0.061836
DMU15	1	1	0.017858	0.574317
DMU16	0.248806	1	0.073677	0.127894
DMU17	0.915081	0.6943	0.021754	0.402804
DMU18	0.174679	0.881834	0.035708	0.084963
DMU19	1	0.458064	0.023763	0.312764
DMU20	1	0.438001	0.024341	0.321823

Based on the findings, the studied units, 15 and 12, have the bests and worst conditions, respectively. The below figure shows the chain's overall efficiency and the stage efficiency. Note that the values of the above table are the reverse of values obtained from the model. These values are between 0 and 1, and comparing them is easier, and they represent efficiency.

According to *Fig. 10*, a comparison between the aggregate efficiency and the stage efficiency can be made.

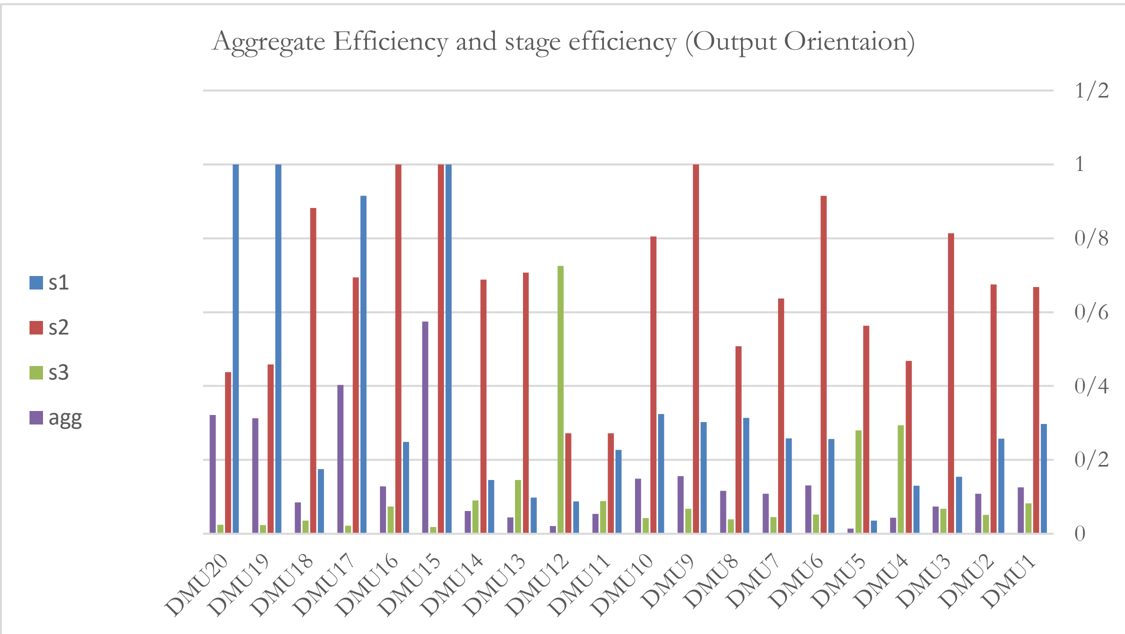

Fig. 10. Aggregate efficiency and stage efficiency.

5 | Conclusion

The studies in the literature of DEA for performance evaluation of petrochemicals sector, the existing competitive situation in petrochemicals sector and increasing negotiating power among customers in internal and international environment indicate importance of paying attention to this sector.

In this research, the efficiency and workability of petrochemical units were investigated. This study aimed to consider the network structure of the efficiency measurement by considering reversible equations. The important issue in the efficiency measurement and using data envelopment analysis is the correct selection of indicators and the contextual factors to create a meaningful model. To do this, in this paper the processes of the production system in olefin, tetramer, and dodecylbenzene were specified, and reviewing the literature and interviewing the expert the most suitable indicators were gathered for performance evaluation. During this research, there were many limitations in data collection, index definition, and conducting research. The limitations were the unavailability of information on petrochemical units to evaluate performance and ranking units. Also, inefficient and efficient units were not named due to data confidentiality and limitation in collecting them. Notably, the findings of this research are not validated permanently and are limited to the time of data collection.

The purpose of this study is to evaluate the performance of 20 companies involved in petrochemical production. The methodology used in this research can be used in all gas and oil refineries in addition to the gas transmission regions, etc. Also, the weakness and problems in production in triple processes of petrochemical were identified to find solutions, and units with better situations were supported and encouraged.

Future studies based on the outcomes of this research are suggested below:

The utilized methodology of this research can be implemented in all gas and oil refineries in addition to the gas transmission regions, etc.

We can also evaluate the supply chain of the petrochemical units. The supply chain evaluation should correspond to the strategy of these organizations. First, a strategy is written for petrochemical units. Then, they are designed according to the network structure and are finally examined. Modeling should be conducted by considering the hybrid orientation of the SBM model to evaluate the overall and stage efficiency of each component of the chain.

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Paper Type: Research Paper

Ranking of Companies in Generating Operating Cash Flows Based on Data Envelopment Analysis

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Abstract

The purpose of this research is to evaluate and rank the efficiency of pharmaceutical companies in creating operational cash flows in line with the objectives of financial reporting. The research method for collecting theoretical bases and research data is library studies. In this research, in order to evaluate the efficiency of pharmaceutical companies in creating operational cash flow, the Data Envelopment Analysis (DEA) model with weight limit is used. The results of this research show that Farabi pharmaceutical company has the highest efficiency score in creating Operating Cash Flows (OCFs) and Loqman pharmaceutical company has the lowest efficiency score. The findings of this research confirm that DEA is a suitable technique for evaluating the performance of companies in creating operational cash flow. Also, this technique, along with traditional financial analysis, can be considered a useful instrument for deciding and evaluating the performance and efficiency of companies. This article can make analysts more familiar; financial and accounting researchers with DEA applications in financial and accounting analysis. Also, this research can expand the use of scientific models in financial and accounting research.

Keywords: Company efficiency, Operating cash flow, Data envelopment analysis, Pharmaceutical industry.

1 | Introduction

One of the objectives of financial reporting is to provide accounting information to it's users about the amount and timing of future cash flows. Operating Cash Flows (OCFs) is a critical economic resource for each business entity. In this regard, making cash from operations to pay profits, repaying debts, paying expenses, and making investments are of paramount importance for companies to continue their activities [1]. Moreover, previous studies have documented the effect of the company's potential in creating OCF and profit on the company's value. The outcomes of these studies confirm that the direct method is valuable to investors when predicting future cash flows and earning [2]. OCF is assumed as a basis to calculate the credit rating of companies and provides increasing information content in forecasting companies' performance indices and credit risk [3]. OCF is a useful tool in assessing the quality of accruals and profit quality [4]. OCFs are considered to calculate Free Cash Flows (FCFs), which are considered to evaluate managers' performance and determine the value of the business entity from investors' perspective [5].

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Given the significance of OCFs in analyzing companies' performance and financial position, the present study aimed to employ the Data Envelopment Analysis (DEA) to develop a method to evaluate companies' performance according to the data obtained from the operational section of the cash flows statement. For this purpose, the direct OCF calculation method is adopted. In direct method, usually real cash inflows and outflows taken directly from company operations. This means it measures cash as its received or paid, rather than using the accrual accounting method. In indirect method, accountants adjust net income to calculate OCFs. This is adjusted as needed using information from the asset and liability accounts on the balance sheet to arrive at cash flows. According to some researchers, the direct OCF method provides more reliable data than the indirect method; hence, the former is of higher quality in financial reporting and is more powerful in forecasting the future OCF [6]. In a similar vein, some researchers have stated that the direct method decreases information asymmetry, enhances the liquidity of the shares, and reduces the transaction cost. Moreover, companies preparing the cash flow statement using the direct method have higher future earnings response coefficients than other companies [7].

Our motivation for choosing DEA is the efficiency of this method for financial analysis. We select DEA technique since it is a strong and flexible method in measuring financial performance. This is because it simultaneously considers several variables as input and output variables to evaluate the company's efficiency. Moreover, DEA is a decision-making tool and can provide further information compared to traditional analysis methods (e.g., financial ratios) [8]. This technique compares companies in the same industry in terms of efficiency. The comparison is acceptable since the traditional analysis compares each company' efficiency with the mean or the median of all companies in the research sample [9]. DEA can overcome complications aroused by the lack of a common measurement scale [1].

The difference between this research and the previous studies is that they evaluate the manager's efficiency based on accrual accounting indicators. Using DEA, most of the previous studies analyzed the manager's performance regarding the employed resources (i.e., assets as an input and accrued sales as an output) [9]. The present study, however, aimed to evaluate the manager's efficiency in creating OCF on a cash basis according to the cash flow statements. Consistent with the objectives of financial reporting, this assessment provides useful information about the capability to generate future OCF for accounting information users. We believe this issue can improve perception of financial analysts, investors and creditors about firms performance in line with the objectives of financial analysis. The findings of the present study can also expands the use of DEA in accounting research. Since accounting researchers are less familiar with DEA, the present study can make them more familiar with the application of DEA in the financial and accounting analysis. The following sections describe the theoretical foundations of the study, methodology, data analysis, and conclusions.

2 | Theoretical Foundations

2.1 | Operating Cash Flow

OCFs are the inflows and outflows resulting from the business entity's main income-making operations. OCFs indicate to what extent a business entity can generate cash from its main operation. Accordingly, the assessment of a company's capability to generate OCF provides financial analysts and other users with useful information for financial reporting. Moreover, information about past cash flow can be used to evaluate the amount, timing, and ambiguity of future cash flows [10]. OCFs provide useful information in line with the objectives of financial analysis for financial analysts [11]. OCF is a critical criterion in forecasting companies' financial health [12].

Compared to profit, OCF is less affected by management's discretion; hence, it is an efficient measure of the profit quality [13]. When companies are in a financial crisis, OCF is more strongly correlated with stock returns than profit, suggesting that investors in the capital market in a financial crisis give higher importance to information about OCF than profit [14]. This reveals the positive and significant effect of OCF on the profit sharing policy [15]. OCF can provide creditors with useful information about the company's capability in meeting its financial obligations [16]. Evaluating the future OCFs of a business entity is of paramount importance for financial institutions and banks when making loan decisions [17].

2.2 | DEA in Financial Performance Appraisal

As a non-parametric method, DEA separately appraises the economic entities' performance. In DEA, performance appraisal is performed based on input and output data, and there is no need to determine the type of relationships between those data in advance. Using linear programming, DEA assesses the relative efficiency of economic entities and allows comparing the performance of those entities using multiple input and output data. Technical efficiency in DEA refers to the ability to reach maximum output from a given set of inputs. Efficiency is the transformation of inputs into outputs and is determined for each decision-making entity using real data [9].

DEA is highly efficient in assessing companies' performance. The traditional financial analysis methods use financial ratios (e.g., Return on Sales (ROS), Return on Assets (ROA), and Asset Turnover (ATO)) separately for the performance appraisal of each economic entity's. If an economic entity possesses several inputs in the output creation process, DEA can simply determine each economic entity's efficiency using the optimal combination of the output and the input. This implies that DEA simultaneously combes several accounting variables as input and output data to calculate an optimal numerical criterion for companies' financial performance. DEA is a non-parametric frontier estimation methodology underpinned by the optimization principle as this technique can evaluate efficiency over time and need no presumption on the efficiency frontier. DEA allows managers to have a correct appraisal of their entities and make correct and rational decisions about the optimal allocation of resources [18].

2.3 | Research Background

DEA is an appropriate technique for converting different ratios and financial data into a single and comparable standard, called "efficiency". DEA is a suitable supplement for the traditional analysis of financial statements using financial ratios [19]. Using DEA to appraise companies' efficiency, researchers have indicated that DEA can strongly determine and classify efficient and inefficient enterprises. Research findings have also documented the higher efficiency of DEA compared to other traditional methods [20]. Another study used the DEA window analysis to evaluate the efficiency of pharmaceutical companies. The findings revealed that the DEA window analysis provided more useful information about performance appraisal compared to the traditional analysis technique [21], [22].

Studies on the time-based DEA of the financial statements of the companies accepted in the Tehran stock exchange suggest that only Iran auto parts company could maintain its full efficiency during six years of appraisal [23]. Furthermore, some researchers have introduced cash flow management as one of the useful criteria for companies' performance appraisal in the value chain. These researchers employed the reverse DEA and showed the effectiveness of measuring companies' efficiency using cash flows in the value chain [24]. In a survey on the performance appraisal of investment companies using DEA, the findings suggested that out of 34 investment companies, only four companies with an efficiency score of one were efficient, implying that 12% of investment companies were efficient and 88% were ineffective [25]. Another study examined the efficiency of audit firms using DEA. The researchers employed the cross-efficiency DEA and ranked Behmand, Azmoudeh Karan, and Arvin Arqam Pars accounting firms first to third in terms of efficiency, respectively. They proposed DEA as an appropriate method for analyzing auditing institutions' efficiency to evaluate auditors' work quality [26].

In China, results of a study show that procurement and application of funds in Chinese companies have an average growth rate of 2.75% and fund procurement gains more importance than fund application [27]. In another study, by using of DEA corporate social performance and financial performance is investigated. The results show that corporate social performance impacts negatively and significantly financial performance [28]. Financial ratios examine only one aspect of the company's performance, and therefore, the company's performance cannot be calculated correctly. Also, due to the multitude of financial ratios, sometimes the evaluation results will be misleading [29].

According to the literature, traditional methods using financial ratios to measure and evaluate efficiency have disadvantages such as one-dimensionality and suffer from limitations and inadequacies of financial ratios. In other words, the evaluation of efficiency using such methods may result in less accurate and objective findings and mislead users. Some researchers evaluated companies' efficiency and ranking using the DEA model underpinned by negative variables. They found out that quick ratios and the profit margin are main coefficients among the financial ratios [30]. The super efficiency model of DEA was also studied in primary and developed profitability models for 15 banks during a two-year period. The research results exhibited no strong correlation between efficiency values and financial ratios in the main profitability model; however, their relationship was strong in the developed profitability model [31].

In another study, the researchers also examined the effectiveness of the banking network using DEA with bootstrap data. They reported the 56.1% efficiency for the country's banking network, indicating no favorable efficiency [32]. DEA confirmed the higher annual efficiency of participation banks compared to other banks [33]. Some studies on companies' bankruptcy have also confirmed DEA as a useful technique and tool to forecast the likelihood of bankruptcy and predict companies' financial health. In the bankruptcy prediction model, companies' efficiency is one of the effective variables [34].

Furthermore, some studies used DEA and reported that the operational efficiency of multinational companies was higher than other companies. They suggested the positive and significant effect of intellectual capital on companies' performance [35]. Another study addressed banks' efficiency using the network DEA technique and reported their low efficiency. The researchers used this technique and examined some scenarios to promote banks' efficiency. Finally, they suggested improving banks' efficiency using DEA [36]. Research on financial distress reveals that the frontier DEA model is highly acceptable for detecting decision-making entities under different financial conditions over several periods as it can provide solutions to promote the performance of such entities with financial problems [37].

3 | Research Methodology

In this study, the statistical sample encompassed 19 companies in the pharmaceutical industry in Iran. The research was a case study in analyzing the data. The library method was also used to research the theoretical foundations of the study. Because the study was carried out in a real environment (i.e., stock exchange companies), it was a field study conducted in 2021. The pharmaceutical industry was selected because of the high significance of operating cash in this industry. In the pharmaceutical industry, the sales prices of pharmaceutical products are determined by the government, and the concerned companies have many demands from drug distribution companies and the Ministry of Health for selling their pharmaceutical products. Moreover, the raw materials of this industry are mostly imported; hence high foreign exchange funds are required. This implies that the importance of cash management in the pharmaceutical industry is as high as in other industries. *Table 1* indicates that the non-significant value of the average cash received from customers versus the average cash paid to the parties to the company's contract (namely suppliers and sellers of raw materials), employees for salaries wages, the government for taxes, and others.

In the present study, input variables were cash paid to employees for salaries and wages, to suppliers for the purchase of goods and services, to the government for taxes, and to others for other operating expenses. Moreover, the cash received from customers for the sale of goods and services was the output variable. Lingo software was used to analyze the data and solve the model.

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4 | Research Findings

4.1 | Descriptive Statistics

In *Table 1*, descriptive statistics of input and output variables (i.e., mean, median, standard deviation, minimum, and maximum) are presented. The table shows the average cash received from customers for the sale of goods and services (3983531 Million Rials), the average cash payment for salaries and wages (9646655 Million Rials), cash paid for tax (114597 Million Rials), cash paid for other operational expenses (279479 Million Rials), and cash paid for the purchase of goods and services (3622698 Million Rials). Also, this table shows that the most payments for salaries and the purchase of materials and goods were from sellers.

Table 1. Descriptive statistics of variables (values are in Million Rials).

	Output Variable	Input Variable			
	Cash Received from the Sale of Goods and Services	Cash Payment for the Purchase of Goods and Services	Cash Paid for other Operating Expenses	Cash Payment for Taxes	Cash Payment for Salaries and Wages
Average	3983531	3622698	279479	114597	6466559
Median	3939872	2382464	258793	83145	5019112
Std.	2335023	2409824	127647	87278	5038664
Min	693793	980510	103101	10773	1126882
Max	8887906	10310087	634722	293526	21323156

4.2 | Efficiency Appraisal Model

In this section, a DEA model is described to evaluate the efficiency of decision making entities. The model is in the field of efficiency appraisal with weight limit. Suppose n decision making entities with m inputs and s outputs. $DMU_j(j = 1,2, \dots, n)$ converts the input vector with components $x_{ij}(i = 1, \dots, m)$ into the output vector with components $y_{rj}(r = 1, \dots, s)$. The efficiency of the oth entity $(o = 1,2, \dots, n)$ is calculated using the multiple CCR model as follows:

$$
\begin{aligned}
& \min \sum_{i=1}^{m} u_r\, y_{r0,} \\
& \textstyle\sum_{i=1}^{m} v_i\, x_{i0} = 1, \\
& \sum_{i=1}^{m} u_r\, y_{rj} - \sum_{i=1}^{m} v_i\, x_{ij} \le 0, \quad j = 1, \dots, n, \\
& u_r \ge 0, \quad r = 1, \dots, s, \\
& v_i \ge 0, \quad i = 1, \dots, m,
\end{aligned} \tag{1}
$$

where the variables are

x_{ij}: Input rate i for jth entity i = 1, ..., m.

y_{rj}: Output rate r for the jth entity r = 1, ..., s.

v_i: The weight given to the ith input.

u_r: The weight given to the rth output.

The following definition is also proposed:

Definition 1. DMU_o is CCR efficiency when the optimal value of the objective function of *Model (1)* is 1; otherwise, it is inefficient.

The problem with *Model (1)* is the optimistic assessment of efficiency as several entities may be efficient as such ranking is not possible. Accordingly, cross-efficiency DEA models are used to solve such a problem. This section provides a DEA model with a higher discriminating power for ranking than traditional DEA models.

The multi-criteria DEA model was proposed to promote the discrimination power in classical DEA [38]. The first objective function (d_0) defines the efficiency of a decision-making entity classically. The other two objective functions (d_j and M) offer more limiting functions, respectively. In the proposed multi-criteria DEA model, three objective functions are analyzed separately with no priority for the functions. Since d_j and M tend to provide less efficient entities compared to the first objective function, these two functions in the multi-criteria DEA, compared to the classic DEA, offer higher discrimination power. The proposed DEA model of presented multi-criteria data is defined as follows [38]:

$$
\begin{aligned}
&\max \sum_{i=1}^{m} u_r y_{r0} \, (\text{or } \min d_o),\\
&\min M,\\
&\min \sum_{j=1}^{n} d_j,\\
&\textstyle\sum_{i=1}^{m} v_i x_{io} = 1,\\
&\sum_{i=1}^{m} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} + d_j = 0, \quad j = 1, \dots, n,\\
&M - d_j \geq 0, \quad j = 1, \dots, n,\\
&u_r \geq 0, \quad r = 1, \dots, s,\\
&v_i \geq 0, \quad i = 1, \dots, m,\\
&d_j \geq 0, \quad j = 1, \dots, n.
\end{aligned}
\tag{2}
$$

In this three-objective problem, the efficiency value of the first objective is from DMU_0, which should be maximized. The second objective minimizes the maximum inefficiency of DMU_j, and the last objective minimizes the total inefficiency of DMU_j. Ideal planning is used to solve this three-objective problem. The first target ideal is 1, and this value for the second and third goals is zero. Accordingly, the single-objective ideal planning model is as follows:

$$
\begin{aligned}
&\min(d_1^+ + d_2^- + d_3^-),\\
&\textstyle\sum_{i=1}^{m} u_r y_{r0} + d_1^- - d_1^+ = 1,\\
&M + d_2^- - d_2^+ = 0,\\
&\textstyle\sum_{j=1}^{n} d_j + d_3^- - d_3^+ = 0,\\
&\textstyle\sum_{i=1}^{m} v_i x_{io} = 1,\\
&\sum_{i=1}^{m} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} + d_j = 0, \quad j = 1, \dots, n,\\
&M - d_j \geq 0, \quad j = 1, \dots, n,\\
&u_r \geq 0, \quad r = 1, \dots, s,\\
&v_i \geq 0, \quad i = 1, \dots, m,\\
&d_j \geq 0, \quad j = 1, \dots, n,\\
&d_1^-, d_1^+, d_2^-, d_2^+, d_3^-, d_3^+ \geq 0.
\end{aligned}
\tag{3}
$$

Regarding the second and sixth constraints, *Model (3)* is as follows:

$$
\begin{aligned}
&\min(d_1^+ + d_2^- + d_3^-),\\
&\textstyle\sum_{i=1}^{m} u_r y_{r0} + d_1^- - d_1^+ = 1,\\
&\textstyle\sum_{j=1}^{n} d_j + d_3^- - d_3^+ = 0,\\
&\sum_{i=1}^{m} v_i x_{io} = 1,\\
&\sum_{i=1}^{m} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} + d_j = 0, \quad j = 1, \dots, n,\\
&d_2^+ - d_2^- - d_j \geq 0, \quad j = 1, \dots, n,\\
&u_r \geq 0, \quad r = 1, \dots, s,\\
&v_i \geq 0, \quad i = 1, \dots, m,\\
&d_j \geq 0, \quad j = 1, \dots, n,\\
&d_1^-, d_1^+, d_2^-, d_2^+, d_3^-, d_3^+ \geq 0.
\end{aligned} \tag{4}
$$

This model is called CCR-GP, in which d d_1^+ ،d_2^- و d_3^- are deviation from the first to the third ideal, respectively. When this model is implemented, the efficiency of DMU_0 can be calculated as follows:

$$
\theta = \frac{\sum_{i=1}^{m} u_r^* y_{rj}}{\sum_{i=1}^{m} v_i^* x_{ij}}, \tag{5}
$$

where v_i^*, u_r^* are the optimal weights obtained from *Model (4).*

One of the problems of DEA models is optimistic assessment, which evaluates DMUs in the best case. In such an assessment, several DMUs may be efficient, and no ranking is possible. One solution to this problem in DEA is weight constraints [39]. This section offers a weight limit method for the CCR-GP model. The presented model is as follows:

$$
\begin{aligned}
&\max \alpha - (d_1^+ + d_2^- + d_3^-),\\
&\sum_{i=1}^{m} u_r y_{r0} + d_1^- - d_1^+ = 1,\\
&\sum_{j=1}^{n} d_j + d_3^- - d_3^+ = 0,\\
&\textstyle\sum_{i=1}^{m} v_i x_{io} = 1,\\
&\sum_{i=1}^{m} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} + d_j = 0, \quad j = 1, \dots, n,\\
&d_2^+ - d_2^- - d_j \geq 0, \quad j = 1, \dots, n,\\
&u_r \geq \alpha, \quad r = 1, \dots, s,\\
&v_i \geq \alpha, \quad i = 1, \dots, m,\\
&d_j \geq 0, \quad j = 1, \dots, n,\\
&d_1^-, d_1^+, d_2^-, d_2^+, d_3^-, d_3^+ \geq 0.
\end{aligned} \tag{6}
$$

This linear programming model is to evaluate efficiency with weight constraints; hence, the first priority is applied to the weight constraint since the goal is to first apply the weight limit and then appraise the performance in the best conditions with the weight limit.

4.3 | Data Analysis

In this section, the companies' efficiency is evaluated. *Table 2* presents the input and output information of the concerned companies. In this table, cash received from the sale of goods and services is input variable, and cash payment for the purchase of goods and services, cash paid for other operating expenses, cash payment for taxes and cash payment for wages are output variables.

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Table 2. Inputs and outputs (in Millions of Rials).

Output Variable	Input Variable				
Cash Received from the Sale of Goods and Services	Cash Payment for the Purchase of Goods and Services	Cash Paid for Other Operating Expenses	Cash Payment for Taxes	Cash Payment for Wages	Company Name
6831114	6317377	249904	83145	1281120	Alborz Daru
3939872	4544421	266446	40717	9479622	Pars Daru
4245050	5227064	327999	77175	11007530	Aburaihan Daru
2084875	2382464	407658	142738	5480295	Osve Daru
8887906	7178272	472421	259084	15113316	Exir Daru
3224458	2080694	252233	43338	6180008	Jaber Ebne Hayyan Daru
5379799	5577668	334660	293526	1577087	Razak Daru
4070940	2851693	155559	114363	6404481	Daru Zahravi
5001814	4516238	103101	167876	1126882	Farabi Daru
693793	2359568	294809	24992	5045655	Loghman Daru
1740067	2116651	233203	31018	4662198	Cosar Daru
1473135	1666540	258793	10773	4051172	Rouz Daru
1760439	2238440	156703	54136	4624652	Zagros Pharmed Pars Daru
4130296	1869858	322332	166890	4982175	Sina Daru
2776768	1857975	163934	64265	4019981	Shimi Darupakhsh
2115939	980510	176730	169992	2165131	Fraravarde Tazrighi
7626365	10310087	634722	276439	21323156	Darupakhsh Karkhanejat
7290461	3237613	334172	55293	9321045	Darupakhsh Mavvad
2414003	1518140	164724	101591	5019112	Kimi Daru

Model (6) is used to evaluate efficiency. Lingo software is used to solve the model, the results of which are presented in *Table 3*.

Table 3. Efficiency scores in Model (6).

Efficiency Scores	Company	Number
0.987	Alborz Daru	1
0.724	Pars Daru	2
0.424	Aburaihan Daru	3
0.426	Osve Daru	4
0.656	Exir Daru	5
0.688	Jaber Ebne Hayyan Daru	6
0.806	Razak Daru	7
0.707	Daru Zahravi	8
1	Farabi Daru	9
0.209	Loghman Daru	10
0.445	Cosar Daru	11
0.246	Rouz Daru	12
0.248	Zagros Pharmed Pars Daru	13
0.562	Sina Daru	14
0.454	Shimi Darupakhsh	15
0.605	Fraravarde Tazrighi	16
0.234	Darupakhsh Karkhanejat	17
0.563	Darupakhsh Mavvad	18
0.354	Kimi Daru	19

According to *Table 3*, Farabi pharmaceutical company is efficient, and others companies are inefficient. Companies can be ranked by efficiency scores. Regarding the scores, Farabi pharmaceutical company, with the highest efficiency score, is ranked first, and Loqman pharmaceutical company, with the smallest efficiency score, is ranked last. Alborz Daru and Razak Daru are ranked second and third respectively. In other words, the findings of this study show that the mentioned companies have performed better in creating operational cash flows compared to other companies. This method allows unique ranking, and that is what traditional DEA methods fail to do.

5 | Discussion and Conclusion

In the present study, pharmaceutical companies were ranked in terms of efficiency in generating OCF using DEA. According to the findings, Farabi and Loqman pharmaceutical companies have the highest and the lowest efficiency scores in the first and the last ranking in terms of generating OCF, respectively. The findings implly that DEA is a suitable technique for performance appraisal. Besides traditional financial analysis, DEA can be regarded a useful tool for decision-making and evaluating companies' performance and efficiency. Since DEA compares each company's efficiency with its industry counterparts and can propose a model to appraise efficiency simply, financial analysts and managers can correctly assess their decision-making entities and make correct and rational decisions for the optimal allocation of resources.

The present findings also confirm that DEA is a suitable supplement for assessing the financial performance of business entities in line with the goals of financial reporting; hence, this technique can remove the limitations and inadequacies of financial ratios. Because DEA can provide more reliable and accurate findings compared to traditional financial ratios, it can provide financial information users with appropriate and reliable information. The findings of this paper show that DEA is a strong mathematical model for financial analysis. This method can be used alongside traditional analysis to evaluate financial performance. It seems that this method can improve traditional financial analysis. In line with financial reporting objectives, we argue that DEA provides new insights into financial analysis. Also, DEA can be considered as a move towards matching traditional ratio analysis with decision usefulness theory. This paper emphasizes that DEA can be a useful tool for cash flows management. Therefore, the results of this research can be consistent with the theoretical foundations and previous studies (36, 37). The findings of this study emphasize that creating and managing cash flows is very important for financial performance. Cash flows, sales growth and profitability are the resources of companies for continued activity and survival. So, this paper expands the awareness of financial and accounting analysts and researchers about the applications of DEA in financial and accounting analyses. In other words, the research findings can expand scientific models on financial and accounting research. Capital market analysts can use the efficiency scores obtained from the DEA to evaluate the market performance and the financial performance of companies at the same time. Also, these scores can be a criterion for evaluating the financial health of companies.

Financial analysts and other financial information users are recommended to calculate companies' efficiency in making money and OCFs and include them in their decision-making models to assess companies' performance. Future researchers are suggested to examine the effectiveness of companies in creating profit and OCF comparatively using DEA.

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Development a Forward-Reverse Network Optimization Model with Delay Reduction and Multistage Fuzzy Demand Satisfaction Policy

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Abstract

Supply chain management is a process in which a number of organizations work together as a supply chain until the raw materials reach the manufacturer and finally, a valuable product is provided to the end consumer. With the increase in population and the increase in environmental sensitivities, the forward-reverse supply chain has attracted a lot of attention, which pursues goals such as optimization, customer satisfaction, responding to their needs in the shortest time with the lowest cost and high quality. In this paper, a forward- reverse multi-product and multi-period network is designed under the condition of uncertainty in the demand parameter. The purpose of the proposed model is to maximize profit by considering customer satisfaction simultaneously and reducing delay and the fuzzy approach has been used to solve the model under conditions of uncertainty. The proposed model is mixed-integer linear programming and for its validation and applicability, it has been solved by GAMS software, a numerical example using simulated data in deterministic and uncertain state. The results of the analysis of the numerical example show that the show that with increasing uncertainty in the demand parameter, the optimal value of the objective function decreases.

Keywords: Forward-reverse supply chain, Demand satisfaction, Delay reduction, Fuzzy.

1 | Introduction

A supply chain consists a network of organizations and facilities that work in tandem to transform raw materials into finished products for customers and fulfills the demands of customers and satisfies the customer [1], [2]. Supply chain management serves manufacturing industries to create a low cost program that guarantees the productivity and success of companies [3]. The forward supply chain starts from the supplier and during that, the manufactured product reaches the final customer. The reverse supply chain starts from the final customer and during that the returned product from the customer is either sent to the market through the process of reproduction or removed from the chain.

By remanufacturing and recycling products that have expired, the structure of supply chain networks has changed and closed-loop supply chains are gaining popularity [4], [5]. Product recovery includes reproduction, reuse and recycling. Remanufacturing is the process of disassembly of the collected

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product and the manufacturer can choose to repair or remanufacture. Reuse is a process that repairs defective parts or removes healthy parts from a product that has failed and uses it in a product that has defective parts [6]. Simultaneous attention of scheduling the production and distribution and assigning delivery times could lead to reduced costs and as a result increase in profit [7]. Due to their complex nature, supply chains face a high degree of uncertainty that can affect the quality of their performance such as uncertainty in purchasing, processing, market and other stages of the closed loop supply chain which has greatly increased the complexity of remanufacturing and reduced the efficiency of the process, hindering the sustainable development of industries and the the economy becomes cyclical [8], [9]. In recent years, a large number of companies, in addition to economic goals, have followed the goals of meeting the needs of customers in the design of the supply chain network [10]. Customer satisfaction is a key factor in the formation of customers' future purchase intention. Every customer has expectations, if the services provided are less than their expectations, it leads to their dissatisfaction [11], [12]. Generally, there are two different demand satisfaction policies: 1) satisfying the entire demand for retailers [13]-[15], and 2) satisfying a proportion of demand and dismissing the rest as lost sales or the fulfilling the remaining demand is delayed [16], [17].

In forward-reverse supply chain design, most of the presented models consider minimizing purchase costs, inventory costs, and ordering costs, and a dyadic network structure that mainly includes manufacturers, distributors or retailers. Rarely do we find studies that have been conducted in the analysis of CLSC with network structure, in which multiple factors are coordinated and consider maximum profit and demand satisfaction at the same time. Based on mentioned points, purpose of this paper to consider the development of a closed loop supply chain model in which the demand parameter is considered non-deterministic. The strength of the presented model is considering the goal of profit maximization, which is considered in this goal of satisfying demand and reducing delay. In this research, our customers are retailers, which are considered to be maximum three modes to satisfying their demand, depending on the type of product. For example, retailer 1 in two stages, retailer 2 in three stages and retailer 3 in one stage, its demand is fulfilled. *Table 1* shows steps to meet demand. The policy of satisfying the demand is to meet the demand of retailers and includes lost sales and delays.

Table 1. The Satisfaction demand modes for the example.

Retailer	Modes to Meet Demand		
1	140	120	80
2	120	100	
3	150		
4	60	50	90

This paper is organized as follows: literature review and mathematical modeling is presented in Sections 2 and 3. In Section 4, we summarize the basic concepts of fuzzy logic and introduce its computational procedure. In Section 5, in order to demonstrate the applicability and validation of the proposed model, a numerical example is solved and the computational process details are expressed. Finally, in Section 6 conclusion and suggests future is expressed.

2 | Literature Review

Most of the studies conducted in the field of logistics network design include different facility location models using mixed-integer linear programming. The common goal of these models is cost minimization and profit maximization. Below are some of the things done in this field.

2.1 | Supply Chain Network Design

Ramezani et al. [18] presented a multi-objective probabilistic model for the problem of integrated logistics network design, under conditions of uncertainty, in which the objectives of the model are considered to be profit maximization, customer responsiveness, and quality. Bushuev et al. [19] in

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research entitled improving supply chain delivery performance for several delivery time distributions, investigated the changes of delivery time distribution parameters on the expected cost of untimely delivery. They presented strategies to improve delivery performance by using mean and variance of delivery time distribution for distribution. Hong et al. [20] developed the problem of designing a supply chain network including production, distribution and customer centers. The proposed model minimizes the total fixed costs of center establishment and transportation costs with the aim of customer satisfaction. In order to solve the proposed model, they used the optimization algorithm of ants. Shojaie et al. [21] presented a novel optimization model based two meta heuristics of PSO and SA then the facility lauout problems is solved by using the modified algorithm manufacturer in other hand, Shojaie et al. [22] presented a research wprk on solving a bi-objective flexible flow shop problem with transporter preventive maintenance planning by NSGA II and MOPSO. Zhu et al. [23] presented a three-stage supply chain with asymmetric information under uncertainty with several suppliers and one. Zaid et al. [24] investigated the role of Supply Chain Integration (SCI) including suppliers, customers and internal integration in creating customer loyalty. The research conducted showed that SCI has a significant direct impact on operational performance and customer satisfaction. The results of the analysis also provided information that operational performance and customer satisfaction can mediate the effect of SCI. Salmannejad et al. [25] proposed a mixed integer linear programming model for medical management and information flow. The objectives of the proposed model are to minimize the cost of purchase, maintenance, manpower and drug expiration and to minimize drug shortages. The results of solving the model showed that the decisions related to the purchase and procurement of drugs have a great impact on the lack of drugs and the control of various costs related to drug inventory in this hospital.

2.2 | Supply Chain Network Design

Zhao et al. [26] considered a two-step, fuzzy closed-loop supply chain. In this model, price-dependent demand and coordination in a CLSC under balanced and unbalanced information have been investigated. Kim et al. [27] proposed a deterministic mixed integer optimization model to deal with the uncertainty of recycled products and customer demand in a closed-loop supply chain. The proposed model considers the uncertainty in the budget to reflect the uncertainty in the mathematical model. Pant et al. [28] designed a closed loop supply chain network to minimize the total cost including four parts (suppliers, manufacturer, distribution center and customer) in the direct direction and six parts (collection center, repair center, reconditioning center, separation center, destruction center and secondary market) in the opposite direction. A mixed linear programming model is developed to solve the proposed network. Mahmoodirad et al. [29] considered multi-state demand satisfaction in a closed-loop supply chain and then solved it in a fuzzy environment using trapezoidal fuzzy parameters. Sonu Rajak et al. [30] investigated a multi-objective mixed integer linear programming approach to minimize environmental impacts and maximize product net profit to transform a traditional linear supply chain into a closed-loop supply chain to achieve sustainability and thereby satisfy sustainable development goals. Safaei et al. [31] investigated a multi-layered mixed integer linear programming model considering factories' vehicles and transport companies' leased vehicles to minimize the costs of a multi-period, multi-layer closed-loop supply chain network. In the proposed model, the demand for products is estimated using the time series model of automatic regressive integrated moving average to reduce the shortage that may occur in the entire supply chain network. Numerical results showed that the proposed model is closer to the real situation and can achieve a reasonable solution in terms of service level, shortage, etc. Zhang et al. [32] investigated in research aimed at recycling defective recycled products and waste recycled products in order to calculate the total profit in a two-channel closed loop supply chain and compare it with inconsistent models. The results of the research showed that the proposed coordinating mechanism in price discounts to retailers leads to the improvement of the financial performance of the supply chain.

3 | Defining the Research Problem

The network proposed in this research is an integrated forward-reverse network with non-deterministic demand that has four levels in the forward direction (suppliers, plants, distributors and retailers) and three

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levels in the reverse direction (collection centers, recycling and destruction). In the forward flow, first the production centers receive the raw materials they need from the suppliers. The final products made in the production centers are sent to the distribution centers, then the products are sent to the retail centers according to the demand of the retailers. In the reverse direction, retailers send a percentage of defective and unusable products to collection centers, then the products are transferred from collection centers to recycling centers. In the recycling centers, the parts of the products are separated and those parts that can be used are sent to production centers for re-production, and those parts that are unusable are transferred to destruction centers for disposal.

3.1 | Assumptions

The following assumptions are made in the network proposed configuration:

I. The model is designed for multi-product, multi-level and multi-period model. In which the demand parameter is considered under uncertainty conditions.
II. Demand satisfaction policies include meeting the demand of retailers and they can supply all these products from a distribution center or any of them from separate distribution centers.
III. Binary variable is considered for retailer demand, so that if a retailer's demand is fulfilled, the value is one, otherwise it is zero.

3.2 | Mathematical Model

The network proposed can be formulated as a mixed-integer linear programming model. Indices, decision variables and model parameters are presented as follows:

Sets

g	Index of mode satisfaction g = 1, …, G.
j	Set of products j = 1, …, J.
i	Set of parts i = 1, …, I.
o	Set of plant o = 1, ..., O.
m	Set of collection center m = 1, 2, …, M.
n	Set of distributor n = 1, ..., N.
l	Set of recycle center l =1, ..., L.
k	Set of suppliers k = 1, …, K.
h	Set of disposal center h = 1, ..., H
t	Set of time periods t =1, …., T.
r	Set of retailer r =1, …, R.

Parameters

$\tilde{D}_{jrgt}$	Demand for the product j the retail center r for satisfaction mode g in period t.
D_{pjrt}	Sale price of each unit of product j at retailer center r in period t.
Q_{ij}	Units of part i in product j.
MC_{jm}	Operational cost of each unit of product j at collection center m.
MP_{jo}	Operational cost of each unit of product j at plant center o.
MD_{jn}	Operational cost of each unit of product j at distribution center n
RC_{il}	Operational cost for recycled of parts i at recycling center l
WDC_{ih}	Operational cost for destruction of parts i at disposal center h.

PR_{it}	Profit by recycling of parts i in period t.
PC_{ik}	Cost of purchasing parts i from suppliers k.
SC_{jm}	Set-up cost of collection center m for product j.
SR_{il}	Set-up cost of recycling center l for parts i.
SU_{jn}	Set-up cost of distribution center n for product j.
SX_{ih}	Set-up cost of disposal center h for parts i.
SO_{jo}	Set-up cost of plant center o for product j.
ST_{jr}	Set-up cost of retailer center r for product j.
TCO_{jon}	Transportation cost for each unit of product j from plant o to distribution center n.
TCD_{jml}	Transportation cost for each unit of product j from collection center m to recycling center l.
TCR_{ilo}	Transportation cost for each unit of parts i from recycling center l to plant center o.
TCK_{ilh}	Transportation cost for each unit of parts i from recycle center l to disposal center h.
TCM_{jnr}	Transportation cost for each unit of product j from distribution center n to retailer center r.
TCU_{iko}	Transportation cost for raw material i from supplier k to plant center o.
TCN_{jmr}	Transportation cost for each unit of product j from retailer center r to collection center m.
λ_i	Maximum percentage of part i recycled.
CU_{jmt}	Capacity of collection center m for product j in period t.
CR_{ilt}	Capacity of recycle center l for parts i in period t.
CD_{jnt}	Capacity of distribution center n for product j in period t.
CG_{iht}	Capacity of disposal center h for parts i in period t.
CP_{jot}	Capacity of plant center o for producing product j in period t.
CM_{jrgt}	Capacity of retailer center r for product j for each satisfaction mode g in period t.
$M1_{ij}$	Conversion rate of the product j to the raw materials or parts i.
TCR_{ilo}	Cost of transporting parts i from recycling center l to plant center o.
TCK_{ilh}	Cost of transporting parts i from the recycling center l to the disposal center h.
TLSjrt	Cost of lost sales per unit of product j retailer center r in period t.
TXIikot	Delivery time of parts i from supplier k to plant center o in period t.
TXPjont	Delivery time of product j from plant center o to distribution n in period t.
TXN_{jnrt}	Delivery time of product j from distributor n to retailer r in period t.
MXIikt	Return parts i to supply center k in period t.
MXPjot	Return product j to production center o in period t.
MXNjnt	Return product j to distribution center n in period t.
$TBmax_{jont}$	Maximum time allowed to receive product j from plant center o by distributor n in period t.
$TOmax_{jnrt}$	Maximum time allowed to receive product j from distributor n by retailer center r in period t.
$TAmax_{ikot}$	Maximum time allowed to receive parts i from supplier k by plant center o in period t.

Decision variables

LS_{jrt}	Amount of lost sales of product j at retail center r in period t.
P_{jot}	Amount of product j to be produced at plant o in period t.
C_{jmt}	Amount of product j at collection center m in period t.
S_{ikt}	Amount of purchased parts i from the supplier k in period t.
B_{jnt}	Amount of product j at distribution center n in period t.

Y_{ilt}	Amount of parts i at recycling center l in period t.
DR_{jr}	Amount of product j retailer center r in period t.
W_{ith}	Amount of parts i destroyed at the destruction center h in period t.
TU_{jnrt}	Amount of product j transported from the distribution center n to the retailer center r in period t.
TC_{jmrt}	Amount of product j transported from the retailer center r to the collection center m in period t.
X_{ikot}	Amount of parts i transported from the supplier k to plant o in period t.
TR_{jont}	Amount of product j transported from the plant o to distribution center n in period t.
TL_{jmlt}	Amount of product j transported from the collection center m to recycling center l in period t.
F_{ilot}	Amount of parts i transported from the recycling center l to plant o in period t.
E_{iht}	Amount of parts i transported from the recycling center l to disposal center h in period t.
A_{jmt}	Binary variable equal to 1 if collection centers m is established for product j in period t, otherwise 0.
V_{ilt}	Binary variable equal to 1 if recycle center l is established for parts i in period t, otherwise 0.
U_{jnt}	Binary variable equal to 1 if distribution center n is established for product j in period t, otherwise 0.
RH_{iht}	Binary variable equal to 1 if disposal center h is established for parts i in period t, otherwise to 0.
GH_{jot}	Binary variable equal to 1 if plant o is established for product j in period t, otherwise to 0.
BH_{grjt}	Binary variable equal to 1 if demand of retailer r is satisfied in mode g in period t, otherwise to 0.
CH_{jrt}	Binary variable equal to 1 if retailer r established for product j in period t, otherwise 0.
TA_{ikot}	Time of receipt of parts i from supplier k by plant center o in period t.
TB_{jont}	Time of receipt of product j from plant center o by distribution center n in period t.
TO_{jnrt}	Time of receipt of product j from distribution center n by retailer center r in period t.

$$
\begin{aligned}
&\text{Max imaize} \\
&\sum_{t}\sum_{g}\sum_{r}\sum_{j}\left(DP_{jrt} * \tilde{D}_{jrgt} * BH_{jrgt}\right) + \sum_{t}\sum_{o}\sum_{l}\sum_{i}(RP)_{iot} * F_{ilot} - \sum_{t}\sum_{k}\sum_{i}(PC)_{ik} * S_{ikt} \\
&-\sum_{t}\sum_{o}\sum_{j}(MP)_{jo} * P_{jot} - \sum_{t}\sum_{n}\sum_{j}(MD)_{jn} * B_{jnt} - \sum_{t}\sum_{m}\sum_{j}(MC)_{jm} * C_{jmt} - \sum_{i}\sum_{l}\sum_{t}(RC)_{il} * Y_{ilt} \\
&-\sum_{t}\sum_{h}\sum_{i}(WDC)_{ih} * W_{iht} - \sum_{t}\sum_{o}\sum_{j}(SO)_{jo} * GH_{jot} - \sum_{t}\sum_{n}\sum_{j}(SU)_{jn} * U_{jnt} - \sum_{t}\sum_{r}\sum_{j}(ST)_{jr} * CH_{jrt} \\
&-\sum_{t}\sum_{m}\sum_{j}(SC)_{jm} * A_{jmt} - \sum_{t}\sum_{l}\sum_{i}(SR)_{il} * V_{ilt} - \sum_{t}\sum_{h}\sum_{i}(SX)_{ih} * RH_{iht} - \sum_{t}\sum_{o}\sum_{k}\sum_{i}(TCU)_{iko} * X_{ikot} \qquad (1) \\
&-\sum_{t}\sum_{n}\sum_{o}\sum_{j}(TCO)_{jon} * TR_{jont} - \sum_{j}\sum_{n}\sum_{r}\sum_{t}(TCM)_{jnrt} * TU_{jnrt} - \sum_{j}\sum_{m}\sum_{r}\sum_{t}(TCN)_{jmrt} * TC_{jmrt} \\
&-\sum_{t}\sum_{j}\sum_{m}\sum_{l}(TCD)_{jml} * TL_{jmlt} - \sum_{t}\sum_{o}\sum_{l}\sum_{i}(TCR)_{ilo} * F_{ilot} - \sum_{t}\sum_{h}\sum_{l}\sum_{i}(TCK)_{ilh} * E_{ilht} \\
&-\sum_{t}\sum_{r}\sum_{j}(TLS)_{jrt} * LS_{jrt}.
\end{aligned}
$$

The objective *Function (1)* of the proposed model includes maximizing profit and customer satisfaction. In the first row computed by multiplying amount of demand and the selling price per unit of product in binary variable for rretailer satisfaction, amount of recycled parts and profit from their recycling, amount of purchased parts and the cost of their purchase. In the second and third rows operating costs (production, distribution, collection, recycling and disposal centers) are included in each of the centers.

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The fourth row is the fixed costs of setting up the facility, and the fifth and sixth rows include transportation costs between the facilities. Finally, the number of lost sales has been stated.

$$S_{ikt} = \sum_{o} X_{ikot} \quad \text{for all } i,k,t. \tag{2}$$

$$C_{jmt} = \sum_{r} TC_{jmrt} \quad \text{for all } j,m,t. \tag{3}$$

$$\sum_{o} TR_{jont} = B_{jnt} \quad \text{for all } j,n,t. \tag{4}$$

$$\sum_{m} TC_{jmrt} = \sum_{l} TL_{jmlt} \quad \text{for all } j,m,t. \tag{5}$$

$$\sum_{o} TR_{jont} = \sum_{l} TU_{jnrt} \quad \text{for all } j,n,t. \tag{6}$$

$$\sum_{o} TL_{jmlt} = \sum_{o} F_{ilot} + \sum_{h} E_{ilht} \quad \text{for all } j,l,i,t. \tag{7}$$

$$\sum_{l} E_{ilht} = W_{iht} \quad \text{for all } i,h,t. \tag{8}$$

$$\sum_{g} BH_{jrgt} = 1 \quad \text{for all } j,r,t. \tag{9}$$

Constraint (2) ensures that the number of parts purchased from the supplier is equal to the parts shipped from the supplier to the production center. *Constraints (3)* to *(8)* assure the balance flow at production, distribution, retailing, collection, recycling and disposal centers. *Constraint (9)* ensures that for each retailer, demand satisfaction is selected.

$$\sum_{m} TL_{jmlt} * M1_{ij} = Y_{ilt} \quad \text{for all } i,j,l,t. \tag{10}$$

$$\sum_{n} TU_{jnrt} + LS_{jrt} = \sum_{g} \tilde{D}_{jrgt} * BH_{jrgt} \quad \text{for all } j,r,t. \tag{11}$$

Constraint (10) shows that the number of parts in the recycling center is equal to the product of the amount of the product transported from the collection center to recycling in the conversion rate of the product to parts. *Constraint (11)* represents the amount of product shipped from the distribution center to retail and the number of lost sales is equal to the amount of retail demand for each of the states of satisfaction multiplied by zero variable and one retailer demand.

$$P_{jot} \le (CP)_{jot} * GH_{jot} \quad \text{for all } j,o,t. \tag{12}$$

$$B_{jnt} \le (CD)_{jnt} * U_{jnt} \quad \text{for all } j,n,t. \tag{13}$$

$$DR_{jrgt} \le CM_{jrt} * CH_{jrt} \quad \text{for all } j,r,g,t. \tag{14}$$

$$C_{jmt} \le CU_{jmt} * A_{jmt} \quad \text{for all } j,m,t. \tag{15}$$

$$Y_{ilt} \le (CR)_{ilt} * V_{ilt} \quad \text{for all } i,l,t. \tag{16}$$

$$W_{iht} \le (CG)_{iht} * RH_{iht} \quad \text{for all } i,h,t. \tag{17}$$

$$Q_{ij} * P_{jot} \le \sum_{k} X_{ikot} + \sum_{l} F_{ilot} \quad \text{for all } i,j,o,t. \tag{18}$$

Constraints (12) to *(15)* indicate the amount of product in each center in relation to its capacity. *Constraints (16)* and *(17)* show the number of parts in the recycling and destruction centers in relation to its capacity and *Constraint (18)* shows the amount of parts in the manufactured product in relation to the amount of parts shipped from the supplier and recycling centers.

$$\sum_{n} TR_{jont} \le P_{jot} \quad \text{for all } j,o,t. \tag{19}$$

$$\sum_{m} TD_{jmrt} \le DR_{jrt} \quad \text{for all } j,r,t. \tag{20}$$

$$C_{jmt} \geq \sum_{l} TL_{jmlt} \qquad \text{for all } j,m,t. \tag{21}$$

Constraint (19) shows the relationship between the amount of product produced in the production center and the amount of product transported from the production center to distribution. *Constraints (20)* and *(21)* indicate the amount of product transported with the amount of product in each of the retail, collection and recycling centers.

$$\sum_{h} E_{ilht} \leq (1-\lambda_{i} * Y_{ilt}) \qquad \text{for all } i,l,t. \tag{22}$$

$$\sum_{n} TU_{jnrt} \geq \sum_{l} TC_{jmrt} \qquad \text{for all } j,r,t. \tag{23}$$

Constraint (22) shows the relationship between the number of parts transported from the recycling center to destruction with the product of the inventory level of parts in the recycling center with the maximum parts that are recycled. *Constraint (23)* shows the relationship between the amount of product transported from the distribution center to the retail center and the product transported from the retail center to the collection center.

$$TXI_{ikot} * (X_{ikot} / MXI_{ikt}) \leq TA\max_{ikot} \qquad \text{for all } i,k,o,t. \tag{24}$$

$$TXP_{jont} * (TR_{jont} / MXP_{jot}) \leq TB\max_{jont} \qquad \text{for all } j,o,n,t. \tag{25}$$

$$TXN_{jnrt} * (TU_{jnrt} / MXN_{jnt}) \leq TO\max_{jnrt} \qquad \text{for all } j,n,r,t. \tag{26}$$

Constraint (24) shows the relationship between the delivery time of the raw material and the maximum time allowed to receive the raw material from the suppliers by the production centers, and *Constraint (25)* shows the relationship between the product delivery time and the maximum time allowed to receive the product from the production centers to the distribution centers. *Constraint (26)* shows the relationship between the product delivery time and the maximum time allowed to receive the product from distribution centers to retail centers.

$$P_{jot}, C_{jmt}, S_{ikt}, B_{jnt}, Y_{ilt}, DR_{jrt}, W_{iht}, TU_{jnrt}, TC_{jmrt}, X_{ikot}, TR_{jont}, TL_{jmlt}, F_{ilot}, E_{ilht} \geq 0 \\ \text{for all } j,o,t,m,i,k,l,h,r,n,g. \tag{27}$$

$$A_{jmt}, V_{ilt}, U_{jnt}, RH_{iht}, GH_{jot}, CH_{jrt}, BH_{jrgt} \in \{0,1\} \qquad \text{for all } j,o,t,m,i,l,h,r,n,g. \tag{28}$$

Constraint (27) shows the non-negativity constraint on the decision variables and *Constraint (28)* shows the binary variables related to facility construction and demand fulfillment.

4 | Solution Methodology

In this paper the methodology proposed by Jiménez et al. [33] is used to find out the optimal solution. This method is based on the definition of expected value and expected interval. The steps in solving the proposed model are shown below:

I. Convert the fuzzy objective function to crisp using the expected value of the corresponding parameter.
II. Determine the triangular fuzzy number for the demand parameter.
III. Determine the minimum acceptable degree decision vector (Δ) and converting fuzzy constraints to crisp.
IV. Getting a linear membership function for the demand parameter in the constraints.

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$$
\begin{aligned}
&\text{Max imaize} \\
&\sum_t\sum_g\sum_r\sum_j \left(DP_{jrt} * \left(\frac{D_{1jrgt} + 2\times D_{jrgt} + D_{2jrgt}}{4}\right) * BH_{jrgt}\right) + \sum_t\sum_o\sum_l\sum_i (RP)_{it} * F_{ilot} \\
&-\sum_t\sum_k\sum_i (PC)_{ik} * S_{ikt} - \sum_t\sum_o\sum_j (MP)_{jo} * P_{jot} - \sum_t\sum_n\sum_j (MD)_{jn} * B_{jnt} \\
&-\sum_t\sum_m\sum_j (MC)_{jm} * C_{jmt} - \sum_i\sum_l\sum_t (RC)_{il} * Y_{ilt} - \sum_t\sum_h\sum_i (WDC)_{ih} * W_{iht} \\
&-\sum_t\sum_o\sum_j (SO)_{jo} * GH_{jot} - \sum_t\sum_n\sum_j (SU)_{jn} * U_{jnt} - \sum_t\sum_r\sum_j (ST)_{jr} * CH_{jrt} \\
&-\sum_t\sum_m\sum_j (SC)_{jm} * A_{jmt} - \sum_t\sum_l\sum_i (SR)_{il} * V_{ilt} - \sum_t\sum_h\sum_i (SX)_{ih} * RH_{iht} \\
&-\sum_t\sum_o\sum_k\sum_i (TCU)_{iko} * X_{ikot} - \sum_t\sum_n\sum_o\sum_j (TCO)_{jon} * TR_{jont} \\
&-\sum_j\sum_n\sum_r\sum_t (TCM)_{jnrt} * TU_{jnrt} - \sum_j\sum_m\sum_r\sum_t (TCN)_{jmrt} * TC_{jmrt} \\
&-\sum_t\sum_j\sum_m\sum_l (TCD)_{jml} * TL_{jmlt} - \sum_t\sum_o\sum_l\sum_i (TCR)_{ilo} * F_{ilot} \\
&-\sum_t\sum_h\sum_l\sum_i (TCK)_{ilh} * E_{ilht}.
\end{aligned} \tag{29}
$$

Assuming d=(d1, d2, d3) a triangular fuzzy number, d1 represents the smallest and d2 is the largest number of sets is the desired number that can be changed between them and its membership function will be *Eq. (30)*.

$$
\mu_{\tilde{d}}(x) = \begin{cases} 0, & x \le d_1, \\ \dfrac{(x-d_1)}{(d_2-d_1)}, & d_1 \le x \le d_2, \\ \dfrac{(d_3-x)}{(d_3-d_2)}, & d_2 \le x \le d_3, \\ 0, & x > d_3. \end{cases} \tag{30}
$$

The triangular membership function is as shown in *Fig. 1*.

Given the presented membership function, the expected interval and its expected value are according to *Eqs. (31)* and *(32)* stated [34].

$$
El(\tilde{d}) = \left(E_1^d + E_2^d\right) = \left(\frac{d_1+d_2}{2}, \frac{d_2+d_3}{2}\right). \tag{31}
$$

$$
Ev(\tilde{d}) = \left(\frac{E_1^d + E_2^d}{2}\right) = \left(\frac{d_1+2d_2+d_3}{4}\right). \tag{32}
$$

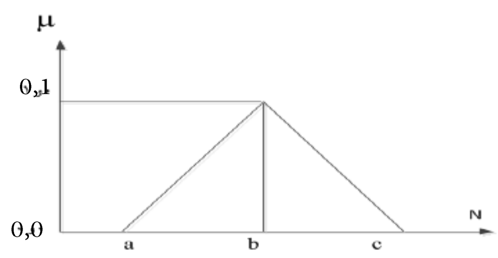

Fig. 1. Triangular membership function.

According to the Jiménez et al. [33] ranking method for each pair of fuzzy numbers *a* and *b* the magnitude of the number *a* relative to the number *b* represented by the μM (*a*, *b*) the *Eq. (33)* is calculated [35].

$$\mu_{\tilde{M}}(\tilde{a},\tilde{b})=\begin{cases}0, & \text{if} \quad E_2^a-E_1^b\leq 0,\\ \dfrac{E_2^a-E_1^b}{E_2^a-E_1^b-(E_1^a-E_2^b)}, & \text{if} \quad 0\in\left[E_1^a-E_2^b,\ \right]E_2^a-E_1^b\leq 0,\\ 1, & \text{if} \quad E_1^a-E_2^b.\end{cases} \tag{33}$$

The decision vector x is displayed as follows:

$$\tilde{a}_i x\leq \tilde{b}_i. \tag{34}$$

This vector is justified based on the definition of rating given with the degree Δ if:

$$\left[(1-\Delta)E_2^a+\Delta E_1^a\right]x\leq \Delta E_2^b+(1-\Delta)E_1^b. \tag{35}$$

The general form of the model can be defined as *Eq. (36)*.

$$\begin{aligned}&\text{Max } Z=EV(\tilde{C}^t)X\\ &\text{s.t.}\\ &\left[(1-\Delta)E_2^a+\Delta E_1^a\right]x\leq \Delta E_2^b+(1-\Delta)E_1^b,\\ &X\geq 0.\end{aligned} \tag{36}$$

The corresponding deterministic limit with the proposed model, the *Eq. (37)*.

$$\mu_{\tilde{D}_{jrgt}}=\begin{cases}1-\dfrac{(\sum_g D_{jrgt}*BH_{jrgt})-\sum_n TU_{jnrt}}{\alpha_{jrgt}}, & \sum_g D_{jrgt}*BH_{jrgt}-\alpha_{jrgt}\leq\sum_n TU_{jnrt}\leq\sum_g D_{jrgt}*BH_{jrgt},\\ 1-\dfrac{\sum_n TU_{jnrt}-(\sum_g D_{jrgt}*BH_{jrgt})}{\beta_{jrgt}}, & \sum_g D_{jrgt}*BH_{jrgt}\leq\sum_n TU_{jnrt}\leq\sum_g D_{jrgt}*BH_{jrgt}+\beta_{jrgt},\\ 0, & \text{otherwise.}\end{cases} \tag{37}$$

$$\sum_g D_{jrgt}*BH_{jrgt}-\alpha_{jrgt}\leq\sum_n TU_{jnrt}\leq\sum_g D_{jrgt}*BH_{jrgt}. \tag{38}$$

$$\sum_g D_{jrgt}*BH_{jrgt}\leq\sum_n TU_{jnrt}\leq\sum_g D_{jrgt}*BH_{jrgt}+\beta_{jrgt}. \tag{39}$$

The *Eqs. (38)* and *(39)* are the final determined form of the demand fuzzy parameter, which are placed in the constraints in the proposed model.

5 | Numerical Example

In this section, to show the applicability of the proposed model, an example in five different sizes has been designed and analyzed by GAMS software, first in a deterministic way and then in the state of uncertainty, and finally the numerical results have been analyzed. In this example, there are different suppliers and the supply centers are considered specific and definite. For the demand parameter, which is considered as triangular fuzzy number, three pessimistic, probable and optimistic points are $\tilde{d}_{jrgt}=(d_{1\,jrgt},\ d_{2\,jrgt},\ d_{3\,jrgt})$ created and then, by determining the acceptable degree of decision-making vector (Δ), the fuzzy limitation of demand becomes crisp. The value of the objective function for (Δ) is shown in *Table 2*.

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Table 2. Random information related to parameters numerical example.

Parameter	Random Distribution	Parameter	Random Distribution
$\tilde{D}_{jrgt}$	(300)	Q_{ij}	(5)
PR_{it}	(20)	$M1_{ij}$	(0.45)
λ_i	(0.7)	PC_{ik}	uniform (5,7)
WDC_{ih}	uniform (3,4)	MP_{jo}	uniform (4,7)
MD_{jn}	uniform (3,3)	RC_{il}	uniform (3,6)
DP_{jrt}	uniform (800,950)	MC_{jm}	uniform (1.5,3)
CM_{jrt}	uniform (800,730)	CD_{jnt}	uniform (500,750)
CG_{iht}	uniform (400,550)	CR_{ilt}	uniform (700,650)
CP_{jot}	uniform (2000,45)	CU_{jmt}	uniform (450,600)
SC_{jm}	uniform (18000,23000)	SX_{ih}	uniform (32000,5000)
SR_{il}	uniform (16000,24000)	ST_{jrt}	uniform (14000,1700
SU_{jn}	uniform (15500,25500)	SO_{jo}	uniform (25000,37000)
TCM_{jnrt}, TCN_{jmrt}, TCU_{iko}, TCK_{ilh}, TCR_{ilo}, TCO_{jon}, TCD_{jml}			uniform (6,15)

Table 3. Objective function for different problems in the deterministic state.

Problem	Problem Size |j|*|i|*|k|*|m|*|n|*|l|*|t|*|h|*|o|*|r|*|g|	Objective Function Values
1	2, 2, 2, 2, 3, 2, 2, 2, 2, 3, 1	1024545
2	3, 3, 2, 3, 3, 3, 2, 3, 3, 3, 2	1651435
3	4, 3, 3, 4, 4, 3, 3, 3, 3, 4, 3	6543064
4	5, 4, 3, 5, 5, 3, 3, 3, 4, 5, 3	11530697
5	6, 4, 4, 5, 5, 4, 3, 4, 4, 5, 3	14352385

Table 4. Objective function for different problems in the deterministic state.

Problem	Problem Size	Uncertaintylevel (Δ)	Objective Function Values under Uncertainty
1	2, 2, 2, 2, 3, 2, 2, 2, 2, 3, 1	0.1, 0.3, 0.6, 0.9	3592832, 3592508, 3592022, 3591536
2	3, 3, 2, 3, 3, 3, 2, 3, 3, 3, 2	0.1, 0.3, 0.6, 0.9	5897274, 5896734, 5895924, 5895114
3	4, 3, 3, 4, 4, 3, 3, 3, 3, 4, 3	0.1, 0.3, 0.6, 0.9	22376908, 22374856, 22371778, 22368700
4	5, 4, 3, 5, 5, 3, 3, 3, 4, 5, 3	0.1, 0.3, 0.6, 0.9	38961104, 38957540, 38952194, 38946848
5	6, 4, 4, 5, 5, 4, 3, 4, 4, 5, 3	0.1, 0.3, 0.6, 0.9	48333948, 48329640, 48323178, 48316716

It has been shown the objective function crisp using the values of *Table 2* in *Fig. 3* and states that the optimal values of the objective function also increase with the increase in the size of the problem. *Fig. 4* shows the objective function for different delta values in different sizes and states that the optimal amount of the objective function also decreases with the incre ase of uncertainty in the demand value.

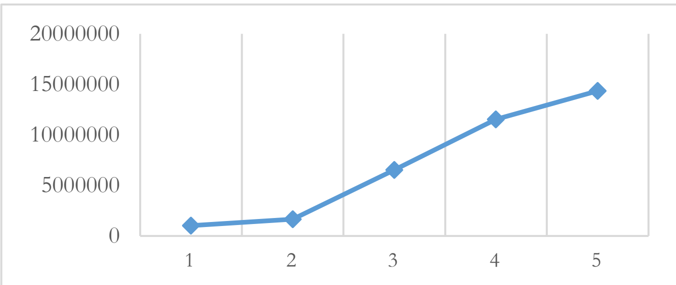

Fig. 2. Optimal values of objective function.

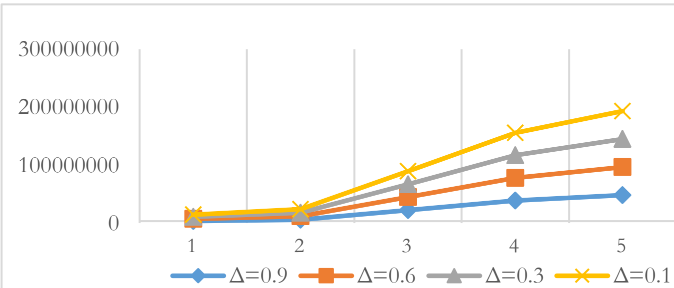

Fig. 3. Optimal values of objective function in uncertainty mode.

6 | Conclusion and Future Research Directions

All logistics activities related to the collection and handling of second-hand consumer goods, parts and materials in order to ensure the desired quality are related to this field also sometimes government restrictions are imposed on supply chains, and in others, there are concerns about the environment and its maintenance. The mentioned conditions lead companies to use forward and reverse integrated logistics network. Increasing attention to environmental issues has multiplied the importance of designing closed-loop supply chain networks that include production, recycling, and waste disposal. When both forward and reverse supply chains are considered with the goal of creating value for the product, the resulting network will be a closed-loop supply chain and designing a proper supply chain network can provide many benefits to improve supply chain performance. In this research presented a multi-product and multi-period forward/reverse integrated supply chain problem with the aim of maximizing profit and meeting customer satisfaction simultaneously and the fuzzy approach has been used to address the lack of certainty in the product. How to collect damaged products, the decision to recycle or destroy the goods, the maximum time to receive parts and products and the lost sales are among the practical dimensions that are in line with this research. In this research to show the applicability of the proposed model, a numerical example has been solved in different size using GAMS optimization software and computational results showed that the total network costs are affected by the amount of demand, return, as well as recycling and disposal for a given capacity of the network, and the optimal value of the objective function decreases by increasing the uncertainty in the amount of demand. As the size of the problem becomes enlarged becomes more computing volume and running time, solve complex issues in the short time is possible by using meta-heuristic methods and this can be the next research work in this field. In addition to the above, other objective can be considered in the model, such as considering warranty for products and maximizing the level of service, consider financial risks in the process, increasing the quality level of output products, and considering environmental factors.

Conflicts of Interest

All co-authors have seen and agree with the contents of the manuscript and there is no financial interest to report. We certify that the submission is original work and is not under review at any other publication.

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Paper Type: Research Paper

Evaluation of Household Expenditure in the United States: Pre-Covid and Post-Covid Statistical Analysis

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Abstract

The COVID-19 pandemic has caused unprecedented disruption to the global economic structure, resulting in significant changes in spending patterns for households worldwide. Developed countries like the United States have been affected as well, struggling to return to pre-pandemic stable economic situations. This study focuses on the impact of the pandemic on household expenditure in the United States, using ANOVA to compare household expenses between the pre-COVID period in 2018 and the post-COVID period in 2021. The results of the study showed a significant increase in all types of household expenditure from pre-COVID to post-COVID periods, highlighting the correlation between the pandemic and changes in spending habits. This trend is further fueled by price increases in daily necessities, inflation of the dollar, and scarcity of goods. The analysis also revealed that the trend was increasing, emphasizing the need for immediate policy interventions to address the issue. Further research is needed to identify the specific types of expenditure driving this increase and the underlying reasons behind it. The implications of the study are significant for policymakers and economists as they underscore the need for effective interventions to stabilize household expenditure and promote economic recovery in the wake of the pandemic. The findings also highlight the importance of utilizing statistical methods such as ANOVA to evaluate complex economic systems and guide evidence-based policy interventions. As future research continues to explore the impact of the pandemic on economic structures worldwide, this study provides valuable insights into the specific changes in household expenditure in the United States, emphasizing the urgent need for targeted policy interventions to address these changes.

Keywords: Pre-post covid analysis, Household expense, Statistical analysis, Covid-19 economic impact, One-way ANOVA.

1 | Introduction

The COVID-19 pandemic began to impact the United States in early 2020 when the first virus cases were reported in the country. As the virus spread and cases increased, the US government declared a national emergency in March 2020, and many states implemented stay-at-home orders to slow the spread of the virus [1].

The COVID-19 pandemic has had a major impact on the global supply chain, leading to disruptions and challenges in multiple industries. Some of the effects include supply chain disruptions with lockdowns and border closures in place; many businesses have struggled to maintain their supply chains, leading to shortages of essential goods and supplies [2]. Additionally, the pandemic has increased demand for certain products, such as Personal Protective Equipment (PPE), household essentials, and cleaning supplies, leading to shortages and price increases [3]-[5]. Then, delays in

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production and shipping with businesses operating at reduced capacity or shut down entirely have been significant delays in production and shipping, affecting the availability of goods and the speed at which they can be delivered [6], [7]. Again, in response to the pandemic, many businesses have shifted to local sourcing to reduce dependence on global supply chains and minimize the risk of supply chain disruptions [8]. Also, the pandemic has led to significant air, and sea transportation disruptions, with many flights and ships canceled, affecting the ability of goods to be transported and delivered [9], [10]. Overall, the COVID-19 pandemic has significantly impacted the global supply chain, leading to disruptions, delays, and challenges for businesses across multiple industries. However, there are also signs of recovery as the pandemic response continues and supply chains began to return to normal in 2021.

However, the pandemic left profoundly impacted the US economy and household finances [11]. According to a US Bureau of Labor Statistics report, household spending on groceries increased by nearly 20% between February and April 2020 as people stockpiled supplies and ate more meals at home [12]. At the same time, spending on travel and entertainment plummeted, with many businesses and venues closing to prevent the virus's spread. Healthcare costs also rose for many households as people sought medical treatment for COVID-19 and purchased PPE such as face masks and hand sanitizer. For those who lost their jobs or saw their hours reduced, the cost of housing became a significant burden as they struggled to make rent or mortgage payments. The shift to remote work and online learning has also increased demand for home office equipment, such as laptops and desks, and delivery services for groceries and other essentials. This has led to higher household expenses as they seek to adapt to the new normal created by the pandemic. The COVID-19 pandemic has significantly impacted household expenses in the United States, with changes to spending patterns driven by the need to stay safe, work and learn from home, and access essential goods and services.

Despite subsidies from the state and federal governments in 2020, the economic condition was alarming. Later the free covid vaccination and the spread of the pandemic reduction provided a much-stabilized livelihood for the people of the United States from 2021, which could be recognized as post covid period [13]. However, the expenses are still high, and this economic downturn would be portrayed in this paper. Domestic services expenditure, entertainment expenditure, housing expenditure, electricity expenditure, transportation expenditure, food expenditure, gasoline, and motor oil expenditure, health care expenditure, life insurance expenditure, maintenance, and repairs expenditure, mortgage interest expenditure, telephone services expenditure, televisions, radios, and sound equipment expenditure, utilities, fuels, and public services expenditure was evaluated from pre to post covid period (2018 to 2021). Evaluating data from 2018 would portray any natural temporal trend in these expenses as well as the unusual changes in expense change due to the pandemic.

The economic evaluation for expense changes was mostly done with algebraic equations in past studies. Very few studies focus on evaluating the change in expense using statistical methods. Hence, the objective of this study was to identify the expenditure change as an impact of the covid pandemic using the ANOVA method for households in the United States. Besides, the total household expense was also evaluated similarly to validate the trend of overall expenditure change.

A recent article published in the Review of Finance highlighted the importance of studying the impact of the pandemic on household finances [14]. The authors note that the pandemic has resulted in significant changes in employment, income, and consumption patterns, and these changes have disproportionately impacted low-income households. Understanding the changes in household expenditure patterns can help policymakers design effective policies to mitigate the economic impact of the pandemic. This paper is especially relevant given the unprecedented nature of the pandemic, and the resulting economic uncertainty will provide a detailed analysis of the changes in household expenditure patterns in the United States during the pandemic. The paper's statistical analysis provides valuable insights into the factors influencing household spending and can inform policymaking and financial planning decisions.

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2 | ANOVA for Comaprison

Alireza et al. [15] randomized controlled clinical experiment was to evaluate how family members' presence—whether trained or not—affects patients' anxiety levels during invasive procedures in an emergency room. Ninety eligible patients participated in the study, which was divided into three equal groups: A (family member is trained), B (family member is untrained), and C (family member is absent). Before and after the operations, anxiety levels were assessed using the Spielberger State-Trait Anxiety Inventory (STAI). The findings showed that following the therapies, anxiety levels generally declined dramatically in all groups.

Singh et al. [16] investigated how, during these lockdown times, internet search activity connected to alcohol changed. Utilizing Google Trends and concentrating on four primary themes—alcoholic beverage types, ways to obtain alcohol, problems resulting from the disruption of the alcohol supply, and seeking assistance for alcohol use disorders—the study examined average relative search volumes during the pre-lockdown, lockdown 1.0, and lockdown 2.0 phases. A statistical technique called Analysis of Variance (ANOVA) is used to determine if the means of three or more independent groups differ in any statistically meaningful ways. ANOVA could be used to examine the differences in mean relative search volumes over the three specified time periods: pre-lockdown, lockdown 1.0, and lockdown 2.0, in the context of the study you described on the changes in online search interest for alcohol-related keywords during different lockdown periods in India. When compared to the pre-lockdown phase, the data showed a notable increase in internet interest related to the purchase of alcohol during lockdown 1.0, but not during lockdown 2.0.

Many other studies used one-way ANOVA to compare financial performances during Covid 19, for stock market performance [17], for exchange-traded funds [18], financial life of working individuals [19], education enterprise profit level [20], income [21]-[24], and so on.

3 | Data Preparation and Analysis

Almost 11500 public survey interview data was extracted from the US Bureau of Labor Statistics website. The data were merged and cleaned using python standard data cleaning format for data analysis (removing the duplicate entries, unusual entries, and null values) [25].

One-way ANOVA was used to find a significant difference between the expense level over the years. Data were grouped based on the year. Hence, there were four groups of interview data. Assumptions of the ANOVA method were tested before conducting the ANOVA analysis [26]. The data was independent, given that data were randomly collected from United States residents, the residuals of the dataset had consistent variance and mean, and the dataset was normally distributed. Statistical Package for the Social Sciences (SPSS) was used to conduct the one-way ANOVA to determine whether there are any statistically significant differences between the means of these four independent (unrelated) groups. The formula for one-way ANOVA was mentioned by Bewick et al. [27]:

$$F = \frac{SS_{between} - df_{between}}{SS_{within} - df_{within}},$$

where, F= F-statistic,

$SS_{between}$ = sum of squares between groups,

SS_{within} = sum of squares within groups,

$df_{between}$ = degrees of freedom between groups,

df_{within} = degrees of freedom within groups.

4 | Result

Table 1 shows the mean expenditure for 2018 and 2019 and the p-value for mean comparison, *Table 2* shows the mean expenditure for 2019 and 2020 and the p-value for mean comparison, and *Table 3* shows the mean expenditure for 2020 and 2021 and p-value for mean comparison for the different household expense. In *Table 1*, only for alcoholic beverage expenditure and maintenance and repairs expenditure, the expense of 2019 was significantly higher than 2018 (p-values < 0.04).

However, in *Table 2*, expenditure for 2020 was significantly higher for housing, electricity, transportation, food, gasoline and motor oil, healthcare, telephone services, utilities, fuels, and public services (p-values < 0.05). Expenditure for households kept increasing in 2021 as well. For all types of household expenses. For 2021 (*Table 3*), all types of household expenditures significantly increased from 2020.

Fig. 1 shows a stacked bar chart for different types of expenditure, indicating that total household expenditure increased with time and increased severely because of covid and portrayed post-covid economic struggles. *Fig. 2* shows the temporal rise of different household expenditures in a granular form.

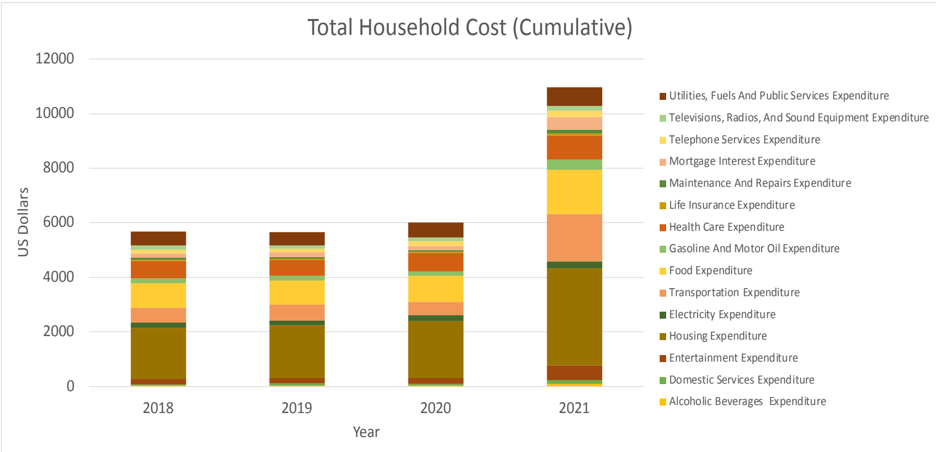
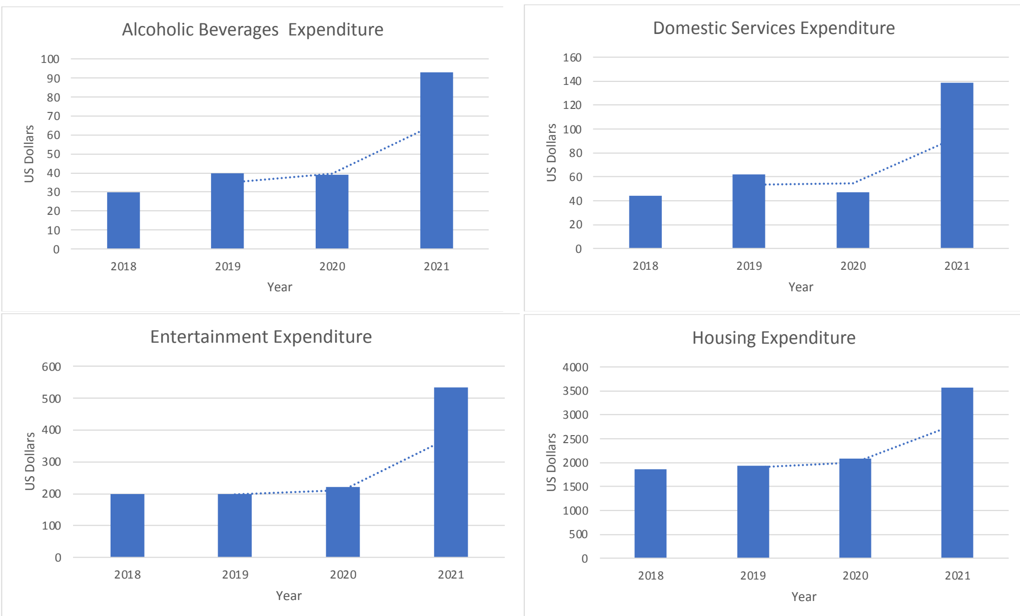


Fig. 1. Stacked bar chart showing the temporal rise of household expenditures.



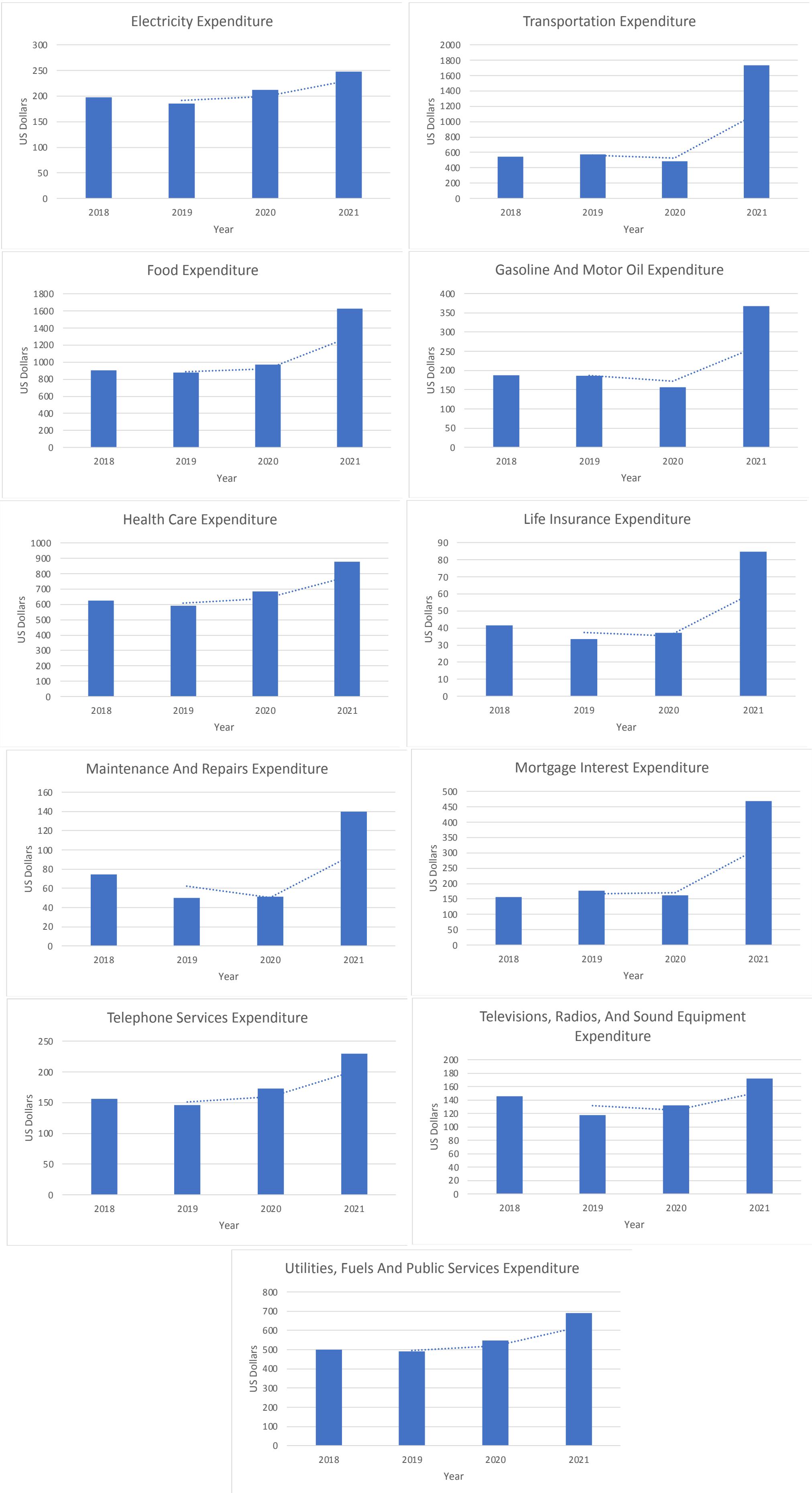


Fig. 2. Bar chart showing the temporal rise of different types of household expenditures.

Table 1. Statistical analysis of household expenditure for years 2018 and 2019.

Year	2018	2019	p-Value
Alcoholic beverages expenditure	29.794	39.883	0.032*
Domestic services expenditure	44.139	62.155	0.153
Entertainment expenditure	197.846	198.884	0.939
Housing expenditure	1860.826	1929.871	0.356
Electricity expenditure	198.080	185.200	0.129
Transportation expenditure	544.836	576.297	0.495
Food expenditure	903.342	876.952	0.445
Gasoline and motor oil expenditure	187.125	186.797	0.977
Health care expenditure	626.398	591.307	0.344
Life insurance expenditure	41.500	33.530	0.218
Maintenance and repairs expenditure	74.715	49.931	0.035*
Mortgage interest expenditure	156.103	177.483	0.349
Telephone services expenditure	155.954	146.193	0.197
Televisions, radios, and sound equipment expenditure	146.193	117.540	0.966
Utilities, fuels and public services expenditure	499.062	491.558	0.698

* p-values < 0.05

Table 2. Statistical analysis of household expenditure for years 2019 and 2020.

Year	2019	2020	p-Value
Alcoholic beverages expenditure	39.883	39.281	0.910
Domestic services expenditure	62.155	47.374	0.287
Entertainment expenditure	198.884	221.991	0.156
Housing expenditure	1929.871	2088.663	0.045*
Electricity expenditure	185.200	212.540	0.003*
Transportation expenditure	576.297	481.107	0.048*
Food expenditure	876.952	969.346	0.014*
Gasoline and motor oil expenditure	186.797	156.977	0.010*
Health care expenditure	591.307	683.854	0.015*
Life insurance expenditure	33.530	37.240	0.575
Maintenance and repairs expenditure	49.931	51.167	0.903
Mortgage interest expenditure	177.483	162.113	0.501*
Telephone services expenditure	146.193	172.889	0.001*
Televisions, radios, and sound equipment expenditure	117.540	132.221	0.078
Utilities, fuels and public services expenditure	491.558	547.915	0.005*

* p-values < 0.05

Table 3. Statistical analysis of household expenditure of year 2020 and 2021.

Year	2020	2021	p-Value
Alcoholic beverages expenditure	39.281	92.996	<0.001*
Domestic services expenditure	47.374	138.476	<0.001*
Entertainment expenditure	221.991	533.944	0.001*
Housing expenditure	2088.663	3566.852	<0.001*
Electricity expenditure	212.540	247.620	<0.001*
Transportation expenditure	481.107	1735.793	<0.001*
Food expenditure	969.346	1627.559	<0.001*
Gasoline and motor oil expenditure	156.977	366.956	<0.001*
Health care expenditure	683.854	877.348	<0.001*
Life insurance expenditure	37.240	84.580	0.033*
Maintenance and repairs expenditure	51.167	140.176	<0.001*
Mortgage interest expenditure	162.113	468.633	<0.001*
Telephone services expenditure	172.889	229.344	<0.001*
Televisions, radios, and sound equipment expenditure	132.221	172.204	0.001*
Utilities, fuels and public services expenditure	547.915	689.431	<0.001*

* p-values < 0.05

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5 | Discussion

Covid 19 pandemic has largely affected the socioeconomic conditions of the United States. Post-covid period adopted more virtual and self-sufficient methods for production, business, services, and so on. The alarming point is, replacing globalization with localization has caused the pandemic's structurally damaging effects on the global economy [28]. Even developed countries like the United States face the adverse effects of dollar inflation, job scarcity, supply chain demand, and rising cost for daily necessities [29], [30].

Similarly, household costs in the United States skyrocketed, especially after a prolonged disruption in the economic structure price of everything increased. The survey data showed that in 2021, all types of household expenditure increased. The expenditure at the family level did not return to the pre-covid condition, rather higher than in the previous year, 2020. The alcoholic beverages, life insurance, and motor oil cost was reported to be significantly higher than the previous year in 2021.

From this study, we can observe the trend of expenditure increment. This price hike in people's daily life is also impacting social life [31], so economic conditions and cost need to be stabilized as soon as possible. However, what type of expenditure affected the total expenditure and the reason for the increase in different types of expenditure should be evaluated.

6 | Conclusion

The covid-19 pandemic has caused significant disruptions in the US economy, leading to widespread job losses and reduced economic activity. Despite this, family household expenditure has actually increased in the post-covid period, surpassing previous years. This trend highlights the need for effective policies to maintain a balanced social and economic footing. The authorities must consider the short and long-term impacts of their policies on households, businesses, and the economy as a whole. The development of policies that address income inequality, support small businesses, and promote economic growth will be critical to ensuring a stable and sustainable recovery from the pandemic.

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A New Efficient Genetic Algorithm-Taguchi-Based Approach for Multi-Period Inventory Routing Problem

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Abstract

The inventory routing problem arises from the combination of the vehicle routing problem and the vendor-managed inventory problem. In this paper, we present a mathematical model and a novel genetic algorithm for solving the multi-period inventory routing problem. The objective is to supply products to scattered customers within a given time horizon while managing customer inventories to avoid shortages and minimize total inventory and transportation costs. To represent solutions for this problem, we introduce a new chromosomal structure. This structure offers simplicity in encoding and decoding solutions, maintains feasibility after crossover and mutation operations, addresses both routing and inventory management in a single step, and consolidates information about each solution method comprehensively. The algorithm parameters, including crossover and mutation rates, population size, number of iterations, and selection pressure, are fine-tuned using the Taguchi method. To assess algorithm efficiency, we utilize standard instances from the literature. Our results demonstrate that the proposed algorithm performs favorably compared to previous approaches.

Keywords: Inventory routing problem, Genetic algorithm, Metaheuristic, Optimization.

1 | Introduction

Supply chain management is a complex system of interrelated components that can significantly impact each other. Two critical components of supply chain management are transportation and inventory management [1]. In the literature, the coordination between transportation and inventory management is often referred to as the Inventory Routing Problem (IRP). The IRP aims to minimize costs by determining the optimal distribution and inventory strategy [2]. It involves three main decisions: how much and when to deliver to each customer, what routes to use for each delivery, and what inventory levels to maintain at each location [3]. The IRP is a challenging and practical problem that arises in various industries and contexts [4].

This paper focuses on the multi-period IRP, which involves determining the optimal inventory levels for each period. Additionally, this problem is classified as an NP-hard problem, meaning that exact methods cannot efficiently solve large-scale and complex instances [5]. Therefore, metaheuristic algorithms, which are approximate methods inspired by natural phenomena or human behavior, are

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often employed to find good solutions for the IRP. These metaheuristic algorithms are widely used across different fields to tackle complex real-world problems [6]. One of the most popular metaheuristic algorithms is the Genetic Algorithm (GA), which mimics the process of biological evolution [7]. The GA operates on a population of solutions called chromosomes, applying operators such as selection, crossover, and mutation to improve them over generations. The key elements of metaheuristic algorithms are intensification and diversification [8]. The GA's performance depends on several parameters, including population size, crossover rate, and mutation rate [9]. In this paper, we propose a new chromosome representation for the IRP and use the Taguchi method to fine-tune the GA parameters. We tested our approach on several benchmark instances and compared it with existing methods from the literature. We demonstrate that our approach can find better or more competitive solutions for the IRP in a reasonable time.

The rest of the paper is organized as follows: Section 2 reviews the related work on the IRP and metaheuristic algorithms. Section 3 presents the mathematical model and the notation for the IRP. Section 4 describes the proposed GA and its components in detail. Section 5 reports the numerical results and the analysis of our approach. Finally, Section 6 concludes the paper and suggests some directions for future research.

2 | Literature Review

The first time IRP was presented by Bell et al. [10], and it has been studied by researchers from various aspects since then. The field of IRP is broad, and researchers have addressed this problem from various angles, such as time periods, transportation fleet, inventory policies, and algorithms. The multi-period inventory routing problem considering the carbon emission regulations was proposed by Cheng et al. [11]. Perishable products were studied in a multi-period inventory routing problem [12]. Xiao and Rao [13] presented the multi-product and multi-period IRP, considering the time window. Alinaghian et al. [14] presented a piecewise linearized green multi-period IRP with time windows. There are other cases that are not the subject of this research and have been discussed in detail in the review article [15].

Exact algorithms for IRP were designed by Solyalı and Süral [16], where the branch-and-cut algorithm is represented by Coelho and Laporte [17], and branch-and-price-and-cut by Andersson et al. [18]. Since exact methods are not efficient for large dimensions of the IRP, the solution to this problem was presented by Archetti et al. [19] using an efficient matheuristic algorithm. Yu et al. [20] introduced a distance-based clustering method based on an ant colony optimization approach. Cordeau et al. [21] designed a decomposition approach to tackle large-scale instances of the IRP. Su et al. [22] integrated a local search-based metaheuristic with mathematical programming. An augmented Tabu Search (TS) algorithm and a Differential Evolution (DE) algorithm for IRP were implemented by Alinaghian et al. [14]. A two-stage hybrid metaheuristic algorithm was proposed by Wu et al. [23] for a multi-period location-inventory-routing problem with time windows and fuel consumption.

John Holland [24] developed the first GA in the early 1970s. To obtain good solutions for multi-period IRP, various GA approaches have been developed [25]. A GA for the IRP with lost sales proposed in [26] uses two matrix chromosomes: the first to determine clustering and the second to determine the route. Using the GA method, Othman et al. [27] proposed simulation optimization modeling for the IRP. The stochastic periodic Can-Deliver policy, which permits early replenishment, serves as the foundation for the IRP simulation model. The chromosomal structure in this study is determined by three levels of warehouse replenishment, and the GA presents a classification of customers. Routing in each category is done by a heuristic algorithm.

Hiassat et al. [28] have developed an efficient GA approach to solve the location-inventory-routing problem with perishable products. The authors used a new chromosomal structure in their GA. Azadeh et al. [29] proposed an IRP for a single perishable product, which has been solved using a GA. The proposed algorithm parameters are tuned using the Taguchi method. In the chromosome presented in this study, the initial and final customers for the visit are determined using a matrix. However, the sequence of middle customers is not determined.

Arab et al. [30] proposed a Non-Dominated Sorting Genetic Algorithm (NSGA-II) for solving the multi-objective, multi-period, and multi-product inventory-routing problem. Yavari et al. [31] investigated solving the multi-period location-inventory-routing problem of perishable products with a GA. Amri Sakhri [32] developed a GA to solve IRP with different crossover structures. Mahjoob et al. [4] proposed a Modified Adaptive Genetic Algorithm (MAGA) to solve the multi-product and multi-period IRP. In this research, two matrix structures consisting of real and binary numbers have been used to represent chromosomes. Furthermore, the method uses a GA only for clustering, while routing is done by another heuristic algorithm. For the IRP with deterministic customer demand, Sakhri et al. [33] created a Memetic Algorithm (MA) based on GA and Variable Neighborhood Search (VNS) methods.

A review of the literature shows that the matrix structures presented in previous research are incapable of simultaneously classifying, routing, and managing inventory for the IRP. For this reason, the researchers solved the model using a two-step procedure: clustering first and then routing with VRP heuristic algorithms. Chromosome structures must contain all the necessary information about the problem. However, this issue complicates chromosome structure. The complexity of chromosome structure reduces the speed and accuracy of the GA. A simpler chromosome has a better ability to improve the performance of the GA.

In this research, a new chromosomal structure has been presented for the GA, which addresses the defects in previous algorithms for solving the IRP problem. Among the important features of this structure, the following can be mentioned:

I. Using this matrix structure, there is no need to solve the problem of routing and inventory management in two stages; both problems will be solved simultaneously.
II. All information related to each solution method, including the distribution route, order of distribution, shipping amount, stock level of warehouses, customers not covered, and unused machines, is contained in a simple way.
III. The simplicity of chromosome encoding and decoding increases the speed and accuracy of solving the problem, thereby enhancing the performance of the GA in the IRP problem.
IV. The ability to develop this structure for the Production Routing Problem (PRP) is also one of its other features.

In summary, the chromosomal structure presented for the first time has improved the performance of the GA in solving the IRP by addressing the defects of previous algorithms.

3 | Problem Description and Formulation

In this study, the problem is expressed as a multi-period inventory routing problem. The supplier must decide which products to transfer to customers to meet the demand specified over the finite planning horizon. The problem is periodic: in each period, the beginning inventory for customers is known, and the demand level for each customer in each period is both known and limited. These demands are specific, deterministic, but variable over time periods, and backlogging and split-delivery are not allowed. Capacitated vehicles transfer goods from the supplier to the customers and return them to the supplier. It is assumed that there is enough inventory at the supplier to satisfy all demands.

A homogeneous transportation fleet is used to respond to customer demand. The holding cost is also assumed to remain constant over the planning horizon. The transportation cost per trip consists of a fixed cost incurred on each trip and a variable cost proportional to the distance traveled. To meet customer demands, the Order-Up-to (OU) policy was adopted, which states that the quantity delivered to a customer must equal the inventory capacity policy. The objective of the problem is to minimize the total transportation and inventory holding costs of the system while ensuring there are no unsatisfied demands from customers in each period. Decision variables in the problem include inventories for customers, material delivery levels to customers, and routes for delivering materials in each period.

The mathematical model is proposed as a single-product, multi-period, and single-objective one. The assumptions considered for modeling and evaluation are as follows:

- *Demands from customers must be met completely in each period.*
- *Distances between points are expressed as Euclidean distances.*
- *The location of suppliers and customers is fixed and determined.*
- *Customer demands are known and fixed.*
- *Each customer must be visited once, with the same vehicle, in each period.*
- *Each vehicle can be launched once in each period.*
- *Vehicle capacities and customers' inventories are limited and determined.*
- *Established routes must only begin from the supplier and end at the supplier.*
- *Customer demands remain fixed and consistent across all periods.*

3.1 | Notation

The notation adopted in the current formulation is described as follows:

3.1.1 | Sets and indices

$N = S \cup C$	Set of all nodes.
$S = \{0\}$	Set of suppliers.
$C=\{1,2,\ldots,\lvert C\rvert\}$	Set of customers.
$K=\{1,\ldots,\lvert K\rvert\}$	Set of homogeneous vehicles.
$T=\{1,\ldots,\lvert T\rvert\}$	Set of time periods.
i & j	Supplier and customers index.
k	Vehicle index.
t	Time period index.

3.1.2 | Parameters

d_i^t	Demand of customer i in time period t.
C_{ij}	Distant-dependent travel cost between customer i and j.
h_i	Unit inventory holding cost at the place of customer i.
f	Fixed transportation cost.
Q	Maximum capacity of vehicles.
U_i	Maximum inventory holding capacity of customer i.

3.1.3 | Variables

y_{ij}^{kt}	1, if the path from node i to node j is traversed by vehicle k in time period t, and 0 otherwise.
qs_{ik}^{t}	The amount of transferred product to customer i by vehicle k in time period t.
ql_{ij}^{kt}	The loading amount of vehicle k in route from node i to node j in time period t.
I_{it}	Inventory level of customer i in time period t.

3.2 | Model Formulation

The mathematical formulation is presented in this section. The objective function given in *Eq. (1)* minimizes the total cost. The first section of the objective function minimizes the fixed cost of the

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supplier's used vehicle. The second section illustrates the inventory holding cost at the customer locations. Routing and transshipment costs are represented in the third section.

$$\text{Min } z = \sum_{k\in K}\sum_{t\in T}\sum_{i\in S}\sum_{j\in C} f \times y_{ij}^{kt} + \sum_{i\in C}\sum_{t\in T} h_i \times I_{it} + \sum_{k\in K}\sum_{t\in T}\sum_{(i,j)\in N} c_{ij} \times y_{ij}^{kt}, \tag{1}$$

s.t.

$$I_i^t \le U_i \quad \text{for all } i \in C, t \in T, \tag{2}$$

$$I_i^t = I_i^{t-1} + \sum_{k\in K} qs_{ik}^t - d_i^t \quad \text{for all } t \in T, i \in C, \tag{3}$$

$$\sum_{k\in K} qs_{ik}^t \le U_i - I_i^{t-1} \quad \text{for all } t \in T, i \in C, \tag{4}$$

$$\sum_{i\in S}\sum_{j\in N} y_{ij}^{kt} \le 1 \quad \text{for all } k \in K, t \in T, \tag{5}$$

$$\sum_{i\in N}\sum_{k\in K} y_{ij}^{kt} \le 1 \quad \text{for all } t \in T, j \in C, \tag{6}$$

$$\sum_{i\in N} y_{ij}^{kt} + \sum_{i\in N} y_{ji}^{kt} = 2 \times \sum_{i\in N} y_{ij}^{kt} \quad \text{for all } t \in T, j \in N, k \in K, \tag{7}$$

$$y_{ii}^{kt} = 0 \quad \text{for all } t \in T, i \in N, k \in K, \tag{8}$$

$$\sum_{i\in C} y_{ij}^{kt} = \sum_{i\in C} y_{ji}^{kt} \quad \text{for all } t \in T, k \in K, j \in S, \tag{9}$$

$$\sum_{j\in C} ql_{ij}^{kt} = \sum_{j\in C} qs_{jk}^t \quad \text{for all } t \in T, i \in S, k \in K, \tag{10}$$

$$\sum_{i\in N} ql_{ij}^{kt} - \sum_{i\in N} ql_{ji}^{kt} = qs_{jk}^t \quad \text{for all } t \in T, j \in C, k \in K, \tag{11}$$

$$ql_{ij}^{kt} \le Q \times y_{ij}^{kt} \quad \text{for all } t \in T,\ i,j) \in N, k \in K, \tag{12}$$

$$y_{ij}^{kt} \in \{0,1\}, \tag{13}$$

$$qs_{ik}^t, ql_{ij}^{kt}, I_i^t \ge 0. \tag{14}$$

Eq. (2) ensures the inventory level of each customer never exceeds the maximum inventory holding capacity. The inventory balance constraint at customer i, as shown in *Eq. (3)*, is equal to the inventory level in period t-1, by adding the total quantity transshipped in period t and subtracting the demand in period t. *Eq. (4)* ensures that the quantity delivered to the customer is below the remaining customer's inventory holding capacity. *Eq. (5)* states that each vehicle is not used more than once in each period by the supplier, as *Eq. (6)* ensures that each customer is not visited more than once in each period. *Eqs. (7)* and *(8)* represent the sub-tour elimination constraints. The flow conservation constraints, which confirm the equality of the number of incoming and departing arcs at a vertex, are defined in *Eq. (9)*. *Eq. (10)* enforces that the amount of vehicle loading on a route should not exceed the amount assigned to that route's customers. Furthermore, *Eq. (11)* specifies that if the demand of customer i is not met in period t, no product will be sent to them. Finally, *Eq. (12)* guarantees that the vehicle's capacity is not exceeded. *Constraints (13)* and *(14)* further define restrictions for the variables.

4 | Genetic Algorithm

Biologically motivated approaches are particularly popular in solving complex optimization problems. The GA is a stochastic optimization approach constructed based on evolutionary processes inspired by the process of natural selection. Using operators such as crossover, mutation, and selection, the GA synthesizes the good features of different individuals within the population in order to create individuals who are better suited. It is widely applied to solve different classes of NP-hard problems. The IRP belongs to the NP-hard class of problems, and exact solution methods are highly time-consuming for large-sized problems. Therefore, in the proposed problem, the GA approach is utilized. The flowchart of the general procedure of the proposed GA in this study is shown in *Fig. 1*. This procedure was adapted from the one presented by Gen et al. [34].

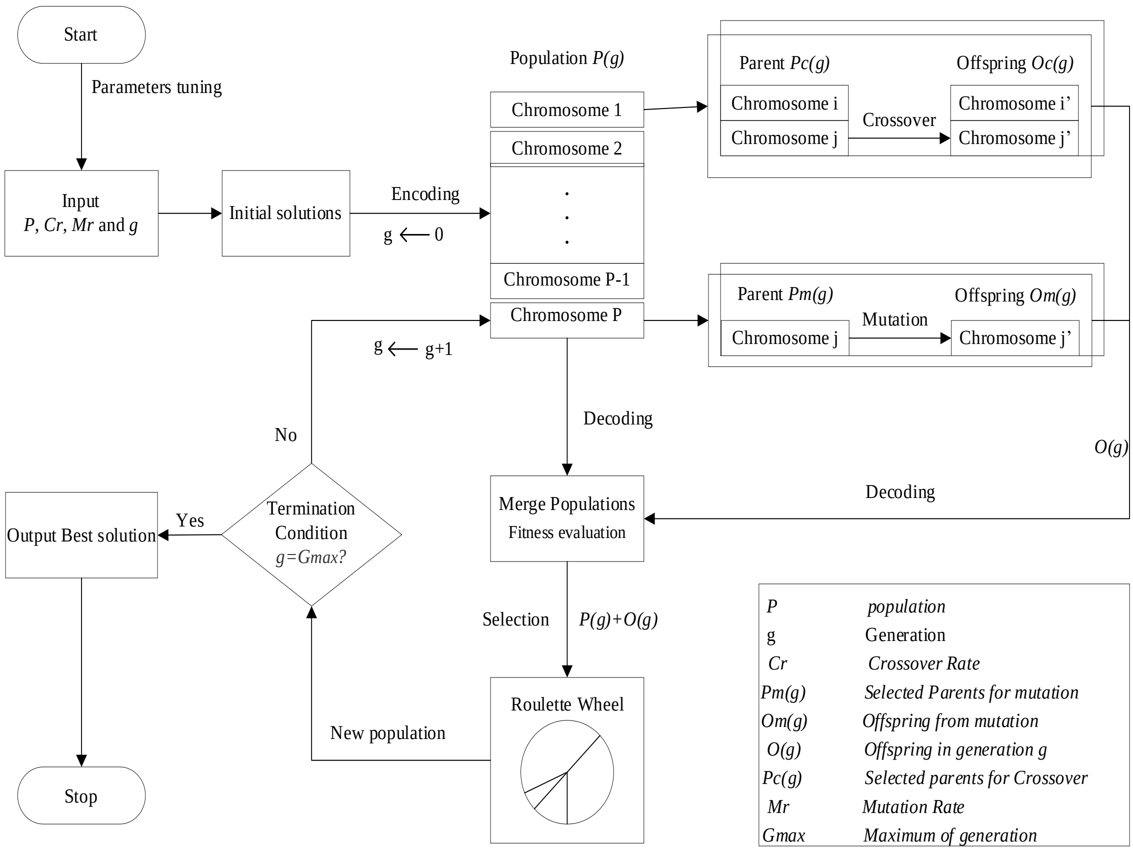

Fig. 1. General procedure of the proposed GA.

GA are initialized by a set of chromosomes (solutions) called the population. These chromosomes progress through successive iterations, known as generations. During each iteration, fitter chromosomes, evaluated by a fitness function, have higher chances of being selected to produce several offspring as new solutions. In the new population, which includes parents and children, the best individuals are chosen based on their fitness. The population size remains constant throughout all iterations. The GA may converge to the best solution after a certain number of iterations. The pseudo code for the GA proposed for the current problem is depicted in *Algorithm 1*.

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Algorithm 1. The pseudo code for general procedure of the proposed GA.

Procedure: GA for IRP
Input: Customers demand, Vehicle capacity, Planning horizon, Taguchi-based tuned parameters
Output: The best solution (vehicle route and amount of products assigned to customers)
begin
g ¬ *0;*
initialize *P(g)* by encoding routine;
evaluate *P(g)* by decoding routine;
while (not termination condition) **do**
create $O_c(g)$ from *P(g)* by crossover routine;
create $O_m(g)$ from *P(g)* by mutation routine;
evaluate *O(g)* by decoding routine;
form *P(g+1)* out of *P(g)* and *O(g);*
set the best current solution;
g +1 ¬ *g*
end
output: The best solution
end

4.1 | Chromosome Representation

The chromosome representation and encoding of a solution is the first and most crucial task when utilizing a GA. Each chromosome must carry all the necessary information about the solution. In this problem, the chromosomes represent both served and unserved customers, as well as the route and amount of products transshipped to customers in each period.

The proposed encoded chromosome takes the form of a matrix with dimensions t *(c+k), where t, c, and k represent the number of time periods, number of customers, and number of vehicles available, respectively. Each row in the matrix is a permutation of real numbers between 1 and c+k, specifically:

- *Numbers 1 to c correspond to customers.*
- *Numbers c +1 to c +k represent vehicles.*

In the t'th row of the chromosome, genes related to customers appear before genes related to vehicles. These represent the customers served by the desired vehicle during period t. The vehicle route starts at the supplier and returns to the supplier after visiting the assigned customers. *Fig. 2* provides an example of the encoding procedure used in this paper.

Vehicle usage

If the first gene value in the chromosome string corresponds to a vehicle or if there are no genes related to customers between the two genes related to vehicles, that particular vehicle will not be used in this specific time period.

Unserviced customers

The gene values after the last vehicle gene up to the end of the chromosome string indicate those customers who were not serviced during this period.

All customers visited

Logically, if there are no other gene values after the last vehicle gene at the end of the chromosome string, it implies that all customers will be visited in this particular period.

This encoding ensures that each chromosome carries essential information about customer service, vehicle routes, and transshipment of products. It's a crucial step in utilizing a GA to solve the problem.

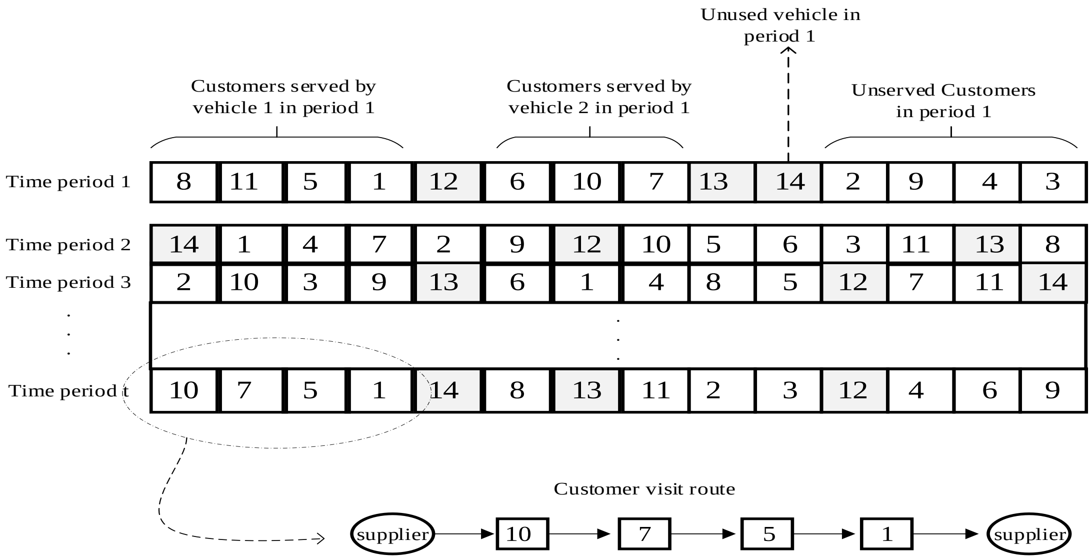

Fig. 2. Chromosome representation.

The example in *Fig. 2* refers to a chromosome with 11 customers and 3 vehicles, operating over t time periods. Gene values 1 to 11 correspond to customers, while genes 12 to 14 represent supplier-available vehicles. In time period 1, the value of genes from the beginning of the chromosome string up to the first gene related to vehicles (i.e., gene 12) represents customers serviced by the first vehicle. The genes after gene 12 up to the second gene related to vehicles (i.e., gene 13) are serviced by the second vehicle. This procedure is repeated for the number of available vehicles (i.e., 3). Customers assigned to each vehicle are served in the order of their appearance on the chromosome. This arrangement determines the route for each vehicle, which starts from the supplier and returns to the supplier after visiting the assigned customers (S-8-11-5-1-S).

4.2 | Initialization

To initiate the exploration of a nearly ideal solution, a population with the desired number of members is generated as an initial solution. Each individual chromosome in the population is represented by a t*(c+v) matrix, where t, c, and v denote the number of time periods, customers, and available vehicles, respectively. Each row of the matrix consists of a random permutation of real numbers between 1 and c+v.

4.3 | Chromosome Decoding

The values of the decision variable, y_{ij}^{kt} are obtained from permutation numbers of the chromosome as follows (k regardless of its gene value, is the kth vehicle's gene in each row):

$$y_{ij}^{kt} = \begin{cases} 1, & \text{if in tth row, gene value j immediately follows gene value i} \\ & \text{in the distance between the kth and k−1th vehicles' gene,} \\ 0, & \text{Otherwise.} \end{cases} \tag{15}$$

The values of qs_{ik}^{t} and I_{it} for the customers according to the values of y_{ij}^{kt} are then determined as follows:

$$qs_{ik}^{t} = \begin{cases} U_i - I_{i,t-1} + d_{i,t}, & \text{if } y_{ij}^{kt}=1, \quad \text{for all i,t,} \\ 0, & \text{Otherwise.} \end{cases} \tag{16}$$

$$I_{i,t} = \begin{cases} U_i, \quad \text{if } y_{ij}^{kt}=1, & \text{for all i,t,} \\ I_{i,t-1} - d_{i,t}, & \text{Otherwise.} \end{cases} \tag{17}$$

Then, the values of ql_{ij}^{kt} which is related to the amount of vehicle loading, are obtained by the chromosome depending on the values of y_{ij}^{kt} and qs_{ik}^{t} as follows:

$$ql_{ij}^{kt} = \sum_{i \in E} qs_{ik}^{t} \qquad \text{for all } j, k, t, \tag{18}$$

E: gene values' set include the gene value j and after that up to kth gene

related to vehicles in row t.

4.4 | Evaluation

The fitness function is calculated according to the objective function to evaluate the solutions in the population. In this model, the fitness value is attained through the cost components of the objective function, along with two penalty terms. These cost components include the routing cost, the cost of using the vehicle (known as the fixed cost), and the inventory holding cost. According to the constraints of the model, penalties related to exceeding the capacity of the vehicle and unsatisfied customer demand in each period are also added to the value of the fitness function. After calculating the fitness function, genetic operators are applied to solutions with better-fitted values, and costly solutions are removed from the population.

4.5 | Genetic Operations

Genetic operators, which are generally categorized as selection, crossover, and mutation, are used to create better solutions and replace them with existing solutions.

4.5.1 | Selection

The chromosome chosen for genetic operations is determined by the roulette wheel operator. Each chromosome in the population is given a selection probability proportional to its fitness value. The fitter chromosomes have lower cost values and subsequently have higher selection probabilities. The selection probabilities are determined based on the total fitness value (F) according to *Eq. (19)*.

$$F = \sum_{h=1}^{popsize} fitness_h. \tag{19}$$

The selection probability Ph for each chromosome h is:

$$Ph = \frac{F - fitness_h}{F \times (popsize - 1)}. \tag{20}$$

Then, a random number r is generated in the range (0,1]. If $qh-1 < r \leq qh$, then chromosome h is selected.

4.5.2 | Crossover

Crossover, the primary GA operator, reproduces individuals by combining the data of parents who were chosen at random in such a way that the resulting offspring exhibit traits from both parents. The fitness function is used to compare these offspring and pass the information on to the following generation. One-point, two-point, and uniform crossover are three different types of crossover that can occur between chromosomes. Cyclic permutations are the best candidates for the two-point crossover. Another method for more quickly covering the search space is to reorganize the chromosomes after the crossover process. In this situation, the Order Crossover (OX) operator, which has proven effective in a variety of routing-related situations [33], is chosen to be used. This kind of crossover occurs as described below:

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I. The first and last genes on the chromosome undergo a two-point crossover. Integer numbers are used to code the genes.
II. The two cut points' locations are chosen at random.
III. The genes between the cut points are first replicated in the offspring in the same order and location.
IV. The genes of the other parent are then copied in the same order, skipping the existing genes, starting from the second cut point of one of the parents.

The OX process to create the first offspring is demonstrated in the example in *Fig. 2*. The exchange of gene sequences between P1 and P2's cut points is what is done first. The first parent's genes from the second cut point are arranged in the following order: 5-7-6-1-3-4-2-8. After genes 7, 1, 4, and 2, which are duplicated in the first child, are deleted, a new sequence, 5-6-3-8, will be copied from the second cut point. The second child goes through the same procedure. The mentioned crossover operator is illustrated in *Fig. 3*.

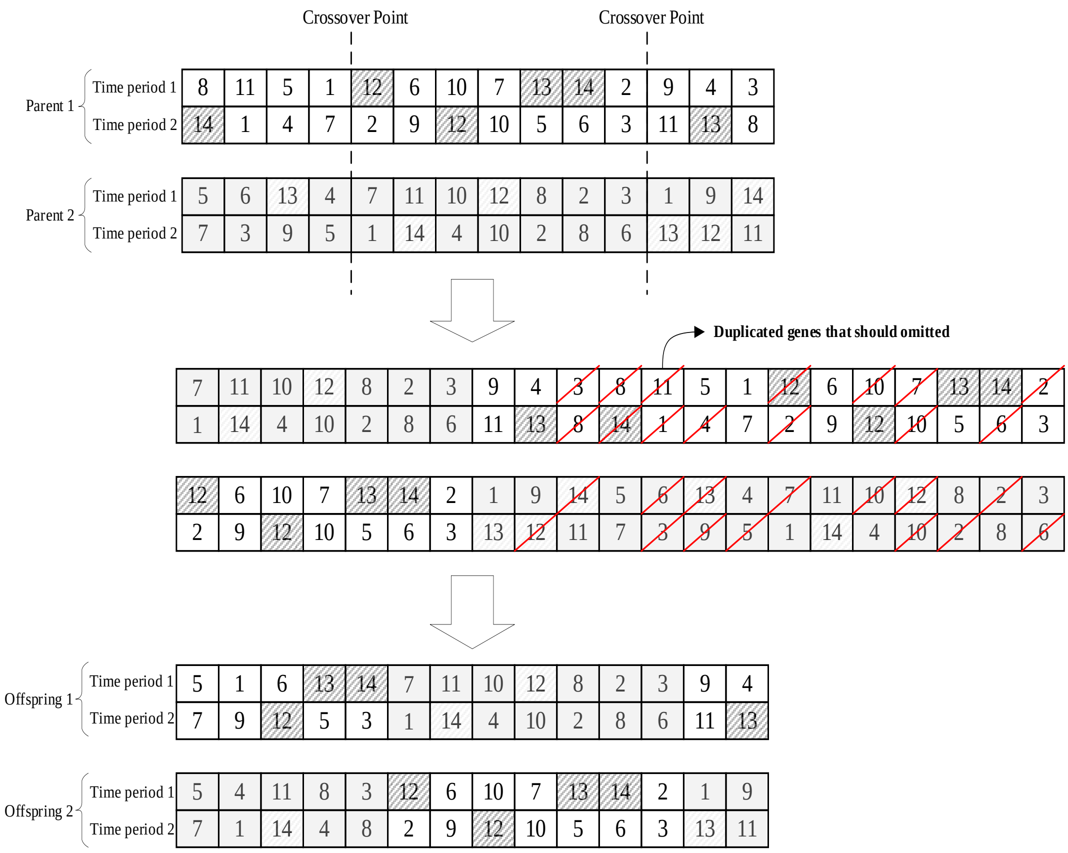

Fig. 3. Order crossover OX process.

4.5.3 | Mutation

Mutation is another genetic operator. The primary purpose of this operator is to investigate novel solutions in the solution space. Additionally, it is used to broaden the search space by randomly changing individual genes to avoid becoming trapped in local optima. In this study, three different mutations are used for the mutation process, as presented below:

Swap mutation

In swap mutation, two genes are selected randomly, and then their values are swapped in all time periods. Permutation is maintained, and perturbation is accomplished using swap mutation.

Reversion mutation

In reversion mutation, a part of the chromosome is reversed. First, two genes are randomly selected, and then all the gene values between the two selected genes are reversed. This procedure is done for all time periods.

Insertion mutation

In the insertion mutation, two genes are randomly selected, and then the value of the first selected gene in all time periods is inserted after the second selected gene.

Each proposed mutation can explore new solutions in the solution space that other mutations are not able to explore. To select the mutation, the roulette wheel operation has been used, which selects mutations with unequal probabilities. The selection probability of mutations is determined according to the performance of the algorithm. The three mentioned mutations are shown in *Fig. 4*.

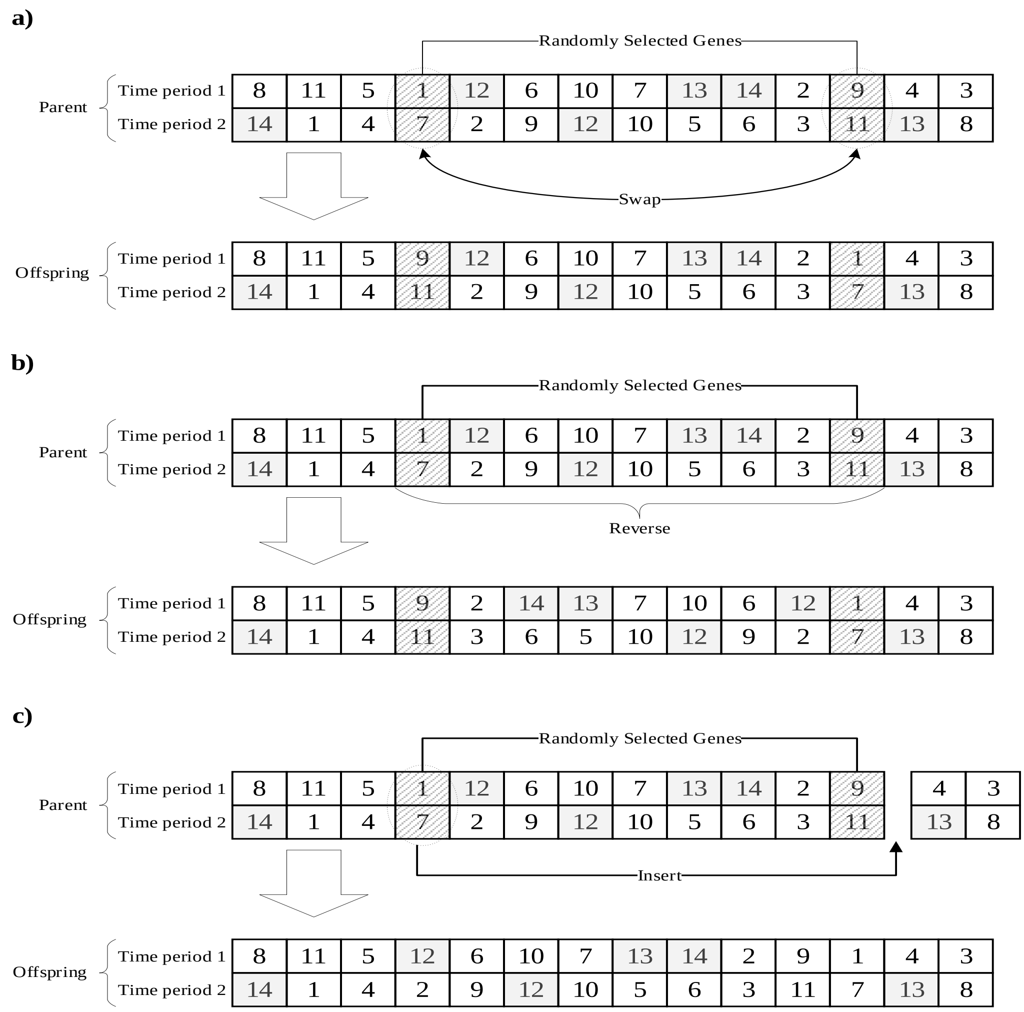

Fig. 4. Mutation operation: a. Swap; b. Reversion; c. Insertion.

4.5.4 | Stop condition

The algorithm stops when a predefined number of generations is reached.

5 | Taguchi-based Parameters Tuning

The appropriate configuration of parameters significantly impacts the effectiveness and efficiency of metaheuristic algorithms. Most research studies rely on either literature-based reference values or trial-and-error to set these parameters. Since the ideal algorithmic parameters vary depending on the specific problem, newly developed algorithms require tailored adjustments to their parameters, resulting in improved algorithmic solutions. To estimate the appropriate algorithmic parameters, the Taguchi method is employed for optimizing the proposed GA's parameters. The Taguchi experimental design has found widespread application in optimization problems. It relies on two main tools: the Orthogonal Array (OA) and the Signal-to-Noise (S/N) ratio. The OA represents a numerical matrix containing experimental plans based on various levels of factors. The robustness of this experimental design is ensured by the S/N ratio, which quantifies variation. In this context, "signal" refers to the desired value (mean response variable), while "noise" corresponds to the undesirable value (standard deviation) [35]. 1) the Taguchi method aims to minimize the impact of noise while simultaneously determining optimal levels for controllable parameters based on robustness [36], and 2) for minimization problems, the goal is to maximize the S/N ratio for each parameter i at its level j, as calculated by *Eq. (21)*.

$$\left(\frac{S}{N\ \text{ratio}}\right)_{ij} = -10\log_{10}\left(\frac{\sum Z_{ij}^2}{n}\right) \qquad \text{for all } i, j, \tag{21}$$

where n is the number of times level j of parameter i is repeated over the runs of all trials and Zij is the objective function value using parameter i on level j. *Algorithm 2* contains the proposed Taguchi method's pseudo code.

Algorithm 2. The pseudo code for Taguchi-based tuning of the proposed GA.

Procedure: Taguchi design for GA parameters
Input: Levels of *Iterarion-N, Population-size, Crossover-R, Mutation-R, Selection-P*
Output: Optimum level of parameters
begin
select $L_{16}(4^5)$ as the suitable OA;
apply GA on each sheme of $L_{16}(4^5)$;
obtain Z for each sheme;
for $i \neg$ 1 to (5) **do**
for $j \neg$ 1 to (4) **do**
calculate S/N ratio;
end
end
determine optimum level of parameters;
end

To implement the selected experimental design, the initial step in parameter configuration involves choosing the parameters that will act as controls and determining their corresponding levels. The proposed GA considers parameters such as population size, number of iterations, crossover rate, mutation rate, and selection pressure for fine-tuning. *Table 1* outlines the different levels in the tuning process for these parameters.

Table 1. Different levels of GA parameters used for turning.

Factors (GA Parameters)	Levels			
	1	2	3	4
Population size	80	100	120	140
Number of iterations	100	150	200	250
Crossover rate	0.7	0.75	0.8	0.85
Mutation rate	0.2	0.25	0.3	0.35
Selection pressure	6	7	8	9

L16 (45) was selected from the standard table of OAs. *Table 2* provides a summary of the sixteen different combinations of the constructed parameters and the outcomes attained through various parameter designs. In this table, the rows represent the parameter levels in each experimental scheme, and the columns represent the particular parameter levels that can be changed for each scheme.

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Table 2. Results obtained from different designs of Taguchi approach.

Design	GA Parameters Population-Size	Iteration-N	Crossover-R	Mutation-R	Selection-P	Fitness Function
1	80	100	0.7	0.2	6	5620.65
2	80	150	0.75	0.25	7	5201.23
3	80	200	0.8	0.3	8	5303.56
4	80	250	0.85	0.35	9	5905.84
5	100	100	0.75	0.3	9	5543.23
6	100	150	0.7	0.35	8	5757.52
7	100	200	0.85	0.2	7	5889.54
8	100	250	0.8	0.25	6	5856.69
9	120	100	0.8	0.35	7	5153.68
10	120	150	0.85	0.3	6	5914.68
11	120	200	0.7	0.25	9	5956.87
12	120	250	0.75	0.2	8	5380.56
13	140	100	0.85	0.25	8	5563.65
14	140	150	0.8	0.2	9	5152.84
15	140	200	0.75	0.35	6	5419.37
16	140	250	0.7	0.3	7	5682.75

Using Minitab 18, the proposed design was applied to each parameter at four levels. According to the mean for the answers and S/N ratio plot shown in *Fig. 5*, a good solution for the population size, number of iterations, mutation and crossover rates, and selection pressure is 120, 150, 0.3, 0.85, and 6, respectively.

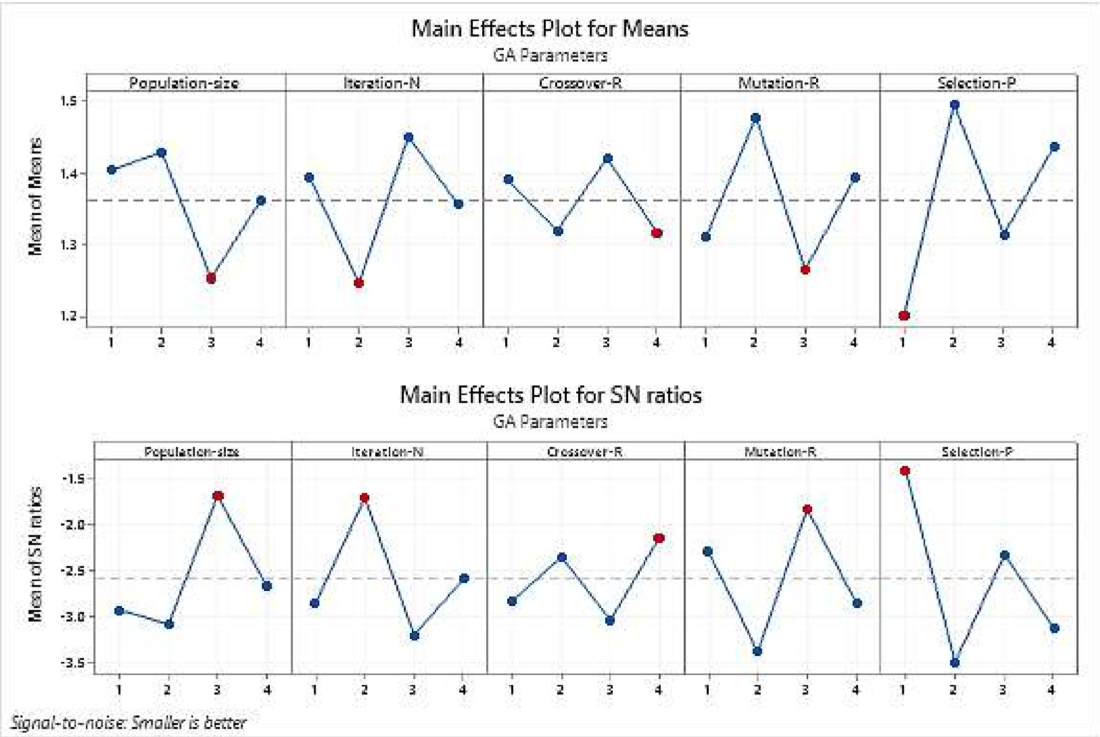

Fig. 5. Comparison of the mean for the answers and S/N ratios of the algorithms.

6 | Results Analysis

The performance of the developed GA was tested in this section. The modified GA has been implemented in MATLAB and run on a Core i5 and 8 GB RAM personal computer. The modified GA was tested on benchmark instances used in [37]. These instances are presented in two sizes: small with 5 to 50 customers and large with 50 to 200 customers. Each of the instances has been proposed over three and six time horizons. The OU policy was intended as a replenishment inventory policy, according to which every visit to a customer brings its inventory to the maximum level. In this research, relevant small instances have been used. The termination criterion is reaching the number of iterations, which is 150 iterations according to parameter tuning. Therefore, the number of iterations is fixed and running times are used for time comparison. To achieve average results of executed instances, each instance was run 30 times. By making the problem more complex, the exact algorithm is less able to solve it in a reasonable time. *Table 3* displays results of developed metaheuristic as well as those from earlier studies that were applied to same benchmark.

Table 3. Average result values obtained by the different resolution methods for the small-instances.

Instance	Optimal Solution	GA-OX	Time(s)	MA	Time(s)	Modified GA	Time(s)	ER
	Number of Time Periods = 3							
Small-5	1418.76	1418.76	71.95	1418.76	83.14	1418.76	68.11	0
Small-10	2228.67	2228.67	81.27	2228.67	98.49	2228.67	76.01	0
Small-15	2493.47	2493.47	118.41	2493.47	179.31	2493.47	99.12	0
Small-20	3053.02	3121.43	160.38	3053.02	221.17	3053.02	132.75	0
Small-25	3451.15	3451.15	220.20	3451.15	238.53	3456.14	185.85	0.144589
Small-30	3643.22	3643.22	232.84	3643.22	253.87	3643.22	201.31	0
Small-35	3846.87	3958.73	248.13	3848.46	281.28	3848.46	213.44	0.041332
Small-40	4125.70	4150.79	275.58	4136.57	296.43	4132.70	249.85	0.169668
Small-45	4270.61	4279.19	293.91	4279.19	311.27	4279.19	286.53	0.200908
Small-50	4810.87	4887.16	364.84	4811.92	409.31	4853.12	301.72	0.87822
	Number of Time Periods = 6							
Small-5	3299.98	3299.98	213.77	3299.98	249.23	3299.98	198.85	0
Small-10	4832.89	4832.89	259.19	4832.89	301.87	4832.89	236.32	0
Small-15	5566.39	5638.59	297.43	5566.39	366.30	5566.39	284.18	0
Small-20	6833.29	6838.42	427.23	6833.29	528.54	6838.42	361.68	0.075074
Small-25	7454.15	7475.88	483.59	7475.88	631.86	7462.11	416.79	0.106786
Small-30	7847.39	7899.12	713.74	7868.36	843.39	7877.52	506.60	0.383949

The size of the instance sets is shown in the first column. The second column shows the optimal solution to the problem solved in [37]. The third, fifth, and seventh columns show the average of the best outcomes obtained with GA-OX and MA of [33] and modified GA developed in this paper, respectively. The average computation time needed to obtain the results from GA-OX, MA, and modified GA is shown in the fourth, sixth, and eighth columns, respectively. The Error Rate (ER) of the modified GA, in the ninth column, was computed with *Eq. (22)*.

$$ER = 100 * \left(\frac{\text{total cost obtained using the modified GA} - \text{optimal solution}}{\text{optimal solution}}\right). \quad (22)$$

As can be seen, the developed algorithm shows better performance in solving instances. The calculated ER is 0% in many cases and up to 0.87% in other cases. The ER in these instances does not even reach 1%, which indicates good performance of the algorithm in the problem due to low time to solve it. Modified GA was able to accurately solve five instance sets of three delivery periods and four instance sets of six delivery periods. By examining the percentage of solution improvement in these instances, it shows the efficiency of the modified GA algorithm. Two algorithms, MA and GA-OX, have cumulative deviations of 1.09% and 9.86% from optimal solution in all instances, respectively. The deviation percentage of proposed algorithm is 1.14%, which is very good compared to GA-OX algorithm. Also, compared to MA algorithm, although difference is very small considering solving time, better performance is obtained from presented algorithm. Better performance of proposed algorithm compared to other two algorithms can be seen not only in value of objective function but also in time taken to reach solution of problem. Modified GA has achieved optimal or close to optimal solution in less time than MA and GA-OX in all instances. This issue can be seen in the diagram in *Fig. 6*. The effect of reducing the solution time will be more pronounced in larger samples.

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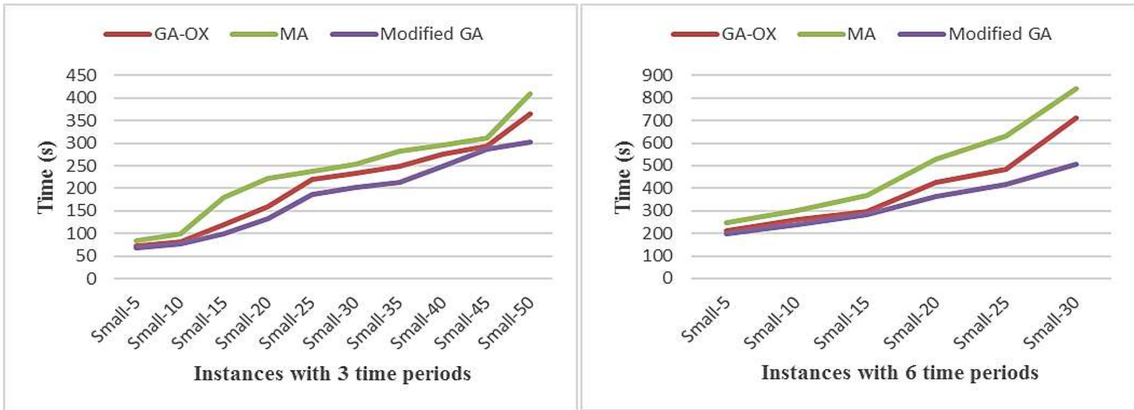

Fig. 6. Comparison of solving times of the algorithms.

Premature convergence in the local optimum is one of the problems of the GA that prevents reaching the global optimum. *Fig. 7* shows the convergence behavior of the proposed GA over 150 generations. This diagram is related to the solution of the first standard example. As it is clear in the figure, the algorithm has converged to local solutions in generations 22 to 30 and 36 to 50, but due to the defined mutation, the algorithm has been able to exit this convergence and continue the solution procedure. The solution converges to the global optimum at 61 generations, which is relatively fast.

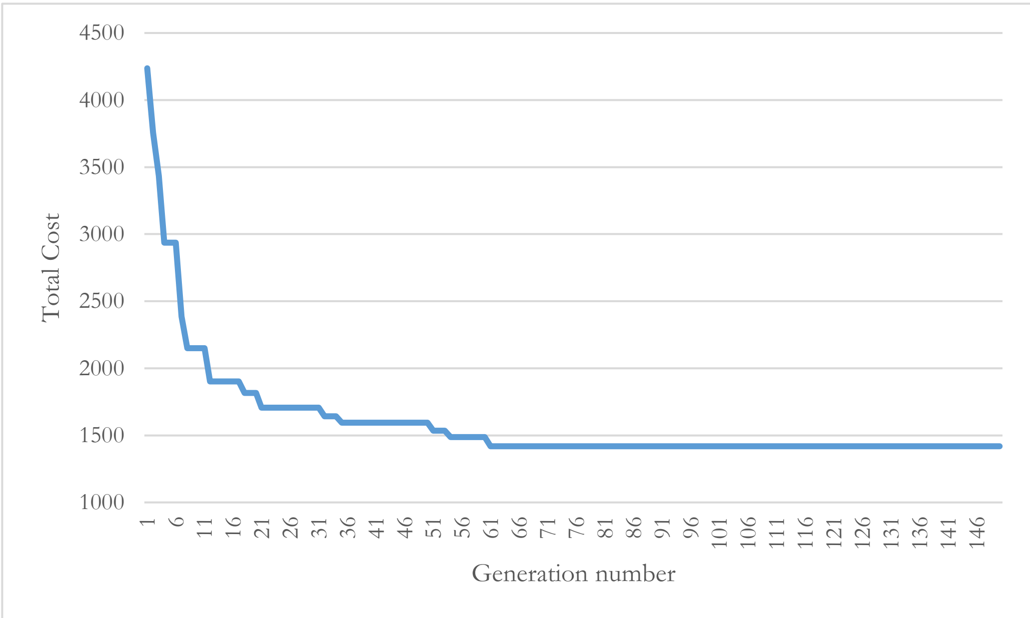

Fig. 7. Convergence behavior of the proposed GA over 150 generations.

7 | Conclusions

The IRP has gained much attention from practitioners in the literature. The problem's complexities imply more use of meta-heuristic algorithms to solve it. The GA is widely used to solve optimization problems. Various approaches have been employed for the chromosomal structure of the GA based on the under-investigation problem, all of which try to improve the performance of the problem algorithm. Introducing a novel chromosomal structure based on IRP, this study could use GA with better conditions. Simple chromosome structure, encrypting and decrypting the chromosome, and higher speed of the algorithm are some characteristics of this structure. To evaluate and investigate the algorithm performance, IRP has been modeled as a linear single-objective mixed integer programming. The parameters of the algorithm are adjusted and determined by using Taguchi method with 16 trials. To compare algorithm efficiency, 26 standard samples with small and large sizes available in the literature were used. Due to the random nature of the algorithm, each sample was solved 30 times, and the average values were used to perform comparisons. A comparison of the results with those in other studies shows that modified algorithm has good performance. In many cases, this algorithm provided higher-quality solutions than its counterparts. In all samples compared, presented algorithm performed better concerning time and could reach solutions in a shorter time. As a suggestion for the future, one can develop the same chromosomal structure for PRP. As well, this algorithm can be combined with others to improve performance.

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A Proportionately Increased Small and Equal Batch Delivery under Consignment Stock Agreement for a Single Vendor, Multiple Buyers Supply Chain System

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Abstract

Consignment stocks agreement had been very useful in inventory control. The benefit ranges from improved cash flow, reduced risk level, savings on investment, reduced ownership cost, low inventory carrying cost and regular restocking to mention few. Also, small batch delivery is an effective strategy for launching a product since it enables a business to assess the market and validate the product before committing to a large production run. In this paper, we combined small batch delivery and consignment stock policy by considering a supply chain setting where a vendor fulfilled the shipment requirement of each buyer sequentially in a single production set up. To achieve this, and as against the equal size shipments policy assumed in literature for different buyers, the vendor sends a smaller shipment first as early entry, followed by n equal shipments. These n shipments are proportionately increased according to the vendor rate of production to each buyer's demand rate. A mathematical cost function is developed to reduce the overall cost of the integrated supply chain system through the optimal cycle time and the optimal numbers of shipments to be delivered to each buyer. Numerical example is given using data from an existing literature, results were compared, and the new distribution policy gives better financial savings in terms of cost over the equal shipment policy assumed in literature. Sensitivity analyses were performed on key parameters to evaluate the robustness of the model.

Keywords: Consignment stock, Economic lot size, Distribution policy, Production rate, Financial savings, Demand rate.

1 | Introduction

Consignment inventory is an arrangement where the vendor stocks is kept in the consumer's repository without the seller transferring ownership to the buyer. It is carried out on a mutual agreement between both parties, who jointly benefit from the policy. Some of the benefits include good financial savings, regular restocking, flexible payment approach (upfront payment/profit sharing), reduced risk level and ease of launching new products. A small batch delivery on the other hand is a useful strategy for product entrance or introduction. It enables a business to test the market and validate the product before funds are invested into the large production run. A small batch delivery typically involves producing a limited quantity of the product and releasing it to a selected group of customers. It helps a company to avoid over production that can freeze up capital in inventory and create cash flow problems. By starting with a small quantity, a company can gauge

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customer demand and adjust production levels accordingly. Other benefits of small batch delivery include provision of platform to identify any production or quality issues before ramping up to full production. This can help ensure that the product meets customer expectations and avoids any potential recalls or quality issues.

Therefore, combining consignment stock agreement with small batch delivery can be a very profitable strategy in reducing the cost of the supply chain system. It allows for flexibility in batch size, while ensuring that the products are available for customers to purchase. This can help reduce the risk of overproduction, since the supplier can adjust production levels based on actual demand. It can also help increase sales and improve cash flow because the supplier can receive payment for the products as they are sold.

In view of the aforesaid, this research aimed at improving costs accruing from consignment stock agreement through a distribution policy that combines both small batch delivery and a proportionately increased equal batch size delivery for a single vendor, many buyers supply chain system. A review of past literature on consignment stock agreement shows that several policies involving equal and constant shipment size had been studied, but none as incorporated a flexible and proportionate shipment size and plan that allow supplier to spontaneously respond to the known buyers demand without shortages or any likely problem associated with bullwhip effect. To achieve this and like Zavanella and Zanoni [1], it is assumed that the vendor produces in a single set up and make delivery to the buyers sequentially. Each buyer consecutively received their total lot size per cycle over n+1 shipments, after which delivery is made to the next buyer. The sequence of customer arrangement is subjective and solely depends on the vendor discretion. The lots size to be deliver to each buyer consist of one (1) small shipment and n equal sized shipments. The equal shipments size is achieved by increasing the first smaller shipment proportionately by the vendor rate of production to the individual buyer's demand rate.

The rest of the paper is arranged as follows: Section 2 reviews literatures that are related to consignment stock policy and integrated modelling. Section 3 defines the problem formally and presents the proposed mathematical formulation. The mathematical formulation is solved, and the obtained results is compared with the equal shipment policy addressed by Zavanella and Zanoni [1] in Section 4. Section 5 presents sensitivity analysis on important parameters. Summary and conclusions are presented in the last Section 6.

2 | Literature Review

Different lot sizing problems had been addressed in literature. This ranges from single-vendor, single-buyer problems to single vendor, many buyers supply chain system. Some models that are applicable to this research work are discussed below.

2.1 | Single Vendor, Single Buyer

Goyal [2], [3] proposed a joint economic lot-size model that is aimed at minimising the overall costs incurred by both the buyer and vendor. Generalisation of this model was done later by Banerjee [4] and [5], Goyal [6] and Goyal et al. [7]. There models assumed equal power between the buyer and the vendor through a contractual deal. Lu [8] developed a model to reduce vendor overall cost at the expense of the highest cost the buyer might incurred.

Later, Hill [9], [10] considered minimising the total costs per year when buyer's demand and frequency of ordering is known to the vendor. This model is suitable when partnership exist between the two parties.

Goyal [11] improved the approach used in finding the best policy for the joint economic lot size problem when capacity constraint through transport equipment is considered. Valentini and Zavanella [12] discussed an industrial setting and a performance review of the consignment stocks agreement. Performance appraisal of the consignment stock agreement using analytical means was proposed by Braglia and Zavanella [13]. Zanoni and Grubbstrom [14] proposed a detailed analytical solution.

Ben-Daya and Hariga [15] assumed stochastic demand and equally defined the lead time as a linear function of the ordered lots and a fixed delay time. A heuristic approach for minimising the holding, lead-time reduction and ordering cost for an integrated system under adjustable lead time assumption between a vendor and a buyer was proposed by Hoque and Goyal [16]. Hill and Omar [17] reviewed literature works on the single vendor, single buyer inventory problem, and through different batch sizes within a replenishment cycle, they suggested a solution methodology that improved the cost attributed to the consignment stock case.

Sarmah et al. [18] identified models addressing buyer–vendor synchronization under deterministic settings. The models were classified based on the issues they addressed and the future line of research. Zhou and Wang [19] proposed a model that requires no numerical difference between buyer's and vendor's unit holding cost. Also, no shipment policy is assumed. The model, however, considered shortages and product deterioration from the buyer's inventory. Islam [20] studied consignment stock policy for a seasonal product. The authors, however, considered the selling period average inventory cost in formulating their profit function. Jaber et al. [21] showed the impact of collection and repairable rate of used items on cost and batch size under consignment stock agreements. The authors modified the remanufacturing, waste disposal and production model through additional purchaser to the vendor's system. Zanoni et al. [22] showed that the consignment stock agreement with Vendor managed Inventory (VMI) policy performed well under the emission trading plan than the traditional joint economic lot size model through the GHG emissions tax and penalty cost that was added to the buyer and vendor cost function. Giri et al. [23] proposed a joint economic lot size model where decision on customer's demand is influenced by the on-hand stock available with the buyer. The model is later generalised to include the space capacity constraint at the buyer's side. The impact of buyer's limited space on the optimal number of shipments, batch size and total cost were studied.

Zahran et al. [24] studied payments delay under consignment stock agreement settings. As a base case situation, an equal payment, equal interval scheme was considered against two possible delay payment plans, with or without interest rate. The results showed that accepting slight delay in payment on the side of the buyer is more advantageous in the system than making payments to the vendor at the time of invoicing. Khan et al. [25] developed a model to investigate the impact of product screening on defectives items and storage cost under both consignment stock agreement and VMI policy. Giri et al. [26], considered equal and unequal sized delivery from a single vendor to a single buyer under VMI and consignment stock policy. The authors considered warranty cost on the side of the vendor whose production system may produce defective items that are discovered during the screening phase by the buyer. The average expected profit is modelled mathematically, and a solution technique is proposed to determine the best possible number of delivery shipment from the vendor. An integrated model that studied consignment stock policy and variable production rate under random demand settings was developed by Aldurgam et al. [27]. The vendor's products that are delivered in full truck load are stored in the manufacturer's warehouse, where they serve as raw material for the manufacturer's product. The model was solved to determine the most economic production lot size, production rate, re-order level and number of full trucks. Islam et al. [28] developed a profit maximizing consignment model that considers realistic factors like shipping time inventory, work in process inventory, selling period inventory of sold products, transhipment cost and several other factors. Hariga et al. [29] investigated the effects of two carbon reduction strategies (carbon tax and carbon cap) on supply chain costs and carbon emissions under vendor managed consignment inventory arrangement. In order to have a more sustainable production process, Zavanella et al. [30] took into consideration energy-related objectives in lot sizing. Gharaei et al. [31] investigated quality control and green policies in a supply chain that was subject to sanctions with vendor-managed inventory and a consignment stock agreement. Giri and Masanta [32] investigated a learning-and-forgetting manufacturer production system with a random return rate for used goods and an inspection method to identify components that qualify for remanufacturing. Early, and late delivery when coordinating a two-level supply chain system were considered by Çömez-Dolgan et al. [33]. Hemmati et al. [34] evaluated bundling and separate sales for two related items under VMI with consignment stock policy. Sen et al. [35] took into account the channel members' (i.e., seller and buyer) warehouse space restrictions under both n-shipment and consignment stock policies in cases when the goods degrade. Marchi et al. [36] revisited consignment stock and delay payment. A methodology for setting appropriate lower and upper inventory limits for the buyer, taking into account the heterogeneity in the supplier base, was developed by Bogaert and Jaarsveld [37]. Zhang et al. [38] considered cap-and-trade regulation in a closed loop supply chain, under consignment stock agreement. Asadkhani et al. [39] integrate coordination, quality requirements and environmental issues in a vendor-buyer supply chain system, under Vendor-Managed Inventory with Consignment Stock (VMI-CS) agreement. The interaction between existing and new products in a diffusion process was studied by Keshavarz and Hamid [40]. Hemmati et al. [41] studied the effect of two points deterioration at the buyer's end under for VMI-CS agreement. The benefit of just-in-time, delay and advance for buyer's delivery under vendor and buyer space limitation were studied by Ambroszkiewicz and Bylka [42].

2.2 | Single Vendor, Multiple Buyer

Like the single-vendor, single-buyer integrated model, the single-vendor, many-buyers model has been of interest to many scholars presently and past. Lai and Staelin [43] proposed a quantity discount model for a seller dealing with many identical buyers. Joglekar [44] improved the work of Lai and Staelin [43] by showing that in many-purchase situation, purchasers' order quantity affects both the revenue stream of the vendor and the cost stream of the manufacturer. Joglekar and Tharthare [43] proposed an independent and logical decision approach for determining the economic lot sizes for a vendor and several buyers. The authors argued that collaboration recommended in literature negate the free enterprise system, and thus support the autonomy of each party to adopt its own independently derived optimal replenishment policy.

Banerjee et al. [45] later proposed a model that coordinates inventory between multiple buyers and one vendor, when trading with a single product, under stochastic demands and lead times via a common cycle approach. The authors centred more on Electronic Data Interchange (EDI) system as a means of having instantaneous communication between the vendor and multiple buyers under a pre-determined agreement and a pre-arranged decision system. Lu [8] object previous assumption that the buyer's ordering and holding costs must be known to the vendor, which are sometimes difficult to estimate. Lu thus proposed, a model that minimises the total cost of the vendor per year, subject to the highest cost the buyer might incurred.

Viswanathan and Piplani [46] developed a model to explore benefits of coordinating supply chain inventories under joint replenishment time for all buyers. The authors, however, failed to include the vendor inventory cost in the model. Woo et al. [47] continued the work of Banerjee [4] by considering a joint investment by the vendor and the buyers to reduce the ordering cost.

Boyaci and Gallego [48] examined the pricing and inventory policies that commonly improve the channel profit of a supply chain system under deterministic price-sensitive customer demand. The authors' showed through consignment stocks agreement how an optimal policy can be implemented jointly. Furthermore, Siajadi et al. [49] showed that for joint economic lot sizing problem, the single shipment policy is less advantageous to the multiple shipment policy. Kim et al. [50] examined a three stage multi echelon supply chain system. The last stage consists of numerous retailers that are interacting with only one buyer, who makes raw material purchase at the first and single-resource level. Zavanella and Zanoni [1] formulated a model that determines the optimal replenishment decision for a single-vendor, multiple-buyers under consignment stock agreement. There results proved that the consignment stock agreement is of higher benefit than the uncoordinated optimization. Controllable lead time under consignment stock agreement was investigated by Srinivas and Rao [51] as a strategy for reducing the expected overall cost of a supply chain system through the optimization of variables like quantities transported, delay deliveries and numbers of transportation. Hariga et al. [52] considered scheduling problems for multiple delivery under single vendor, multiple buyers' consignment stock arrangement. The problem was formulated using nonlinear mixed integer programming and a heuristic procedure was proposed to give a near optimal schedule. The solution obtained gives a significant savings that increases with the number of buyers. Bendaya et al. [53]

studied different vendor, buyer partnership under consignment stock agreement and the result obtained from their formulation showed that CS agreement is more advantageous when the vendor operates a flexible system, and the frequency of shipments is inversely related to the lot size. Mandal and Giri [54] considered both imperfection in the vendor production system and an adjustable lead time through a crashing cost paid by the buyer. Fauza et al. [55] proposed an integrated approach to address food inventory policy and supply chain under different quality characteristics. Variable production rate and imperfect quality of products under consignment stock agreement was considered by Sarker et al. [56]. Omar and Zulkipli [57] assumed the demand rate is positively dependent on the level of items displayed. Vendor imperfect production process as induced by dependent demand was considered by Guchhait et al. [58]. Chan et al. [59] synchronized the lengths of the buyers' ordering cycles and the vendors' production cycles under stochastic demand settings. Bendaya et al. [60] considered the remanufacturing of commercially viable used products recovered from the end consumer in a two-stage single vendor and multiple purchasers closed loop supply chain under a centralized consignment stock agreement. Delivery route and cost of emission of greenhouse gases were considered by Castellano et al. [61]. Agustiandi et al. [62] took into account warehousing, capital, and service level constraints. Castellano et al. [63] study the impact of controllable lead time and backorders-lost sales mixture. Charkraboty et al. [64] study a VMI contract wherein buyers charge the supplier a penalty endogenously on the excess of inventory supplied each time the provider exceeds some predetermined inventory level. Recently, Adegbola [65] studied a single vendor, multiple buyers supply chain problem under stochastic demand, full truck load assumption using simulation optimization. The author evaluated two distribution policies (JRP and VMI) and further showed that the coefficient of variation should be considered as a judgment criterion of when to embrace simulation modeling ahead of other modeling techniques.

Therefore, from the survey, it is obvious that the closest research work is the work of Zavanella and Zanoni [1]. The aim of this research work is to evaluate another coordinated product delivery policy in which a small batch delivery is combined with consignment stock agreement. Each buyer independently received one (1) small size shipment to launch an entry into the market, and n equal shipments that is increased proportionately according to the ratio of the vendor production rate to each manufacturer demand rate. A mathematical model is formulated for the proposed policy and a numerical example is given using the same data from Zavanella and Zanoni [1].

3 | Notations and Assumptions

Like Zavanella and Zanoni [1], the following notations and assumptions are employed in developing the analytical model.

3.1 | Model Assumptions

- *A cycle refers to the period used by the vendor to produce the total lot size required by Y buyers in a one set up.*
- *The cycle is repeated homogeneously within all period considered.*
- *The Vendor production rate outweigh the joint demand rate of the buyers i.e.,* $p > D$ *where* $D = \sum_i^Y d_i$.
- *Each buyer received a smaller lot on first shipment after which they receive an increased equal sized shipment (the numbers of which may differ for different buyers as shown in Fig. 1). Production continues and the final shipment is made immediately production finishes.*
- *The equal shipments received by each buyer after the first smaller shipment is this smaller shipment size increased by the ratio of the vendor production rate to individual buyer's demand rate i.e.,* $k = \frac{P}{d_i}$.

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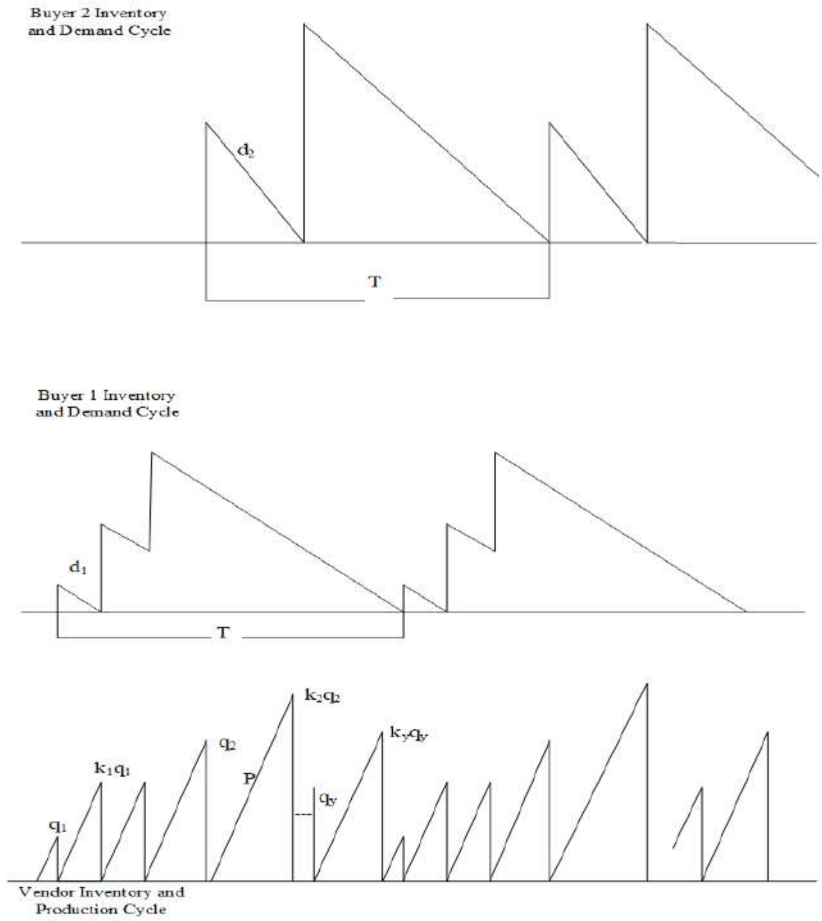

Fig. 1. The proposed inventory profile for the single vendor, multiple buyers' consignment stock policy.

3.2 | Notations

A_1	Vendor set up cost (€/set up).
$A_{2,i}$	Buyer ith ordering/emission cost (€/order).
h_1	Holding cost of vendor per item and per time unit (€/item time unit).
$h_{2,i}$	Holding cost of buyer ith per item and per unit time (€/item time unit).
p	Vendor rate of production (item/time unit).
d_i	Demand rate of buyer i_{th} buyer (continuous rate) (item/time unit).
Y	Number of buyers.
T	Cycle time (time unit).
n_i	Numbers of shipment delivery transported to buyer i per cycle time.
q_i	Buyer i_{th} batch size received per delivery from the vendor.
$T.C$	The average total cost per unit time.

3.3 | Mathematical Model Development

In this part, we derive the total cost function for the proposed single vendor, multiple buyers integrated inventory model in line with the assumptions stated above. For simplicity of modelling, we first consider single-buyer, single-vendor case, a situation where i=1, after which the model is generalized to multiple-buyers case i.e., i=Y. All cost identified in the proposed model are enumerated below.

I. Vendor set up cost: The vendor produces the lot sizes sent to all buyers in a single set up per cycle.

$$\frac{A_1}{T}. \tag{1}$$

II. Vendor holding cost: This is the cost of holding a given level of inventory in stock during each production runs. Since different lot sizes are shipped to each buyer, it consists of one (1) small lot size and n increased lot size to be deliver to each independent buyer.

$$\frac{h_1 T di^4}{2(n_i p + di)^2}\left(\frac{n_i p}{di^2} + \frac{1}{P}\right). \tag{2}$$

The cost incurred by the vendor is the summation of both his set up cost and holding cost.

$$T.C_{vendor} = \frac{A_1}{T} + \frac{h_1 T di^4}{2(n_i p + di)^2}\left(\frac{n_i p}{di^2} + \frac{1}{P}\right). \tag{3}$$

For all Y Buyers, the total cost per unit time incurred by the vendor is.

$$T.C_{\ vendor)} = \frac{A_1}{T} + \sum_{i=1}^{Y} \frac{h_1 T di^4}{2(n_i p + di)^2}\left(\frac{n_i p}{di^2} + \frac{1}{P}\right). \tag{4}$$

III. Ordering emission cost of each buyer:

$$n_i + 1)\frac{A_{2,i}}{T}. \tag{5}$$

IV. Holding cost of each buyer: To compute each buyer average inventory holding cost, the buyers' end of the profile shown in *Fig. 1* was analysed based on the inventory accumulation from the $q_i(1 + n_i k_i)$ lot size received from the vendor. The buyers' profile in *Fig. 1* was partitioned into one (1) small triangle,$\frac{n(n-1)}{2}$ rectangles, (n-1) triangle and one big triangle.

$$\frac{T d_i^2 h_{2,i}}{(n_i p + d_i)^2}\left(\frac{n_i p}{2}\left(\frac{n_i p}{d_i} + 1 - n_i\right) + \frac{d_i}{2}\right). \tag{6}$$

For all Y Buyers, the total cost incurred is the summation of the emission cost and the inventory holding cost for each buyer.

$$TC_{(buyer)} = \sum_{i=1}^{Y}(n_i + 1)\frac{A_{2,i}}{T} + Th_{2,i}\sum_{i=1}^{Y}\frac{d_i^2}{(n_i p + d_i)^2}\left(\frac{n_i p}{2}\left(\frac{n_i p}{d_i} + 1 - n_i\right) + \frac{d_i}{2}\right) \tag{7}$$

The total cost of the integrated supply chain system under the proposed distribution policy is the combined cost of the buyers and the vendor,

$$T.C_{system} = TC_{(vendor)} + TC_{(buyer)}.$$

$$\begin{aligned} TC_{system} = {} & \frac{A_1}{T} + \sum_{i=1}^{Y}\frac{h_1 T d_i^4}{2(n_i p + di)^2}\left(\frac{n_i p}{di^2} + \frac{1}{P}\right) + \sum_{i=1}^{Y}(n_i + 1)\frac{A_{2,i}}{T} \\ & + Th_{2,i}\sum_{i=1}^{Y}\frac{d_i^2}{(n_i p + d_i)^2}\left(\frac{n_i p}{2}\left(\frac{n_i p}{d_i} + 1 - n_i\right) + \frac{d_i}{2}\right). \end{aligned} \tag{8}$$

Furthermore, the objective here is to minimize the cost function, subject to having a complete and sequential delivery to all buyers i.e., the first buyer received his shipments completely before other buyers are considered per cycle. Precedence is however not given to the buyer's arrangement or chronological sequence, as this is subjective and solely depend on the discretion of the vendor. The two decision variables are the cycle time (T) and the number of delivery shipments $(1 + n_i)$ that minimizes the total cost of the integrated supply chain system. A joint optimum solution technique is proposed to find the best possible values for the decision variables, and the results obtained are compared with that from Zavanella and Zanoni [1] using the same data, so as to estimate the financial savings.

4 | Solution Method, Numerical Example and Sensitivity Analysis

4.1 | Joint Optimum Solution Approach

This approach optimizes the cost of the integrated supply chain system centrally. The cost is minimized by taking the first derivative of TC_{system} with respect to T, we have,

$$\frac{\partial T.C}{\partial T} = 0. \tag{9}$$

$$\frac{\partial T.C}{\partial T} = \frac{-1}{T^2}\left(A_1 + \sum_{i=1}^{Y} A_{2,i}\, n_i + 1)\right) + \frac{h_1}{2}\sum_{i=1}^{Y}\left(\frac{n_i P d_i^2}{(n_i p + d_i)^2} + \frac{di^4}{P(n_i p + d_i)^2}\right)$$
$$+ \frac{1}{2}\sum_{i=1}^{Y} h_{2,i}\left(\left(\frac{n_i^2 p^2 d_i}{(n_i p + d_i)^2} + \frac{n_i p di^2}{(n_i p + d_i)^2} - \frac{n_i^2 d_i^2 p}{(n_i p + d_i)^2}\right)\right. \tag{10}$$
$$+ \frac{d_i^3}{(n_i p + d_i)^2}) = 0\,.$$

$$= \frac{-1}{T^2}\left(A_1 + \sum_{i=1}^{Y} A_{2,i}\, n_i + 1)\right) + \sum_{i=1}^{Y}\frac{h_1}{2(n_i p + d_i)^2}\left(n_i P d_i^2 + \frac{di^4}{P}\right)$$
$$+ \sum_{i=1}^{Y}\frac{h_{2,i}}{2(n_i p + d_i)^2}\left(n_i^2 p^2 d_i + n_i p d_i^2 - n_i^2 d_i^2 p + d_i^3\right) = 0. \tag{11}$$

$$\frac{1}{T^2}\left(A_1 + \sum_{i=1}^{Y} A_{2,i}\, n_i + 1)\right)$$
$$= \sum_{i=1}^{Y}\frac{h_1}{2(n_i p + d_i)^2}\left(n_i P d_i^2 + \frac{di^4}{P}\right) \tag{12}$$
$$+ \sum_{i=1}^{Y}\frac{h_{2,i}}{2(n_i p + d_i)^2}\left(n_i^2 p^2 d_i + n_i p d_i^2 - n_i^2 d_i^2 p + d_i^3\right).$$

$$\left(A_1 + \sum_{i=1}^{Y} A_{2,i}\, n_i + 1)\right)$$
$$= T^2\,\frac{\left[h_1 \sum_{i=1}^{Y} d_i^2 \frac{(n_i P^2 + d_i^2)}{n_i P + d_i)^2} + P\left(\frac{\sum_{i=1}^{Y} h_{2,i}\, d_i\left(d_i^2 - Pn_i\; n_i - 1)d_i + n_i^2 p^2\right)}{n_i P + d_i)^2}\right)\right]}{2P} \tag{13}$$

$$T^2 = \frac{\left(A_1 + \sum_{i=1}^{Y} A_{2,i}\, n_i + 1)\right)}{\frac{\left(h_1 \sum_{i=1}^{Y} d_i^2 \frac{(n_i P^2 + d_i^2)}{n_i P + d_i)^2}\right) + P\left(\frac{\sum_{i=1}^{Y} h_{2,i}\, d_i\left(d_i^2 - Pn_i\; n_i - 1)d_i + n_i^2 P^2\right)}{n_i P + d_i)^2}\right)}{2P}}. \tag{14}$$

$$T^* = \sqrt{\frac{2P\left(A_1 + \sum_{i=1}^{Y} A_{2,i}\, n_i + 1)\right)}{\left(h_1 \sum_{i=1}^{Y} d_i^2 \frac{(n_i P^2 + d_i^2)}{n_i P + d_i)^2}\right) + P\left(\frac{\sum_{i=1}^{Y} h_{2,i}\, d_i\left(d_i^2 - Pn_i\; n_i - 1)d_i + n_i^2 P^2\right)}{n_i P + d_i)^2}\right)}} \tag{15}$$

If

$$x=\left(A_1+\sum_{i=1}^{Y}A_{2,i}\ n_i+1)\right),y$$

$$=\frac{\left(h_1\sum_{i=1}^{Y}d_i^2\frac{(n_iP^2+d_i^2)}{n_iP+d_i)^2}\right)+P\left(\frac{\sum_{i=1}^{Y}h_{2,i}\,d_i\left(d_i^2-Pn_i\ n_i-1)d_i+n_i^2P^2\right)}{n_iP+d_i)^2}\right)}{2P}$$

$$\begin{aligned}T.C_{system}=&\frac{\sqrt{y}}{\sqrt{x}}\left(A_1+\sum_{i=1}^{Y}A_{2,i}\ n_i+1)\right)\\&+\frac{\sqrt{x}}{\sqrt{y}}\left(\sum\nolimits_{i=1}^{Y}\frac{h_1}{2(n_ip+d_i)^2}\left(n_iPd_i^2+\frac{di^4}{P}\right)\right.\\&\left.+\sum_{i=1}^{Y}\frac{h_{2,i}}{2(n_ip+d_i)^2}\left(n_i^2p^2d_i+n_ipd_i^2-n_i^2d_i^2p+d_i^3\right)\right).\end{aligned} \tag{16}$$

$$\begin{aligned}&T.C_{system}\\&=\frac{\sqrt{y}}{\sqrt{x}}\left(A_1+\sum_{i=1}^{Y}A_{2,i}\ n_i+1)\right)\\&+\frac{\sqrt{x}}{\sqrt{y}}\left(\frac{\left(h_1\sum_{i=1}^{Y}d_i^2\frac{(n_iP^2+d_i^2)}{n_iP+d_i)^2}\right)+P\left(\frac{\sum_{i=1}^{Y}h_{2,i}\,d_i\left(d_i^2-Pn_i\ n_i-1)d_i+n_i^2P^2\right)}{n_iP+d_i)^2}\right)}{2P}\right).\end{aligned} \tag{17}$$

Since,

$$=\frac{\sqrt{m}}{\sqrt{x}}x+\frac{\sqrt{x}}{\sqrt{m}}m=2\sqrt{xm}.$$

$$T.C_{system}=$$

$$2\sqrt{\frac{1}{2P}\left(\left(A_1+\sum_{i=1}^{Y}A_{2,i}\ n_i+1)\right)\right)\left(\left(h_1\sum_{i=1}^{Y}d_i^2\frac{(n_iP^2+d_i^2)}{n_iP+d_i)^2}\right)+P\left(\frac{\sum_{i=1}^{Y}h_{2,i}\,d_i\left(d_i^2-Pn_i\ n_i-1)d_i+n_i^2P^2\right)}{n_iP+d_i)^2}\right)\right)} \tag{18}$$

To find both the optimal number of shipments n_i received by each buyer, and the cycle time T that minimizes the total cost function, the pseudocode below is used. n_i is an integer that is determined by performing a line search over a reasonable range to minimize the cost function of the integrated supply chain system. The optimal cycle time T is then determined by substituting n_i^* back into *Eq. (15)*.

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Algorithm 1. A pseudocode algorithm proposed to solve the small batch, equal shipment size model.

$T.C^*_{system} = big\ J$

$For\ n_i = 1: step\ size\ of\ 1: n_{max} \quad \forall\ i = 1,2,3, \dots \dots \dots \dots \dots, Y$

$Compute\ T.C_{system}$

$$= 2\sqrt{\frac{1}{2P}\left((A_1 + \sum_{i=1}^{Y} A_{2,i}(n_i+1))\right)\left(\left(h_1 \sum_{i=1}^{Y} d_i^2 \frac{(n_i P^2 + d_i^2)}{(n_i P + d_i)^2}\right) + P\left(\frac{\sum_{i=1}^{Y} h_{2,i} d_i \left(d_i^2 - P n_i (n_i - 1) d_i + n_i^2 P^2\right)}{(n_i P + d_i)^2}\right)\right)}$$

$if\ \ T.C_{system} \le T.C^*_{system}$

$T.C^*_{system} = T.C_{system}\ (n_i^*) \qquad for\ all\ i = 1,2,3, \dots \dots \dots \dots \dots, Y$

$n_i^* = n_i \qquad for\ all\ \ i = 1,2,3, \dots \dots \dots \dots \dots, Y$

$End\ if$

$$Compute\ T^* \quad = \sqrt{\frac{2P\left(A_1 + \sum_{i=1}^{Y} A_{2,i}(n_i^*+1)\right)}{\left(h_1 \sum_{i=1}^{Y} d_i^2 \frac{(n_i^* P^2 + d_i^2)}{(n_i^* P + d_i)^2}\right) + P\left(\frac{\sum_{i=1}^{Y} h_{2,i} d_i \left(d_i^2 - P n_i^* (n_i^* - 1) d_i + n_i^{*2} P^2\right)}{(n_i^* P + d_i)^2}\right)}}$$

$End\ for$

$n_i^*, T^* = argmin\ T.C_{system}$

$min\ \ T.C_{system} = min\{T.C_{system}[T^*, n_i^*]\}\ \ for\ all\ \ i =$

$1,2,3, \dots \dots \dots \dots \dots, Y$

4.2 | Numerical Example

The model was evaluated using data in *Table 1* as extracted from Zavanella and Zanoni [1].

Table 1. Model data acquired from Zavanella and Zanoni [1].

P	3200 units/year
D	1500 units/ year
d_1	500 unit/ year
d_2	1000 unit / year
A_{1}	€400 / setup
A_{21}	€75 /order
A_{22}	€25 / order
h_1	€5/item/year
h_{21}	€4/item/year
h_{22}	€4 /item /year

The results obtained were then compared with that of Zavanella and Zanoni [1] as the base case and it was observed that the proposed model perform better in terms of financial savings.

Table 2. Comparison of results of the equal size shipment policy with those obtained from the proposed small batch, relatively increased equal shipment size policy.

Policy Type	n_1	n_2	T	TC (Vendor)	TC (Buyer 1)	TC (Buyer2)	TC (System)
Equal size shipment policy	1	3	0.424	1134.1	601.7	849.9	2585.7
	$1 + n_1$	$1 + n_2$	T	TC (vendor)	TC (Buyer 1)	TC (Buyer2)	TC(System)
Small batch and relatively increased equal shipment policy	1	3	0.443	1125.0	612.4	744.8	2482.2

Furthermore, *Table 3* shows that the new policy gives better financial savings in terms of cost, and this is about 4% for the whole system, 1% for the vendor and 13% for buyer 2. Buyer 1 however had 2% increment in cost due to high holding cost resulting from his low demand rate, as the cycle time for all participants is relatively constant.

Table 3. Financial savings in percentage as obtained by comparing the proposed gradually increased shipment policy with the equal sized shipment proposed by Zavanella and Zanoni [1].

TC (Saving)	Vendor (Saving)	Buyer 1 (Increment)	Buyer 2 (Saving)
4%	1%	2%	13%

5 | Sensitivity Analysis

Using the previous example as the base case scenario, we change some model parameters to examine their effects on the decision variables and also, to evaluate the robustness of the proposed model.

Effect of vendor holding cost h_1

Table 4 illustrates how the vendor holding cost has an impact on the proposed policy. It is obvious that by raising the vendor unit holding cost, the cost at the buyer decreases, while the cost incurred by the vendor together with the overall cost of the integrated supply chain system increases. The reason for this is that any increase in the unit holding cost of the vendor reduces the joint cycle time. This compelled the vendor to reduce the shipment size delivered to each buyer, so as to prevent a high surge in costs that could amass from holding and sending a bigger constant lot size to the individual buyer. Meanwhile, if we compared the result obtained from the small batch with proportionate increase equal shipment policy with those obtained from the equal and constant sized shipment policy found in literature; the former gives better financial savings in term of cost. The gain ranges from 3.7% to 4.3% depending on the choice of other parameters.

Table 4. Effect of vendor holding cost h_1.

Parameter	Small Batch With Relatively Increased Equal Shipment Policy							Equal Size Shipment Policy							
h_1	$1+n_1$	$1+n_2$	T	TC (Vendor)	TC (Buyer 1)	TC (Buyer 2)	TC (sys)	n_1	n_2	T	TC (Vendor)	TC (Buyer 1)	TC (Buyer 2)	TC (sys)	% Savings
4.5	1	3	0.447	1096	643.97	748.5	2459.8	1	3	0.428	1109.0	603.6	853.7	2566.3	4.30
5	1	3	0.443	1125	641.2	744.8	2482.2	1	3	0.425	1134.1	601.7	849.8	2585.7	4.20
5.5	1	3	0.439	1153	638.5	741.2	2504.3	1	3	0.422	1159.0	599.9	846.2	2605.0	4.00
6	1	3	0.435	1180	635.9	737.8	2526.5	1	3	0.419	1183.5	598.1	842.6	2624.2	3.90
6.5	1	3	0.432	1208	633.5	734.4	2548.0	1	3	0.416	1208.0	596.4	839.1	2643.2	3.70

Effect of vendor set-up cost A_1

The effect of the vendor set up cost (A_1) was studied in *Table 5*, the model responded through an increase in the joint cycle time, which is an indication that the shipment lot size or the number of shipment delivery increases. From *Table 5*, an increase in the vendor set up cost from 350 to 400 as example, retained the number of shipment deliveries, but it increases the size of shipments lots to be delivered at each buyer. This resulted in higher costs at the vendor and buyer, which combined to increase the total cost of the supply chain system. A proportionate increment in the quantities delivered is necessary to balance out the increment in the set-up cost so as to have an economy of scale. *Table 5* further shows that when the vendor set up cost changes, the small batch policy with comparatively increased equal shipment size gives better savings than equal sized shipment.

Table 5. Effect of vendor set-up cost A_1.

Parameter	Small batch with Relatively Increased Equal Shipment Policy							Equal Size Shipment Policy							
A_1	$1+n_1$	$1+n_2$	T	TC (vendor)	TC (Buyer 1)	TC (Buyer 2)	TC (sys)	n_1	n_2	T	TC (vendor)	TC (Buyer 1)	TC (Buyer 2)	TC (sys)	% Savings
350	1	3	0.423	1040.3	627.5	726.3	2366.0	1	3	0.406	1047.7	590.5	827.1	2465.4	4.20
400	1	3	0.443	1125.0	641.2	744.8	2482.0	1	3	0.425	1134.1	601.7	849.8	2585.7	4.16
450	1	3	0.463	1204.5	655.0	763.2	2592.5	1	3	0.444	1012.8	613.1	872.3	2700.7	4.17
500	1	3	0.482	1279.6	668.8	781.4	2698.4	1	4	0.481	1228.0	636.7	944.0	2808.7	4.09
550	2	3	0.554	1244.8	871.2	855.4	2795.6	1	4	0.498	1104.1	648.7	963.5	2910.9	4.12

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Table 6. Effect of buyer holding cost $\left(h_{2,i}\right)$.

Parameter	Small Batch with Relatively Increased Equal Shipment Policy							Equal Size Shipment Policy							
$h_{2,i}$	$1+n_1$	$1+n_2$	T	TC (Vendor)	TC (Buyer 1)	TC (Buyer 2)	TC (sys)	n_1	n_2	T	TC (Vendor)	TC (Buyer 1)	TC (Buyer 2)	TC (sys)	% Savings
3.0	1	3	0.497	1054.1	547.9	635.1	2212.9	1	4	0.563	993.9	525.5	775.1	2294.5	3.69
3.5	1	3	0.468	1089.8	596.3	691.9	2351.0	1	3	0.528	1094.0	560.4	790.1	2444.5	3.98
4.0	1	3	0.443	1125.0	641.2	744.7	2482.2	1	3	0.425	1134.1	601.7	849.8	2585.7	4.12
4.5	1	2	0.422	1159.5	683.3	794.4	2606.4	1	3	0.404	1173.2	640.5	905.9	2719.6	4.34
5.0	1	2	0.385	1305.1	753.8	743.5	2725.0	1	3	0.386	1211.4	677.1	958.8	2847.2	4.48
5.5	1	2	0.371	1336.0	794.1	784.5	2832.5	1	3	0.370	1248.6	711.8	1009.0	2969.4	4.83

Effect of buyer holding cost $h_{2,i}$

The buyer holding cost was equally varied to study how the model responded to a change in this parameter. *Table 6* showed clearly that the joint cycle time and the numbers of delivery.

Shipments reduce when the holding cost at the buyers increases because the vendor tried to save costs by reducing the goods available at the buyers through a reduced shipment lot size or numbers. The total cost of the supply chain and the vendor cost, however, keep growing because of the fixed set up cost, which remains unchanged regardless of the quantities of goods produced by the vendor. Also, from *Table 6*, the small batch with relatively increased equal shipment policy gives better financial savings over equal sized shipment when the holding cost at the buyer is changing.

Table 7. Effect of the vendor production rate (P).

Parameter	Small Batch with Relatively Increased Equal Shipment Policy							Equal Size Shipment Policy							
P	**1 + n_1**	**1 + n_2**	**T**	**TC (Vendor)**	**TC (Buyer 1)**	**TC (Buyer 2)**	**TC (sys)**	**n_1**	**n_2**	**T**	**TC (vendor)**	**TC (Buyer 1)**	**TC (Buyer 2)**	**TC (sys)**	**% Savings**
1600	2	5	0.575	1022.8	536.2	695.6	2345.8	2	6	0.557	972.1	739.2	803.0	2514.3	7.1
2000	2	4	0.538	1036.1	651.7	734.9	2415.7	1	4	0.447	1120.4	626.9	827.4	2571.4	6.4
2400	2	4	0.528	1004.9	724.6	793.5	2459.9	1	4	0.445	1129.7	613.9	837.1	2580.8	4.9
2800	1	4	0.444	1152.0	600.1	714.2	2479.2	1	4	0.444	1098.3	613.2	875.8	2587.4	4.4
3200	1	3	0.443	1125.0	641.2	744.8	2482.2	1	3	0.425	1134.1	601.7	849.8	2585.8	4.2

Effect of production rate

Finally, we studied the production rate effect in *Table 7*. For the small batch, relatively increased equal shipment policy, an increase in the production rate reduces the joint cycle time, and equally increase the shipment lot size to be deliver at the buyer. This is responsible for the reduction in the vendor cost due to the fixed set up cost that is charged for the gradually increased quantities of items produced, which also increase the cost at the buyers through the holding cost that varied with the quantities shipped. Comparing the results from the policy discussed above with those obtained from the equal shipment policy shows that the former policy performed better in terms of savings, which ranges between 7.1% and 4.2%.

6 | Summary and Conclusion

In this research, we further explore the single vendor, multiple buyers' consignment stock inventory problem by considering small batch delivery under known production and demand conditions. As compared to previous work, we proposed a new policy wherein the vendor coordinate shipments by sending smaller lots to the buyer first after which the buyer received an equal shipment that is increased at constant rate $(\frac{P}{d_i})$. The vendor ensures that each buyer is fully served before the next buyer is considered, and the buyer's sequence/arrangement is very flexible and subjective depending on the discretion of the vendor. An integrated mathematical model that described the cost associated with this policy from the vendor and buyer's perspectives was formulated, and like Zavanella and Zanoni [1], a joint optimal solution technique was adopted in solving the problem being the better of the two solution techniques evaluated by the previous authors.

Furthermore, the proposed policy was evaluated using the same data from Zavanella and Zanoni [1], and the results obtained was compared with that from equal shipment, equal interval policy proposed by Zavanella and Zanoni [1] at different holding cost, set up cost and production rate of the vendor and buyers. The percentage difference was computed and the small batch with relatively increased equal shipment policy gives better financial reward than the equal shipment, equal interval policy found in literature. Finally, we studied how the model responded to changes in key parameters through a sensitivity analysis performed on the holding cost, set-up cost and production rate. The impact of these parameters on the number of shipments delivered to each buyer, shipment size, vendors and buyers' cost were investigated and discussed.

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Conflict of Interest and Authorship Conformation

All authors participated in

I. Problem analysis and mathematical model development.
II. Article drafting and revising.

This authors further confirmed that the manuscript has not been submitted to or placed under review with another journal.

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A Vibration Damping Optimization Algorithm to Solve Flexible Job Shop Scheduling Problems with Reverse Flows

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Abstract

The Flexible Job shop Scheduling Problem (FJSP), as a Production Scheduling Problem (PSP), is generally an extension of the Job shop Scheduling Problem (JSP). In this paper, the FJSP with reverse flow consisting of two flows of jobs (direct and reverse) at each stage is studied; the first flow initiates in Stage 1 and goes to Stage C (the last stage), and the second flow starts with Stage c and ends up in Stage 1. The aim is to minimize the makespan of the jobs (the maximum completion time). A Mixed Integer Programming (MIP) is presented to model the problem and the Branch and Bound (B&B) method is used to solve the problem. A numerical small-size problem is presented to demonstrate the applicability, for which the Lingo16 software is employed for a solution. Due to the NP-hardness of the problem, a meta-heuristic, namely the Vibration Damping Optimization (VDO) algorithm with tuned parameters using the Taguchi method, is utilized to solve large-scale problems. To validate the results obtained using the proposed solution algorithm in terms of the solution quality and the required computational time, they are compared with those obtained by the Lingo 16 software for small-size problems. Finally, the performance of the proposed algorithm is compared with a Genetic Algorithm (GA) by solving some randomly generated larger-size test problems, based on which the results are analyzed statistically. Computational results confirm the efficiency and effectiveness of the proposed algorithm and show that the VDO algorithm performs well.

Keywords: Vibration damping optimization, Scheduling, Flexible job shop, Reverse flow, Mathematical programming, Genetic algorithm.

1 | Introduction

Production scheduling is an important challenge in the field of production and operation planning and attracted the attention of many researchers. It can be defined as the determination of a sequence to assign tasks that utilize resources to produce products [1]. Being classified as an NP-hard problem, Job shop Scheduling (JSP) is the most known and complex scheduling problem [2]. Besides, the Flexible Job shop Scheduling Problem (FJSP) is one of the extensions of the JSP with more complication and Mati and Xia [3] proved that it belongs to the class of NP-hard problems as well.

One of the most important issues in the field of FJSP is the so-called reverse flows involved within an assembly/disassembly network, in which non-conforming and used assembled products are returned by the customers. To have a sustainable production line, these products are then disassembled to use their good components in recovery activities. Over the past few years, most of the research on reverse logistics dealt with the return of products in a supply chain [4].

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The first crucial step involved in product recovery operation is disassembly. Disassembly scheduling, as one of the important operational problems, can be generally defined as the problem of determining the quantity and the timing of the end-of-use/life products while satisfying the demand for their parts over a planning horizon [5]. Some disassembly operations are almost always needed in remanufacturing, recycling, and disposal. Several questions arise which increase the uncertainty in disassembly yield: 1) is the component missing? 2) is the component functional? 3) What is the version of the component? If these questions can be answered before the disassembly of the component, unnecessary disassembly of a non-functional or unneeded component can be avoided [6]. In disassembly, the flow process is divergent and there is a high degree of uncertainty in the quality of the returned products. The reusability of parts creates a demand and availability of the reusable parts is significantly less predictable than what is found in the assembly process [7].

As an operational (short-term) decision, the current study seeks to determine proper scheduling for the FJSP of an assembly/disassembly network that involves reverse flows. In what comes in the next section, some relevant works in this area are surveyed in chronological order.

2 | Literature Review

In this section, a few relevant and recent researches on the JSP are first reviewed. Then, some studies on the reverse flow and disassembly problems are surveyed.

Gao et al. [8] addressed the FJSP with two constraints, namely fuzzy processing time and new job insertion. A Two-stage Artificial Bee Colony (TABC) algorithm with several improvements was proposed to solve FJSP with fuzzy processing time and new job insertion constraints.

A JSP was considered in Giglio et al. [9], where the authors proposed a design to integrate an energy-efficient JSP and lot sizing for manufactured raw materials and remanufactured return products. Stating that their Mixed-Integer Linear Programming (MILP) formulation was NP-Hard, they proposed a relax-and-fix heuristic to solve their problem. Wu and Sun [10] formulated a FJSP, in which turn-on/off machines and speed levels were taken into account to save energy. They solved their problem using a Non-dominated Sorted Genetic Algorithm (NSGA-II).

A Flexible Job-shop Rescheduling Problem (FJRP) for new job insertion was addressed by Gao et al. [11]. Their paper deal with bi-objective FJRPs to minimize: 1) instability and 2) one of the following indices: a) makespan; b) total flow time; c) machine workload; and d) total machine workload. A metaheuristic algorithm, named DJaya presented to solve FJRP.

Gong et al. [12] introduced a Double Flexible Job shop Scheduling Problem (DFJSP) model that not only takes into account the processing time but also considers green production measures including environmental protection and human factor indicators. They solved their problem by utilizing a Newly proposed Hybrid Genetic Algorithm (NHGA).

A FJSP to minimize the total energy consumption of the shop was investigated by Meng et al. [13]. The authors solved their proposed models using the CPLEX SOLVER, based on which the effectiveness of their approach was assessed. They showed that their MILP models outperform the existing model that minimizes idle time. Gong et al. [14] developed a many-objective integrated energy- and labor-aware FJS model. The objectives involved in their model included makespan, total labor cost, total energy cost, total workload, and maximal workload. They used an NSGA-III meta-heuristic to solve their problem.

On the subjects of reverse flow and disassembly, we refer interested readers to Dondo and Mendez [15], Osmani and Zhang [16], and Giri et al. [17] for the latest research in reverse flows conducted in supply chains. Here some works related to heuristic and meta-heuristic approaches to solve the reverse flow problem in assembly and disassembly environments are discussed. McGovern and Gupta [7] developed a Genetic Algorithm (GA) for disassembly line balancing problems. Adenso-Diaz et al. [18] presented a path-relinking-based heuristic that seeks the optimal disassembly sequence plan. Duta et al. [19] provided a GA for finding the disassembly sequence with the best financial income. Kim et al. [20] provided a Branch and Bound (B&B) algorithm to meet the demand for end-of-use product components by determining the quality and time of this component's disassembly. Ilgin and Gupta [6] studied the sensors implanted in products on the performance of a washing machine disassembly line. Wan & Gonnuru [21] proposed a GA that provides a disassembly sequence to maximize the benefits by taking into account the recovery value and the disassembly cost. Abdeljaouad et al. [22] proposed a hybrid heuristic algorithm for job-shop scheduling problems with reverse flows for disassembly. Tanimizu et al. [23] proposed a scheduling method for open-shop scheduling problems containing both disassembly and post-processing operations. Finally, Ren et al. [24] proposed a model for the asynchronous Parallel Disassembly Planning (i.e. aPDP) problem. They adopted an efficient meta-heuristic based on GA to identify the disassembly sequence and manipulator allocation. Aghighi et al. [25] investigated open shop scheduling with the reverse flow. A MILP model was developed to solve the problem. The proposed model includes an objective to minimize the maximum completion time of all jobs (makespan). A numerical example is presented and solved by using the GAMS software to validate the proposed mathematical model. Seven meta-heuristic algorithms (VDO, SA, ICA, BAT, HSA, ACO, and CS) were used to solve larger-size examples.

Given the above-mentioned literature, in this paper, a FJSP with reverse flow is studied. A Mixed Integer Programming (MIP) model is presented and the B&B method is used to solve small-size problems. Due to the NP-hardness of the problem, a meta-heuristic algorithm, namely a Vibration Damping Optimization (VDO) algorithm is used to solve large-scale problems. The Taguchi method is used to calibrate the parameters of the algorithm, in an attempt to find better solutions. Finally, the proposed algorithm is compared with the GA by solving some randomly generated test problems.

The remainder of the paper is organized as follows; in Section 3 the problem is defined, the assumptions are made, the notations are defined, and the problem is formulated. The proposed solution approach comes in Section 4. Some problems are solved in Section 5 to validate the proposed algorithm as well as to assess its performance when compared to the one of a GA. Finally, the conclusion and future research recommendations are in Section 6.

3 | The Problem and its Model

In this section, the problem is defined explicitly. Then, notations are defined, based on which the mathematical formulation of the problem is developed.

3.1 | Problem Definition

In this paper, there are n jobs that must be scheduled at c stages on m_k machines in the kth workshop. There are two operating ranges: the direct jobs cover the stages in the order $1, 2, \dots, c$, and the reverse jobs cover the stage in the order $c, c-1, \dots, 2, 1$. The set of E_1 shows the set of direct jobs and E_2 shows the set of reverse jobs. *Fig. 1* shows the direct and reverse flows involved in this job shop.

Note that for any flexible job-shop scheduling problem with reverse flows, where there are two flows of jobs at each stage in opposite directions, optimal solutions are using the following policies [22]:

- *At the first stage, all of the direct jobs should be processed before the reverse jobs (on any machine).*
- *At the last stage, all of the reverse jobs should be processed before the direct jobs (on any machine).*

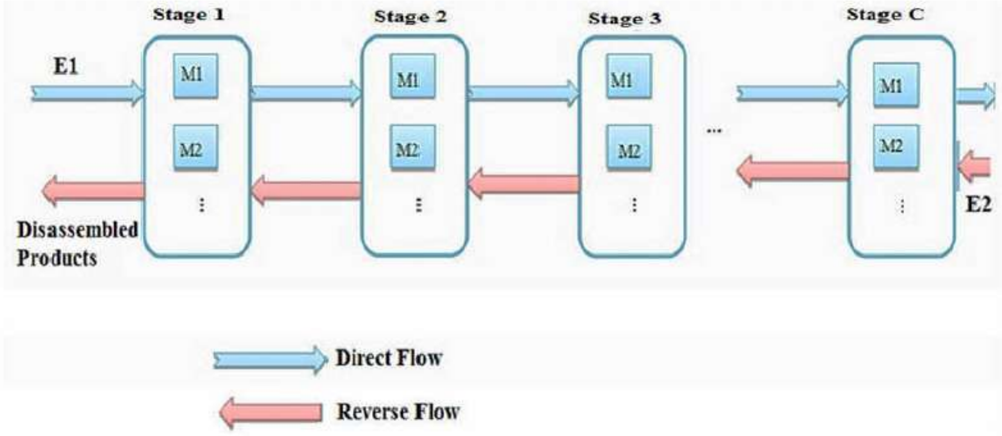

Fig. 1. An overview of the flows.

3.2 | Assumptions

There are two types of jobs (direct and reverse).

Direct and reverse flows enter and exit each stage.

All jobs are available at zero time and are independent.

There are m_k jobs in stage $k; k = 1,2, \dots, c$.

All machines are independent of each other.

All machines are continuously available.

At a time, each machine can process at most one job, and each job can be processed only on one machine.

Each machine can process only one operation.

The machines in each stage are different from each other.

The machines of a direct and a reverse path are the same.

Preemption is not allowed. In other words, the processing of a given job on a machine cannot be interrupted once started.

The processing time can be different in various machines.

The machine setup time is ignored.

3.3 | Notations

The indices, the sets, the parameters, and the decision variables defined to formulate the problem are as follows.

3.3.1 | Indices

j: An index used for a job, $j = 1,2, \dots, n$.

k: An index used for a stage, $k = 1,2, \dots , c$.

s: An index used for a machine in stage k, $s = 1,2, \dots , m_k$.

i: The position index, $i = 1,2, \dots , n$.

h: The previous position index, $h = 1,2, \dots , i$.

S_1: Number of jobs in the direct flow.

$n - S_1$: Number of jobs in the reverse flow.

3.3.2 | The sets

$j = \{j_1, j_2, \dots , j_n\}$: The set of all jobs.

$E_1 = \{j_1, \dots, j_{S_1}\}$: The set of direct jobs.

$E_2 = \{j_{S_1+1}, \dots, j_n\}$: The set of reverse jobs.

3.3.3 | The parameters

n: The number of all jobs.

m: The number of all machines.

m_k: The number of jobs in stage k.

n: The number of positions.

O_{kjs}: The operation of job j on machine s at stage k.

P_{kjs}: The time required to process job j on machine s at stage k.

M: A large positive number.

3.3.4 | Decision variables

t_{kj} : The starting time of job j in stage k.

P'_{kj}: The real processing time (real duration) of job $j$at stage k.

$$X_{jki}^{S} = \begin{cases} 1, & if\ job\ j\ is\ assigned\ to\ position\ i\ at\ stage\ k, \\ 0, & Otherwise. \end{cases}$$

3.4 | Mathematical Modeling

The proposed mathematical model of the problem follows:

$$\text{Min } Z = C_{max}, \tag{1}$$

s.t.

$$\sum_{s=1}^{m_k}\sum_{i=1}^{n} X_{jki}^{S} = 1 \quad \text{for all } j = 1, \ldots, n \;;\; k = 1, \ldots, c, \tag{2}$$

$$\sum_{j=1}^{n} X_{jki}^{S} \leq 1 \quad \text{for all } i = 1, \ldots, n \,;\; k = 1, \ldots, c \,;\; S\epsilon m_k, \tag{3}$$

$$t_{1j} \leq M\left(1 - \sum_{s=1}^{m_1} X_{j11}^{S}\right) \quad \text{for all } j = 1, \ldots, S_1, \tag{4}$$

$$t_{cj} \leq M\left(1 - \sum_{s=1}^{m_c} X_{jc1}^{S}\right) \quad \text{for all } j = S_1 + 1, \ldots, n, \tag{5}$$

$$P'_{kj} = \sum_{i=1}^{n}\sum_{s=1}^{m_k}\left(P_{kjs}\right)X_{jki}^{S} \quad \text{for all } j = 1, \ldots, n;\; k = 1, \ldots, c, \tag{6}$$

$$t_{kj} + P'_{kj} - t_{\,k+1)j} \leq 0 \quad \text{for all } k = 1, \ldots, c-1;\; j = 1, \ldots, S_1, \tag{7}$$

$$t_{kj} + P'_{kj} \text{-} \;\; t_{\,k-1)j} \leq 0 \quad \text{for all } k = 2, \ldots, c;\; j = S_1 + 1, \ldots, n, \tag{8}$$

$$t_{cj} + P'_{cj} \leq C_{max} + M\left(1 - \sum_{s=1}^{m_c} X_{jci}^{S}\right) \quad \text{for all } j = 1, \ldots, S_1 \quad \text{for all } i = 1, \ldots, n, \tag{9}$$

$$t_{1j} + P'_{1j} \leq C_{max} + M\left(1 - \sum_{s=1}^{m_1} X_{j1i}^{S}\right) \quad \text{for all } j = S_1 + 1, \ldots, n \quad \text{for all } i = 1, \ldots, n, \tag{10}$$

$$t_{kq} + M\left(1 - X_{qk\; i+1)}^{S}\right) \geq t_{kj} + P'_{kj} - M\left(1 - X_{jkh}^{S}\right) \quad \text{for all } j, q = 1, \ldots, n$$
$$\text{for all } k = 1, \ldots, c \quad \text{for all } i = 1, \ldots, n-1 \quad \text{for all } h = 1, \ldots i \quad \text{for all } s = 1, \ldots, m_k, \tag{11}$$

$$X_{jki}^{S} \in \{0{,}1\}. \tag{12}$$

The mathematical *Eq. (1)* represents the objective function minimizing the completion time (C_{max}) of the last job on the last machine. *Constraint (2)* defines that each job at each stage can be processed only in one position and only on one machine. *Constraint (3)* ensures that only one job can be processed at a position of one stage on a machine. *Constraint (4)* assures that if a direct job is processed at the first position of the first stage (on each machine), its starting time of operation at the first stage is zero. *Constraint (5)* defines that if a reverse job is processed at the first position of the last stage (on each machine), its starting time of operation at the last stage is zero. *Constraint (6)* shows the real processing time of job *j* at the k^{th} stage. *Constraint (7)* implies that the starting time of a direct job at each stage is equal to its starting time at the previous stage plus its processing time (a pre-required direct job constraint). *Constraint (8)* indicates that the starting time of a reverse job at each stage is obtained by adding its starting time at the next stage with its processing time (pre-required reverse job constraint). *Constraint (7)* and *Constraint (8)* are initial precedence constraints on the operations of the jobs. *Constraint (9)* and *Constraint (10)* determine the finishing times (C_{max}) of the direct and reverse jobs, respectively, obtained by the finishing times of the last job. *Constraint (11)* ensures that if job *q* at stage *k* is processed after job *j*, then the starting time of job *q* must be equal to the starting time of job *j* plus its processing time. *Constraint (12)* defines the type of the variable being used.

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3.5 | An Illustrative Example

A small example is solved in this section using the Lingo 16 software to illustrate the applicability of the model developed in Section 3.4. In this example, there are 3 direct jobs (1, 2, and 3) and 3 reverse jobs (4, 5, and 6) to be scheduled at 5 stages. While there are 2 machines in Stages 1 and 2, in the other three stages, 3 machines are available. *Table 1* shows the duration of each job at each stage on each machine and *Fig. 2* displays the optimal solution.

Table 1. The duration of each job at each stage on each machine [p(k, j, s)=P_{kjs}].

p(1,1,1)= 4	p(2,1,1)=2	p(3,1,1)=3	p(4,1,1)=1	p(5,1,1)=1
p(1,1,2)=5	p(2,1,2)=5	p(3,1,2)=4	p(4,1,2)=1	p(5,1,2)=1
p(1,1,3)=3	p(2,1,3)=3	p(3,1,3)=1	p(4,1,3)=2	p(5,1,3)=1
p(1,2,1)=4	p(2,2,1)=5	p(3,2,1)=3	p(4,2,1)=2	p(5,2,1)= 1
p(1,2,2)=5	p(2,2,2)=2	p(3,2,2)=5	p(4,2,2)=1	p(5,2,2)=1
P(1,2,3)=1	p(2,2,3)=4	p(3,2,3)=4	p(4,2,3)=1	p(5,2,3)=1
p(1,3,1)=1	p(2,3,1)=4	p(3,3,1)=4	p(4,3,1)=1	p(5,3,1)=1
p(1,3,2)=1	p(2,3,2)=1	p(3,3,2)=4	p(4,3,2)=2	p(5,3,2)=1
p(1,3,3)=5	p(2,3,3)=1	p(3,3,3)=3	p(4,3,3)=3	p(5,3,3)=1
p(1,4,1)=3	p(2,4,1)=5	p(3,4,1)=3	p(4,4,1)=3	p(5,4,1)=1
p(1,4,2)=1	p(2,4,2)=1	p(3,4,2)=2	p(4,4,2)=1	p(5,4,2)=1
p(1,4,3)=3	p(2,4,3)=2	p(3,4,3)=3	p(4,4,3)=2	p(5,4,3)=1
p(1,5,1)=1	p(2,5,1)=1	p(3,5,1)=3	p(4,5,1)=1	p(5,5,1)=1
p(1,5,2)=2	p(2,5,2)=4	p(3,5,2)=5	p(4,5,2)=2	p(5,5,2)=1
p(1,5,3)=1	p(2,5,3)=4	p(3,5,3)=5	p(4,5,3)=2	p(5,5,3)=1
p(1,6,1)=3	p(2,6,1)=2	p(3,6,1)=2	p(4,6,1)=2	p(5,6,1)=1
p(1,6,2)=5	p(2,6,2)=5	p(3,6,2)=4	p(4,6,2)=1	p(5,6,2)=1
p(1,6,3)=1	p(2,6,3)=3	p(3,6,3)=5	p(4,6,3)=1	p(5,6,3)=1

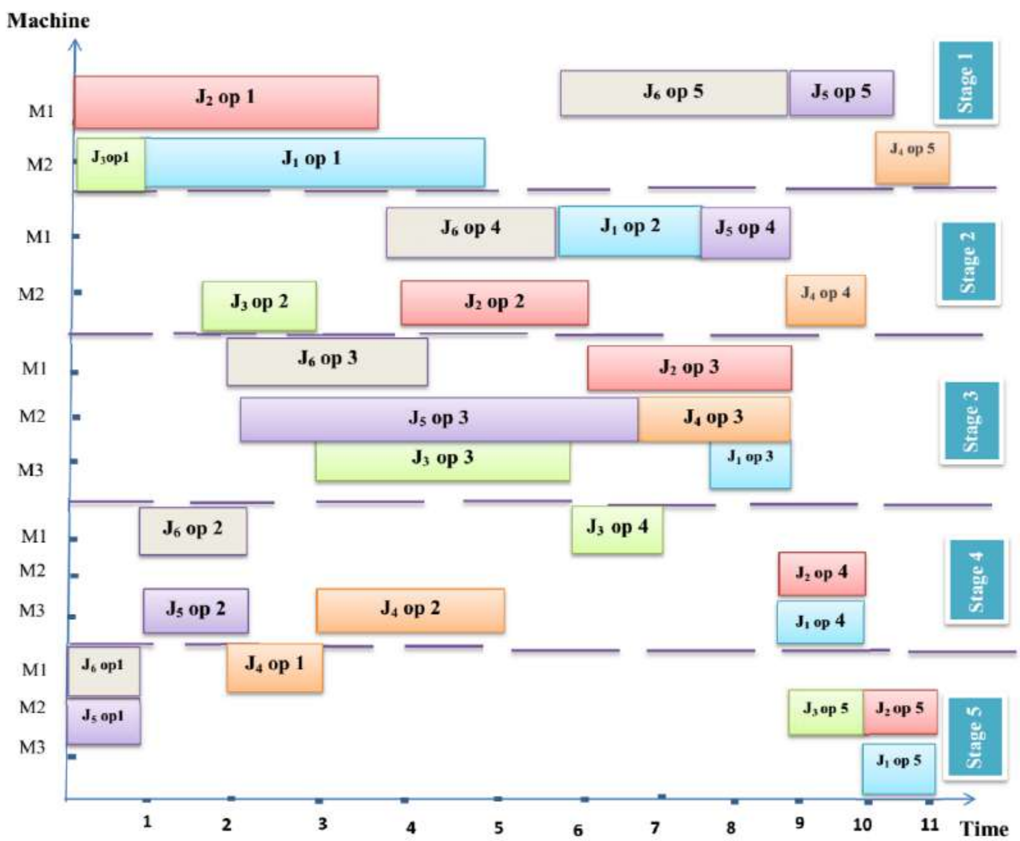

Fig. 2. The optimal solution obtained by Lingo 16.

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4 | The Solution Approach

Due to the NP-hardness of the problem [22], a VDO algorithm is utilized in this section to solve large-scale problems. VDO, was first proposed by Mehdizadeh et al. [26] and has been used to solve many optimization problems especially Production Scheduling Problems (PSPs) so far. Aliabadi et al. [27] studied the scheduling and sequence of multi-product flow-shops problem with sequence-dependent setup times. Three meta-heuristics including a hybrid Particle Swarm Optimization (PSO), a hybrid VDO, and a hybrid GA were used to solve this problem. Mehdizadeh et al. [26] used a VDO algorithm for the identical parallel machine scheduling problem with sequence-independent family setup times to minimize the total weighted completion time and computationally compare the results obtained by the proposed VDO with the results of the GA and branch-and-bound method. Yazdani et al. [28] employed two meta-heuristics, namely SA and VDO to minimize the makespan of a Dual-Resource Constrained Flexible Job-shop Scheduling Problem (DRCFJSP). Alaghebandha et al. [29] considered an efficient Hybrid Vibration Damping Optimization (HVDO) with an Imperialist Competitive Algorithm and Simulated Annealing to solve the economic lot sizing and scheduling problem in distributed permutation flow shop problem with several non-identical factories and machines and used GA and VDO for comparison. Rashidi Komijan et al. [30] applied a VDO algorithm to solve an integrated airline feet assignment and crew scheduling problem. To evaluate the performance of the proposed VDO algorithm, the obtained results were compared with PSO and optimal solutions. Aghighi et al. [25] used the VDO algorithm and six other algorithms including SA, ICA, BAT, HSA, ACO, and CS) to solve larger-size examples of open shop scheduling with the reverse flow. Soofi et al. [31] used the VDO algorithm to solve the Dual-Resource Constrained Flexible Job-shop Scheduling (DRCFJSS) problem under machine breakdown and operational uncertainty.

4.1 | Vibration Damping Optimization Algorithm

There is a useful connection between vibration damping and combinatorial optimization. VDO algorithm is a powerful meta-heuristic algorithm with a stochastic search method based on the concept of vibration damping in mechanical vibration [32], [33]. All bodies possessing mass and elasticity are capable of vibration. Vibrating systems are all subject to damping to some degree because energy is dissipated by friction and other resistances. This process is called vibration damping. At a given amplitude, the probability distribution of the oscillatory system is determined by the Rayleigh probability. At high amplitude, the probability converges to 1 for all energy states. It can also be seen that there exists a small probability that the system might have high energy at low amplitudes. Therefore, the statistical distribution of energies allows the system to escape from a local energy minimum. *Table 2* indicates an analogy between optimization problems and vibration damping.

Table 2. The analogy between optimization problems and VDO algorithm [26].

VDO	Optimization Problems
System states	Feasible solution
Energy	Objective function
Change of state	Neighbouring solution
Amplitude	Control parameter
Vibration damping	Heuristic solution
Degrees of freedom	Number of decision variables

The algorithm consists of a sequence of iterations. Each iteration consists of randomly changing the current solution to create a new solution in the neighborhood of the current solution. The neighborhood is defined by the choice of the generation mechanism. Once a new solution is created the corresponding change in the cost function is computed to decide whether the newly produced solution can be accepted as the current solution. If the change in the cost function is negative the newly produced solution is directly taken as the current solution. Otherwise, it is accepted according to Raleigh's probability. If the difference between the cost function value of the current and the newly produced solutions is equal to or larger than zero, a random number r in [0, 1] is generated from a uniform distribution, and if $r \le p(A)$, then the newly produced solution is accepted as the current solution. If not, the current solution is unchanged. The pseudo-code of the vibration damping algorithm for single objective function problems is shown in *Algorithm 1*.

Algorithm 1. The pseudo-code of the VDO algorithm [26].

Step 1. Generating a feasible initial solution.
Step 2. Initializing the algorithm parameters, which consist of: initial amplitude (A_0), maximum iteration at each amplitude (L), damping coefficient (γ), and standard deviation (σ). Finally, parameter t is set in one (t=1)
Step 3. Calculating the objective value U_0 for the initial solution.
Step 4. Initializing the internal loop. In this step, the internal loop is carried out for l =1 and repeat while l < L.
Step 5. Neighbourhood generation.
Step 6. Accepting the new solution
Set $\Delta = U - U_0$ Now, if $\Delta < 0$, accept the new solution, else if $\Delta > 0$ generate a random number r between [0, 1];
If $r < 1 - exp\left(\frac{-A_t^2}{2\sigma^2}\right)$, then accept a new solution; otherwise, reject the new solution and accept the previous solution.
If $l > L$, then $t + 1 \longrightarrow t$ and go to Step 7; otherwise, $l + 1 \longrightarrow l$ and go back to Step 5.
Step 7. Adjusting the amplitude. In this step, $A_t = A_0 \, exp(\frac{-\gamma t}{2})$ is used for reducing amplitude at each iteration of the outer cycle of the algorithm. If A_t = 0 return to Step 8; otherwise, go back to Step 4.
Step 8. Stopping criteria. In this step, the proposed algorithm will be stopped after pre-specified minimum amplitude A_{min} is achieved. At the end, the best solution is obtained.

4.1.1 | Solution representation

A solution is designed and represented in two steps. In Step 1, direct jobs are placed before the reverse jobs, and in Step 2, the reverse jobs are placed before the direct jobs. Note that the direct and the reverse jobs are selected randomly. In addition, a solution is represented by two $C \times N$ matrices called S and m, where C is the number of stages and N is the permutation of all jobs (direct and reverse). Depending on the rows of the S matrix, the order the jobs are placed in each stage is determined. In addition, the m matrix shows how the machines are assigned to the jobs placed in each stage. *Fig. 3* depicts a solution structure with 10 jobs, 4 stages, and up to 4 machines in each stage. In this figure, the number 3 in the first row of the S matrix and the number 1 in the same row of the m matrix indicate that the third job (direct) at the first stage is processed on Machine 1.

s

1	2	4	5	3	6	9	8	7	10
1	5	2	3	4	9	8	7	10	6
9	6	8	7	10	1	4	5	2	3
8	7	10	9	6	1	2	3	4	5

m

4	3	4	4	1	4	1	3	2	4
1	1	2	3	3	2	2	3	2	1
2	3	3	1	1	3	2	2	1	2
1	2	1	4	3	4	4	1	4	3

Fig. 3. Proposed solution representation.

4.1.2 | Initial solution

Choosing the proper start solution plays an essential role in the quality of the near-optimum solution and the time required. Various methods such as a related priority dispatching rule or a random method can be applied to yield an initial solution. In this research, a random method is employed to select initial solutions for the VDO algorithm.

4.1.3 | Neighborhood structure

Three operators (swap mutation, insertion, and reversion) are used in this paper to construct a neighborhood for the solution representation *S*. In the swap mutation shown in *Fig. 4*, two cells in each row of the *S* matrix corresponding to the job orders in different stages are chosen randomly and then their locations are exchanged.

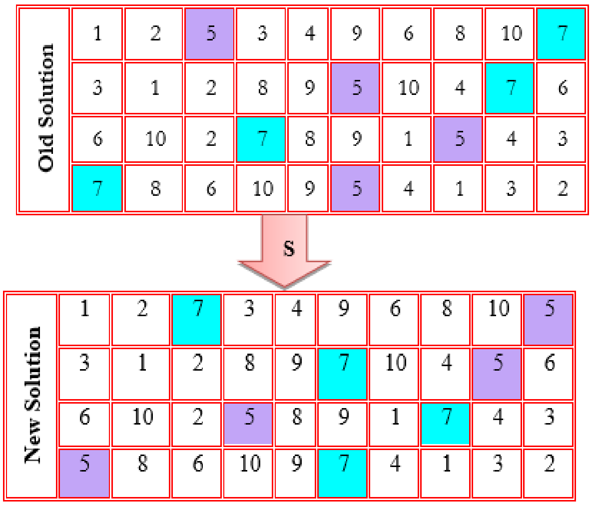

Fig. 4. The considered swap operator.

Next, the fitness of the solution before and after the swap is compared. If the fitness is not improved, then the solution remains unchanged. Similarly, the insertion and reversion operations are employed as shown in *Figs. 5* and *6*, respectively. In addition, the uniform operator is used for the solution representation m (the m matrix), based on which 50% of the cells are selected randomly and then reproduced. *Fig. 7* depicts this operation.

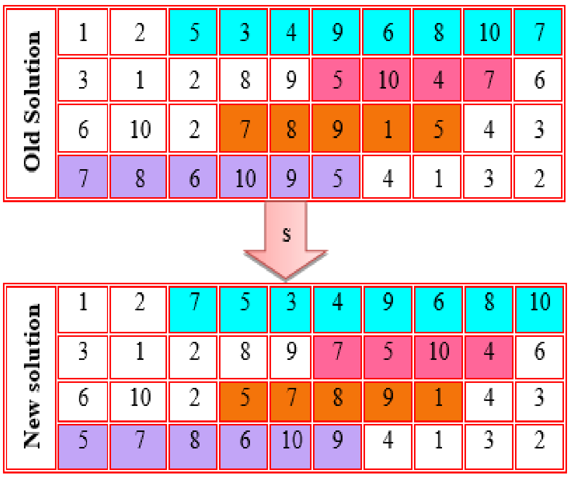

Fig. 5. The insertion operator.

Old solution									
1	2	5	3	4	9	6	8	10	7
3	1	2	8	9	5	10	4	7	6
6	10	2	7	8	9	1	5	4	3
7	8	6	10	9	5	4	1	3	2

s

New solution									
1	2	7	10	8	6	9	4	3	5
3	1	2	8	9	7	4	10	5	6
6	10	2	5	1	9	8	7	4	3
5	9	10	6	8	7	4	1	3	2

Fig. 6. The reversion operator.

Old solution									
4	3	4	4	1	4	1	3	2	4
1	1	2	3	3	2	2	3	2	1
2	3	3	1	1	3	2	2	1	2
1	2	1	4	3	4	4	1	4	3

m

New solution									
3	3	4	2	1	4	2	3	1	3
1	2	1	3	2	2	2	2	2	1
3	3	3	2	1	4	2	4	3	2
1	3	2	4	2	4	3	2	4	2

Fig. 7. The uniform operator.

For a new solution to remain feasible, when the S operator is applied, it is necessary to make sure that each job in each row is replaced only once. Moreover, in calculating the makespan (fitness function), the first operation of each direct job should be scheduled at the first stage and the first operation of each reverse job should be scheduled at the last stage. When the m operator is applied, the new solution remains feasible anyway.

4.1.4 | Reducing amplitude

After each iteration in the main loop of the VDO algorithm, the amplitude is reduced using *Eq. (13)*. This can reduce the probability of accepting neighbors generated to avoid increasing the number of iterations and hence to increase the speed [32].

$$A_t = A_0 e^{-\gamma t/2)}. \tag{13}$$

4.1.5 | Neighborhood acceptance/rejection mechanism

After each generation, a new solution is created based on which the corresponding change in the objective function is computed to decide whether the newly produced solution can be accepted as the current solution [26]. If the new member overcomes the current member, the new member replaces it. Otherwise, the probability in *Eq. (14)* is calculated after that a uniform random number r is generated within [0, 1]. If r is less than the probability obtained by *Eq. (14)*, the new solution will replace the current solution.

$$P = 1 - e^{-\frac{A^2}{2\sigma^2}}. \tag{14}$$

4.1.6 | Repeating the number of inner loops

Another important parameter for finding better solutions is the number of neighborhood searches for each amplitude L repeated for each member of the population. The amplitude is used to reduce them in each iteration [34]. The number of repetitions of the inner loop depends on the success rate of the forced vibration. The number of these repetitions is directly related to the algorithm time, meaning that a large number of the inner loop leads to an increase in the time of the algorithm while not improving the quality of the solutions.

4.1.7 | Stop criterion

While there are different criteria such as the maximum number of iterations available in the literature to stop a meta-heuristic, in this paper the VDO algorithm and other mentioned algorithms are stopped when it achieves a good convergence behavior.

5 | Computational Results

To demonstrate the applicability of the proposed methodology alongside validating the results obtained in terms of the solution quality and computational time, the solutions obtained using the proposed VDO algorithm are compared with those obtained by the Lingo 16 software, when both solve some small-size problems. In addition, a GA [35] is also utilized to analyze and compare the results found by the VDO algorithm when they solve some randomly generated test problems. Before doing this, the parameters of both solution algorithms are first calibrated in the next subsection using the Taguchi method [36] to find better quality solutions in a reasonable time.

5.1 | Parameter Tuning

In this paper, the Taguchi method is implemented to tune the parameters of the VDO algorithm and GA. In this method, a statistical measure, called the Signal-to-Noise (S/N) ratio, is calculated to evaluate the performance of the algorithm based on a specified combination level of its parameters. Here, the term "signal" denotes the mean objective function value which is desirable, and "noise" denotes the standard deviation of the objective function values which is undesirable. The aim is to maximize the signal-to-noise ratio [31], [32].

The parameter levels of the algorithms to solve a typical problem are presented in *Tables 3* and *4*. Besides, the L25 design of the Taguchi method is employed for the proposed algorithms. The algorithms are coded in MATLABR2013a, Version 8.1.0.604, and are executed on a PC with 6 GB RAM and a 2.30 GHz CPU. For each algorithm, the related S/N ratio is obtained using the Minitab 17 software as illustrated in *Figs. 8* and *9*. In these figures, the best level of each parameter is selected to be the one with the highest S/N ratio. As a result, the proper values of the parameters are presented in *Tables 5* and *6*.

Table 3. Parameters and their levels for the GA.

Parameters	Npop	P_c	P_m
	25	0.6	0.1
	50	0.7	0.15
Presented levels	100	0.8	0.20
	150	0.9	0.30
	200	0.95	0.40

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Table 4. Parameters and their levels for the VDO algorithm.

Parameters	A_0	γ	L
	100	0.001	1
	300	0.005	2
Presented levels	500	0.01	3
	700	0.05	4
	1000	0.1	5

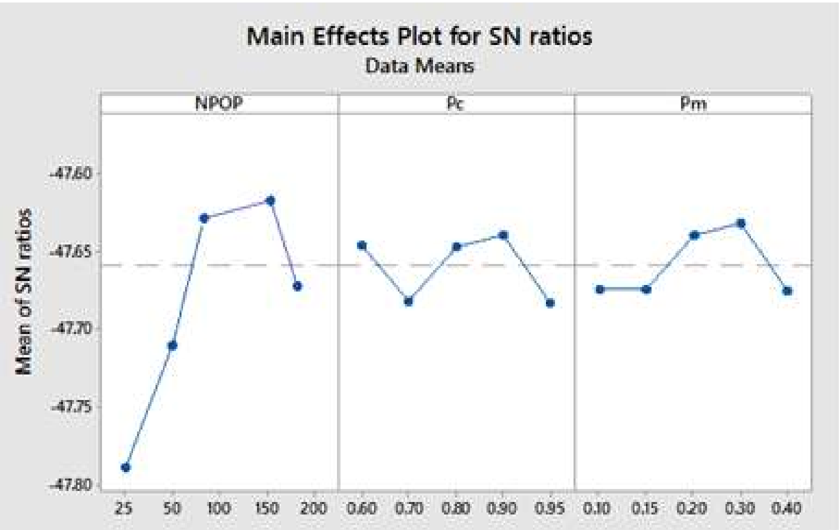

Fig. 8. The mean S/N ratio plot at each level of the parameters for the objective function values found by GA (Larger mean of S/N is better).

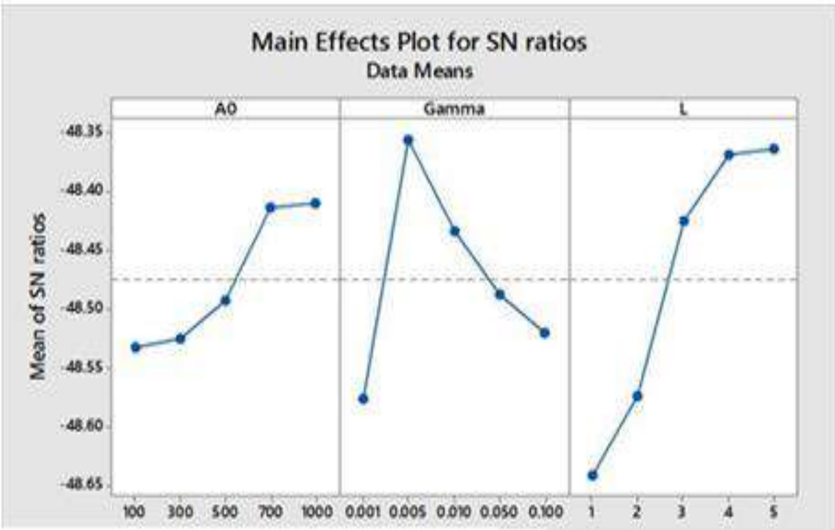

Fig. 9. The mean S/N ratio plot at each level of the parameters for objective function values found by the VDO algorithm (A larger mean of S/N is better).

Table 5. Optimal levels of the GA parameters.

Parameters	Npop	P_c	P_m
Selected levels	Forth level	Forth level	Forth level
The amount of levels	150	0.90	0.30

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Table 6. Optimal levels of the VDO algorithm parameters.

Parameters	A_0	γ	L
Selected levels	Forth level	Second level	Fifth level
The amount of levels	700	0.005	5

5.2 | Numerical Results and Validation

To validate the proposed algorithm in terms of the solution quality (makespan) and computational time, the results found by the proposed VDO algorithm are compared with those obtained by the Lingo 16 software and GA.

5.2.1 | Validation results

At first, the results found by the proposed VDO algorithm are compared with those obtained by Lingo 16 software 14 problems. In the first 10 small-size problems, the numbers of direct and reverse jobs are both 2 and 3, while the number of stages is chosen in the range 2-7. Besides, the maximum number of available machines in each stage is 3 and the processing times are selected randomly from a uniform distribution in [1, 99]. The VDO algorithm stops when it converges to a solution.

The optimal makespans of 14 test problems obtained by Lingo software are shown in the 6th column of *Table 7*. The 8th and the 9th columns of this table present the best and the average solutions found by the VDO algorithm when it solves each problem 5 times. A review of the results in *Table 7* shows that the VDO algorithm and the Lingo software 16 are capable of attaining the optimal solutions for test problems 1-10. In the last four test problems, however, the VDO algorithm works better, whereas Lingo is unable to solve these problems in reasonable computational times.

In addition, the best solutions found by the VDO algorithm for all small-size problems are identical to the optimal solutions obtained by Lingo while the computational time required by Lingo is larger than the one needed in the VDO algorithm to solve each problem. The required computational times for the proposed algorithm are significantly smaller than the Lingo software 16 as shown in *Table 7*. Furthermore, the average solutions found by the VDO algorithm are not far from the best solutions. This somehow validates the proposed VDO algorithm.

5.2.2 | Experimental results

As Lingo is unable to solve medium and large-size problems in a reasonable computational time, in this section, the proposed solution algorithm is compared with (GA) when both solve 30 randomly generated test problems. In these problems, the numbers of direct and reverse jobs range are in 6-20, the number of stages is in the range of 2-3, the maximum number of machines in each stage is 3, and again the processing times are selected randomly from a uniform distribution in [1, 99]. The algorithms are stopped when they converge. In addition, each of the problems is solved by the two algorithms five times. The last six columns of *Table 8* contain the best solution, the average solution, and the computational time of the two algorithms.

Table 7. The result obtained by the proposed VDO and LINGO.

Problem					LINGO Results		VDO Results			
Number	Number of Direct Jobs	Number of Reverse Jobs	Stages	Maximum Number of Machines in Each Stage	Lingo Results (the Best Solution Found)	Lingo Time (s)	Best VDO	Solution Average	Average of CPU-Times (s)	GAP between Lingo and Best VDO
1	2	2	2	3	7	14	7	7.2	3	0
2	2	2	3	3	16	22	16	16.1	6.1	0
3	2	2	4	3	14	35	14	14	14	0
4	2	2	5	3	23	20	23	23.4	16	0
5	2	2	6	3	24	27	24	24.2	23	0
6	2	2	7	3	28	40	28	28	34.1	0
7	3	3	2	3	6	113	6	6.2	50.2	0
8	3	3	3	3	25	150	25	25.3	100.2	0
9	3	3	4	3	23	270	23	23.6	80.6	0
10	3	3	5	3	27	1703	27	27.4	120.2	0
11	4	4	2	3	N/A	3740	10	10.7	214.72	-
12	4	4	3	3	N/A	3867	16	16.6	222.94	-
13	4	4	4	3	N/A	3998	14	14	235.19	-
14	5	5	2	3	N/A	4000	18	18.2	216.77	-

Table 8. Results obtained by the proposed VDO and GA.

Problem					VDO Results			GA Results		
Number	Number of Direct Jobs	Number of Reverse Jobs	Stages	Maximum Number of Machines in Each Stage	Best Solution	Average Solution	Average CPU-Time (s)	Best Solution	Average Solution	Average CPU-Times (s)
1	6	6	2	3	27	27.2	27.42	28.40	28.56	42.88
2	6	6	3	3	37.70	38.08	50.31	48	48.96	84.53
3	7	7	2	3	29.11	29.65	99.65	51	51.68	117.70
4	7	7	3	3	31.20	31.82	24.30	83.20	83.78	53.66
5	8	8	2	3	29.90	32.09	84.51	68	68.54	105.12
6	8	8	3	3	52.10	52.77	80.40	135.31	136.77	182.326
7	9	9	2	3	36	36.17	55.32	46.8	47.05	95.49
8	9	9	3	3	52	52.50	87.13	170	170.27	123.54
9	10	10	2	3	36.10	36.72	96.23	72	72.89	147.17
10	10	10	3	3	29.1	30.19	114.21	214	214.88	164.103
11	11	11	2	3	39	39.44	214.72	118.10	118.59	259.63
12	11	11	3	3	63.11	63.92	222.94	99	99.28	261.03
13	12	12	2	3	47	47.6	235.19	56.11	56.58	299.8
14	12	12	3	3	83.91	84.04	216.77	84	84.47	238.33
15	13	13	2	3	49	49.23	225.39	279	279.61	247.56
16	13	13	3	3	59.89	61.04	239.51	303.99	304.91	269.22
17	14	14	2	3	55	55.48	248.23	305.12	305.456	302.31
18	14	14	3	3	55.97	56.03	264.23	274	274.17	332.6
19	15	15	2	3	63	63.41	285.33	77.11	77.52	354.6
20	15	15	3	3	92	92.46	304.23	121	121.58	399.02
21	16	16	2	3	76	76.43	684.22	230	230.92	768.3
22	16	16	3	3	82.99	83.50	680.42	229.11	229.57	798.4
23	17	17	2	3	84	84.40	699.54	263	263.57	814.3
24	17	17	3	3	115.96	116.14	704.26	289	289.40	825.4
25	18	18	2	3	96	96.7	725.33	135.2	135.73	840.3
26	18	18	3	3	128	128.17	739.22	202	202.14	869.69
27	19	19	2	3	153	153.68	804.20	206.96	207.26	902.33
28	19	19	3	3	155.10	155.57	822.25	218	218.42	934.5
29	20	20	2	3	188	188.21	840.50	294	294.57	955.4
30	20	20	3	3	200	201.13	891.23	314	314.12	999.12

The results in *Table 8* show that the VDO algorithm has a better performance in terms of solution quality and computational time compared to GA. To further compare the two solution algorithms, another measure, namely the Relative Percentage Deviations (RPD) is defined in *Eq. (15)*.

$$\mathrm{RPD} = \left|\frac{\mathrm{Alg_{sol}} - \mathrm{Min_{sol}}}{\mathrm{Min_{sol}}}\right| * 10. \tag{15}$$

In *Eq. (13)*, Alg_{sol} is a solution found for a problem by either of the two solution algorithms and Min_{sol} is the smallest solution when both algorithms solve the problem 5 times. The average RPDs of the VDO algorithm and GA for each of the 30 test problems are reported in *Table 9* and the graphical comparison of the relative deviation index is shown in *Fig. 10*. The results in *Table 9* as well as in *Fig. 10* indicate that VDO algorithm is the better algorithm in terms of RPD.

Table 9. Averag RPD of the VDO algorithm and GA.

Problem Number	Algorithm	
	GA	VDO
1	0.05	0.00
2	3.71	0.40
3	0.90	0.09
4	2.08	0.17
5	1.52	0.18
6	7.03	0.94
7	0.73	0.33
8	5.26	0.93
9	1.68	0.35
10	6.90	0.11
11	3.36	0.45
12	2.65	1.35
13	9.28	0.75
14	10.21	2.09
15	1.08	0.81
16	2.11	1.24
17	9.08	1.04
18	10.23	1.06
19	1.85	1.33
20	3.47	2.40
21	8.49	1.81
22	7.44	2.07
23	8.69	2.09
24	9.64	3.27
25	3.99	2.59
26	6.43	3.71
27	6.62	4.65
28	7.03	4.72
29	9.83	5.91
30	10.54	6.39
Average	5.40	1.77

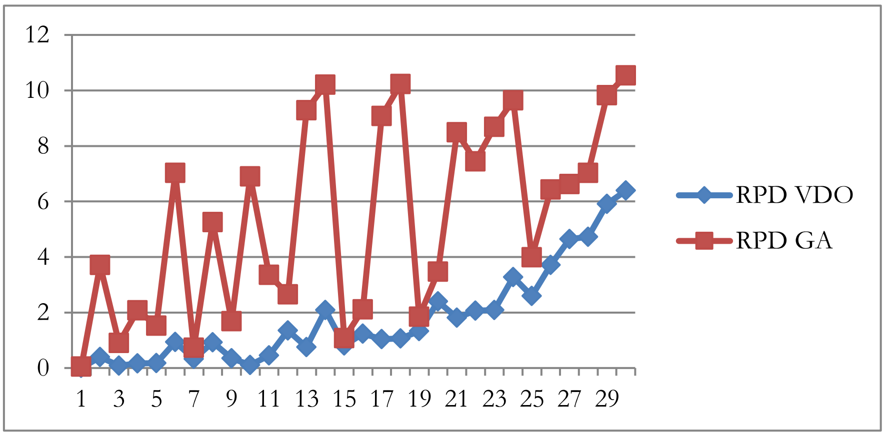

Fig. 10. Graphical comparisons of the RPD index.

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To verify the validity of the algorithms in terms of average RPD statistically, their means are compared using a two-sample Student's t-test. The samples have been accomplished using the MINITAB 17.0 software and are summarized in *Table 10*, where the p-value of the test statistic in comparing the two means becomes 0.000. It means that the two RPD means are significantly different at almost 100% confidence level. In other words, the VDO is the better algorithm in terms of the average *RPD* for sure. In addition, the mean plot and the LSD intervals of the two algorithms in *Fig. 11* reveal this fact better.

Table 10. Statistical comparison of VDO and GA in terms of average RPD.

Factor	N	Mean	St.Dev	Se Mean	P-Value	T-Value	Result
VDO	30	1.77	1.76	0.32	0.000	5.12	H_0 is rejected
GA	30	5.40	3.46	0.63			

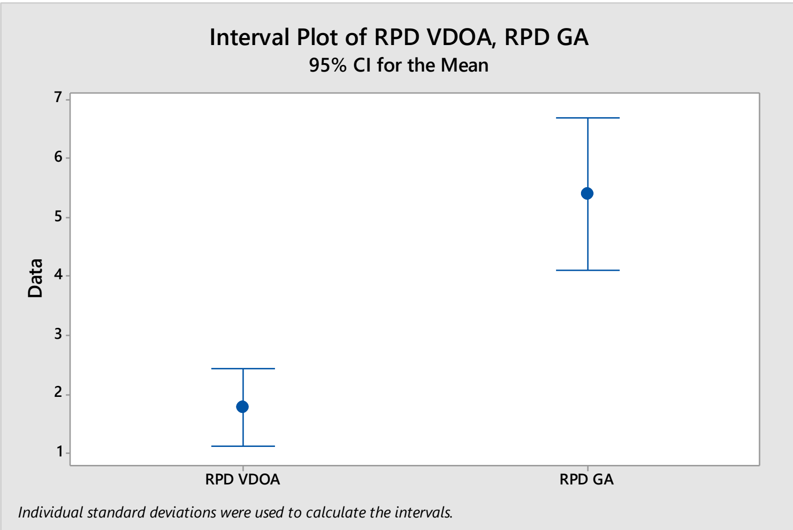

Fig. 11. Mean plot and LSD intervals of RPD for the proposed meta-heuristic algorithms.

6 | Conclusions and Future Research

In this paper, the FJSP with two flows of jobs (direct and reverse) at each stage was investigated. A mathematical programming model was presented and a numerical example was solved by Lingo 16 software to demonstrate the applicability. A small example was solved by using the Lingo 16 software to illustrate the applicability of the developed mathematical. Then the results found by the proposed VDO algorithm were compared with those obtained by Lingo 16 software on 14 problems. A review of the results showed that the VDO algorithm and the Lingo software 16 were capable of attaining the optimal solutions for small test problems. In the last test problems, however, the VDO algorithm worked better, whereas Lingo was unable to solve these problems in reasonable computational times. In addition, the best solutions found by the VDO algorithm for all small-size problems were identical to the optimal solutions obtained by Lingo while the computational time required by Lingo was larger than the one needed in the VDO algorithm to solve each problem. Furthermore, the average solutions found by the VDO algorithm are not far from the best solutions. Due to the NP-hardness of the problem, a metaheuristic algorithm namely the VDO algorithm was applied to solve large-scale problems. To validate the results obtained by the VDO algorithm in terms of solution quality and time, the results were compared with those obtained from Lingo 16 software for small-size problems. Finally, the proposed algorithm was compared with a GA by solving some randomly generated large test problems, based on which the results were analyzed statistically. Computational results showed that the VDO algorithm had better performance than GA. Developing a multi-objective mathematical model for the FJSP with reverse flows, taking into account real conditions such as sequence-dependent setups-times, blocking, etc., and considering the reverse flow concepts in other scheduling environments can be considered as future studies.

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